1.2 Sequences & Series

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.2 Sequences & Series
Difficulty	Very Hard

Time allowed: 90

Score: /71

Percentage: /100

Question 1a

Julie is starting a new web-based subscription business. She sells her subscriptions for $19.50 per month with customers paying at the start of every month. She has 12 customers ready to sign up in the first month. By the fifth month she has 29 customers.

(a) Given that the increase in customers follows an arithmetic sequence, calculate the number of customers Julie will have in the 9th month.

[3 marks]

Question 1b

(b) Calculate the revenue Julie's business will generate by the 17th month. Give your answer correct to the nearest dollar.

[4 marks]

Question 2a

A football team, SME United, have a new stadium with a maximum capacity of 5000 seats, 1500 seats are reserved for the opposition supporters. SME United have 2195 loyal fans who come to all home matches. The manager has predicted that SME United will gain 45 new loyal fans every match who will come to every home match thereafter.

(a) Based on the manager's prediction, work out how many matches will be played before the number of unreserved seats run out.

[4 marks]

Question 2b

A ticket to one of the matches costs $12.

(b) If the the manager's prediction for the increase in loyal fans is correct, and if on average half of the tickets reserved for opposition supporters are sold per game, calculate the revenue that SME United will generate from ticket sales in a 30 match season.

[4 marks]

Question 3a

Ben and Sam are both cyclists competing in a 22.5 km race at the Herne Hill Velodrome in London, England. One lap of the velodrome is 450 m.

Ben takes a total of 42 minutes to complete the race.
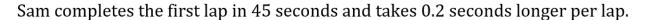
(a) Calculate Ben's mean lap time in seconds.

[2 marks]

Question 3b

(b) Given that each of Ben's laps took him 1% longer to complete than the previous one, calculate how long it took him (in seconds) to complete his first and last laps.

[3 marks]

Question 3c

Sam completes the first lap in 45 seconds and takes 0.2 seconds longer per lap.

(c) Determine who completed the race the first out of Ben and Sam. Justify your answer.

[3 marks]

Question 4a

The first three terms of a geometric sequence are $x + 4$, $6x$ and $2x^2$ respectively, where $x \in \mathbb{R}, \ x \neq 0$.

(a) Find u_5, the fifth term of the sequence. Give your answer as a fraction.

[4 marks]

Question 4b

(b) Find S_7, the sum of the first seven terms of the sequence.

[4 marks]

Question 5a

Lucy is considering two investment strategies.

Strategy A requires an initial deposit of $100. At the start of the second month a deposit of $115 would need to be made, with monthly deposits at the start of each month thereafter that are each $15 more than the deposit in the previous month.

Strategy B requires an initial deposit of $90. At the start of the second month a deposit of $93.60 would need to be made, with monthly deposits at the start of each month thereafter that are each 4% more than the deposit in the previous month.

(a) Write an expression, using sigma notation, to represent the **total amount invested** after n months in

   (i)   Strategy A.
   (ii) Strategy B.

[2 marks]

Question 5b

(b) Find which **monthly deposit** from Strategy A would be the last one that is greater than the corresponding **monthly deposit** from Strategy B.

[3 marks]

Question 5c

(c) Find after which monthly deposit the **total amount invested** in Strategy B would exceed the **total amount invested** in Strategy A.

[3 marks]

Question 6a

Joshua is the owner of a new restaurant which is open in the evening from Monday to Friday. The restaurant has a maximum capacity of 50 guests per evening. During the restaurant's first week they had an average of 24.6 guests per evening and the average spend per guest was $57.55.

(a) Calculate the total amount of revenue the restaurant made in the first week. Give your answer correct to 2 decimal places.

[1 mark]

Question 6b

During the first week Joshua ran a successful marketing campaign and noticed that during the fourth week the restaurant had an average of 33 guests per evening.

(b) Assuming the growth in average guests per evening follows an arithmetic sequence, find the week during which the restaurant will experience capacity issues.

[3 marks]

Question 6c

(c) Calculate the total revenue that the restaurant will generate in its first 10 weeks of being open. Give your answer correct to the nearest dollar.

[4 marks]

Question 7a

The first term of both an arithmetic and a geometric sequence is 1 and both sequences have the same second term. The 20th term of the arithmetic sequence is five times the third term of the geometric sequence.

(a) Find the possible values of the second term.

[4 marks]

Question 7b

(b) Find the possible values of the 10th term for each sequence.

[4 marks]

Question 8a

The first four terms of an arithmetic sequence are $(x + y), (2x + 3), (6y - 1)$ and $(9y - x)$ respectively.

(a) Find the values of x and y.

[4 marks]

Question 8b

Let S_n denote the sum of the first n terms of the sequence.

(b) Find the largest value of n such that $S_n < 800$.

[3 marks]

Question 9a

Guy starts a new job where his base salary is $\$x$ per year and his salary will increase by $\$y$ every year for z years.

(a) Given that at the end of z years Guy will have earned \$367 200 and his salary after z years will be \$37 200, show that

$$734\ 400 = z(x + 37\ 200)$$

[3 marks]

Question 9b

(b) Additionally, given that after $\frac{z}{4}$ years Guy's salary will be \$26 400, show that

$$14\ 400 = zy$$

[2 marks]

Question 9c

(c) Find the values of x, y and z.

[4 marks]