3.6 Matrix Transformations

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.6 Matrix Transformations
Difficulty	Medium

Time allowed: 90

Score: /69

Percentage: /100

Question 1a

(a)

Write down the matrix that represents a rotation of $225°$ clockwise about the origin.

[2 marks]

Question 1b

(b)

Write down the matrix that represents a reflection in the y-axis.

[2 marks]

Question 1c

(c)

Find a single matrix that represents the composite transformation consisting of the transformation in part (a) followed by the transformation in part (b).
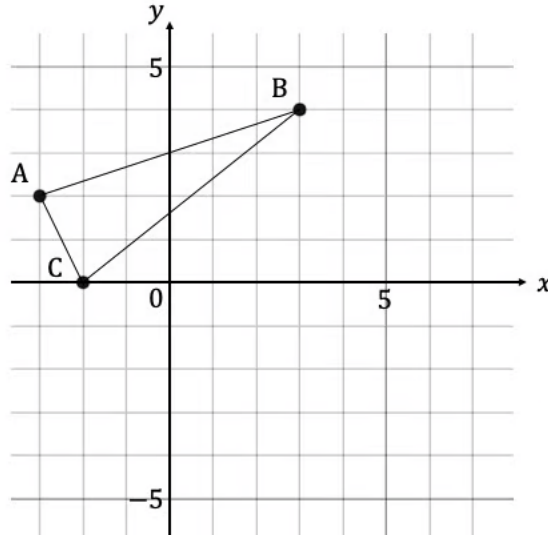
[2 marks]

Question 1d

(d)

Hence find the coordinates of the image of the point $(4, -1)$ after a rotation of $225°$ clockwise about the origin followed by a reflection in the y-axis.

[2 marks]

Question 2a

The diagram below shows a triangle .

(a)
Write down the position matrix of the triangle ABC.

[2 marks]

Question 2b

The triangle ABC is to be mapped to triangle $\mathrm{A'B'C'}$ by a single transformation defined by the transformation matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

(b)
Find the position matrix of the mapped image and draw triangle $\mathrm{A'B'C'}$ on the diagram.

[3 marks]

Question 2c

(c)
Describe fully the transformation that triangle ABC has undergone.

[2 marks]

Question 3a

Points in a plane are subjected to a transformation T that transforms a point $(x,\ y)$ to the point $(x',\ y')$, where T is defined by

$$T: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 0.5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

(a)
Describe in words the transformation T.

[2 marks]

Question 3b

(b)
Find the matrix T^{-1}.

[2 marks]

Question 3c

(c)
Hence find the coordinates of the point $(x,\ y)$ if $(x',\ y') = (12, -4)$.

[2 marks]

Question 4a

A quadrilateral has vertices $(2,\ 1)$, $(4,\ 4)$, $(7,\ 1)$ and $(9,\ 4)$.

(a)
Find the area of the quadrilateral.

[2 marks]

Question 4b

The quadrilateral undergoes a transformation represented by the matrix $\begin{pmatrix} -2 & 1 \\ -4 & 3 \end{pmatrix}$.

(b)
Find the determinant of the transformation matrix.

[2 marks]

Question 4c

(c)
Hence find the area of the image.

[2 marks]

Question 5

An object undergoes a vertical stretch with scale factor 2 followed by a reflection in the x-axis. The position matrix of the image is $\begin{pmatrix} 2 & 5 & 4 \\ 3 & -2 & -1 \end{pmatrix}$.

Use a matrix method to find the coordinates of the object before the transformation.

[5 marks]

Question 6a

The points $A(7, -3)$ and $B(2, 6)$ are transformed to become the points $A'(18, 8)$ and $B'(12, 16)$ respectively.

(a)
Find the 2×2 matrix T that represents the linear transformation.

[5 marks]

Question 6b

(b)

Given that point $C(-4,5)$ is transformed by T^2, find the coordinates of the image point C'.

[3 marks]

Question 7a

An object is reflected in the line $y = \dfrac{\sqrt{3}}{3}x$.

(a)

Write down the matrix that represents the transformation.

[3 marks]

Question 7b

P is a vertex of the object that is being reflected.

(b)

Find the coordinates of P if the coordinates of its image P' are $(2, 2\sqrt{3})$.

[3 marks]

Question 8a

The triangle PQR, with vertices $\mathrm{P}(-1, 1)$, $\mathrm{Q}(5, 3)$ and $\mathrm{R}(-9, -2)$, is translated by the vector $\begin{pmatrix} -7 \\ 2 \end{pmatrix}$ and then enlarged by a scale factor of 3 with the centre of enlargement at the origin.

(a)
Find a single transformation in the form $AX + b$ that maps PQR onto $\mathrm{P'Q'R'}$.

[2 marks]

Question 8b

(b)
Hence determine the coordinates of $\mathrm{P'Q'R'}$.

[3 marks]

Question 9a

a)
Find the 2×2 transformation matrices that represent the following transformations:

(i)
R, a rotation of $\dfrac{\pi}{4}$ radians anti-clockwise

(ii)
S, a reflection in the line $y = x$

(iii)
T, a stretch with scale factor 5 parallel to the y-axis.

[3 marks]

Question 9b

(b)
Find a single transformation matrix that represents the composite transformation

(i)
RT^3

(ii)
R^8STS

[4 marks]

Question 9c

(c)
Find the coordinates of the image of the point $A(3, -1)$ after it has undergone the composite transformation specified in part (b)(i).

[2 marks]

Question 9d

(d)
State the name of the single transformation that is equivalent to the composite transformation specified in part (b)(ii).

[1 mark]

Question 10a

The triangle PQR with position matrix T_0 has vertices $P(5, 2)$, $Q(-3, 1)$ and $R(-5, -4)$.

The triangle is transformed by the transformation matrix $M = \begin{pmatrix} 0 & \dfrac{1}{2} \\ -\dfrac{1}{2} & 0 \end{pmatrix}$.

T_n denotes the position matrix of the image triangle after PQR has been transformed times by matrix M.

(a)
By multiplying two appropriate single transformation matrices together, verify that the matrix M is an enlargement by scale factor $\dfrac{1}{2}$ followed by a $90°$ clockwise rotation.

[3 marks]

Question 10b

(b)

Explain why the area of the triangle with position matrix T_1 will be $\dfrac{1}{4}$ of the area of triangle PQR.

[2 marks]

Question 10c

(c)

Find T_2, and hence the coordinates of the image triangle after triangle PQR is transformed twice by matrix M.

[3 marks]