4.7 Further Probability Distributions

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.7 Further Probability Distributions
Difficulty	Medium

Time allowed: 110

Score: /88

Percentage: /100

Question 1a

A 'lucky dip' bag contains seven bars of chocolate and 5 packets of sweets. Suraya selects two items at random without replacing them.

The probability distribution table for the discrete random variable X, "the number of packets of sweets selected", is shown below.

X	0	1	2
$P(X = x)$	$\frac{21}{66}$	$\frac{7k}{66}$	$\frac{2k}{66}$

a)
Find the value of k.

[3 marks]

Question 1b

b)
Find $E(X)$.

[2 marks]

Question 1c

c)
Find $E(X^2)$.

[2 marks]

Question 1d

d)
Find $\text{Var}(X)$.

[3 marks]

Question 2a

A population of grasshoppers is being studied. It is found that the length of an adult grasshopper, in cm, has PDF

$$f(x) = \begin{cases} kx^2(6-x), & 0 \leq x \leq 6 \\ 0, & \text{otherwise.} \end{cases}$$

a)
Find the value of k.

[4 marks]

Question 2b

b)
Sketch the probability density function.

[2 marks]

Question 2c

c)
Find the probability that a grasshopper picked at random is less than 4 cm in length.

[2 marks]

Question 3a

A game is played with two fair spinners. Each spinner is divided into three sections numbered 1, 2 and 3. A player's score is obtained by spinning both spinners simultaneously and adding together the numbers that they land on.

a)
Complete the table below for the probability distribution of the game.

Score, X					
$P(X = x)$					

[2 marks]

Question 3b

b)
Find the expected score, $E(X)$.

[2 marks]

Question 3c

Jian Wei wants to award prizes such that a player receives $3 for the score that they achieve.

c)
Find the expected prize money for the game.

[2 marks]

Question 4a

A continuous random variable has a probability distribution function

$$f(x) = \begin{cases} \frac{3}{4}(-x^2 + 2x), & 0 \leq x < 2 \\ 0, & \text{otherwise.} \end{cases}$$

a)
Show that the mean of the random variable is equal to 1.

[4 marks]

Question 4b

b)
Find the variance of the random variable.

[6 marks]

Question 4c

c)

Hence, find the standard deviation of the random variable, leaving your answer in the form $\frac{\sqrt{a}}{b}$.

[3 marks]

Question 5a

At a school probability fair, some students create a game using one complete suit from a standard pack of cards. A player must pay $1 to pick a card at random. If their card is a jack, queen or a king they will receive $1 back, if their card is an ace they will receive $5 otherwise if their card is an ordinary number card from 2 to 10, they will receive nothing.

a)
Show that the game is not fair.

[4 marks]

Question 5b

b)
Calculate

(i)
$\mathrm{E}(X^2)$

(ii)
$\mathrm{Var}(X)$

[4 marks]

Question 5c

The students want to make the game fair, so decide to give a prize to anyone who picks an ordinary number card.

c)
Calculate the value of the new prize for choosing an ordinary number card.

[2 marks]

Question 6a

A discrete random variable B has probability distribution given by $\mathrm{B} = ab(b+1)$, where $b = 5, 6, 7$.

a)
Find the value of a.

[3 marks]

Question 6b

b)
Complete the probability distribution table below.

B	5	6	7
$\mathrm{P}(B = b)$			

[2 marks]

Question 6c

c)
Find the mean of B.

[2 marks]

Question 6d

d)
Find the standard deviation of B.

[5 marks]

Question 7a

A continuous random variable has the probability density function given by

$$f(x) = \begin{cases} tx^3 - \frac{x^2}{18} + \frac{7}{36}x, & 0 \le x < 6 \\ 0, & \text{otherwise.} \end{cases}$$

a)
Find the value of t.

[4 marks]

Question 7b

b)
Hence, find the values of

(i)
the mean

(ii)
the mode

(iii)
the median.

[8 marks]

Question 8

A random variable has $\mathrm{E}(X) = 23$ and $\mathrm{Var}(X) = 1.5$.

Find

(i)
$\mathrm{E}(X - 6)$

(ii)
$\mathrm{E}(-2X + 5)$

(iii)
$\mathrm{Var}(X + 7)$

(iv)
$\mathrm{Var}(3X - 3)$

[6 marks]

Question 9a

Consider the function defined by

$$f(x) = \begin{cases} \frac{1}{10}x^2 & 0 \leq x < 2 \\ \frac{82}{135} - \frac{14}{135}x & 2 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

where $f(x)$ is the probability density function of a continuous random variable.

a)
Sketch the graph of $f(x)$

[3 marks]

Question 9b

b)
Find the value of $\mathrm{E}(X)$.

[4 marks]

Question 9c

c)
Find the value of $\text{Var}(X)$.

[4 marks]