4.5 Probability Distributions

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.5 Probability Distributions
Difficulty	Hard

Time allowed: 90

Score: /71

Percentage: /100

Question 1a

Frank plays a game involving a biased six-sided die.
The faces of the die are numbered 1 to 6.
The score of the game, X, is the number which lands face up after the die is rolled.
The following table shows the probability distribution for X.

Score, x	1	2	3	4	5	6
$P(X = x)$	$\dfrac{1}{6}$	$\dfrac{1}{2}p$	$\dfrac{1}{8}$	$\dfrac{3}{2}p$	$\dfrac{1}{12}$	$3p$

(a) Calculate the exact value of p.

[2 marks]

Question 1b

Frank plays the game once.

(b) Calculate the expected score.

[3 marks]

Question 1c

Frank plays the game twice and adds the scores together.

(c) Find the probability Frank has a total score of 4, giving your answer as a fraction.

[3 marks]

Question 2a

A weekly raffle ticket costs k, with three different levels of prizes, S. The grand prize in the first week is $100 and it increases by $5 every week if nobody wins it.
The following table shows the probability distribution for S.

Prize, s	0	20	Grand prize
$P(S = s)$	$9p$	$7p$	$4p$

(a) Find the value of p.

[1 mark]

Question 2b

(b) Given the grand prize is not won, write down an expression for the grand prize, G, in the form $G = a + bn$, where a and b are constants to be found and n is the week of the raffle.

[2 marks]

Question 2c

(c) Given the raffle is a fair game in the fourth week, find the value of k.

[2 marks]

Question 2d

(d) Find the week in which the expected profit for the ticket buyer is $5.

[3 marks]

Question 3a

A shooting target is divided in to three regions A, B and C. Contestants pay $7.50 to enter and get to take one shot.

The probability of hitting each region is given in the following table:

Region	A	B	C	Missed target
Probability	$\dfrac{1}{15}$	$\dfrac{2}{15}$	$\dfrac{a}{15}$	$\dfrac{b}{15}$

It is given $3a = b$ and $a, b \in \mathbb{Z}$.

(a) Find the value of a and b.

[3 marks]

Question 3b

A contestant's prize is dependent on the target they hit.

Region	A	B	C	Missed target
Prize	$35	$k	$7.50	0

(b) Calculate the value of k such that the game is a fair game.

[4 marks]

Question 4a

A game is played where contestants shoot a football at a goal with a goal keeper. The goal is divided into five regions; A, B, C, D and E. Each region is assigned with the scores, X, outlined in the table below.
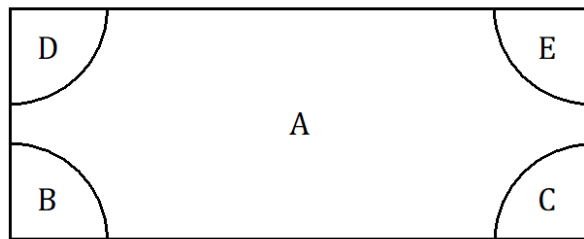
The following table shows the value of X for each region and the probability distribution for X.

Region	A	B	C	D	E	Miss
Score (X)	1	4	4	8	8	-2
$\mathbf{P}(X = x)$	0.3	p	p	q	q	0.4

It is given that $p = 2q$.

(a) Find the exact value of p and q.

[3 marks]

Question 4b

(b) Calculate the expected score.

[2 marks]

Question 4c

(c) Find the probability that a player has a score of 16 after two rounds.

[3 marks]

Question 5a

A spinner has six sections; A, B, C, D, E and F. The table below shows the area of the spinner occupied by each section and their respective pay offs.

Section	A	B	C	D	E	F
Area	$\dfrac{2}{7}$	$\dfrac{5}{21}$	$\dfrac{4}{21}$	$\dfrac{1}{7}$	p	$\dfrac{1}{21}$
Prize	$6	$5	$4	$3	$2	$1

(a) Calculate the value of p.

[1 mark]

Question 5b

The game costs $4 and John says that the expected profit from playing the game is $0.30.

(b) Calculate the percentage error in John's claim.

[4 marks]

Question 6

A biased coin has a probability of showing tails as 0.85. Leon plays a game where he flips the coin. He pays $15 to play. If the coin lands on tails he receives nothing but if it lands on heads he receives 5c$. The game is fair.

Determine the value of c and write down the total prize if he wins.

[5 marks]

Question 7a

Two biased coins are tossed and a fair spinner divided into four equal sectors numbered 1 to 4 is spun.

(a) Write down the total number of possible outcomes when the two coins are tossed and the spinner is spun.

[1 mark]

Question 7b

A random variable, X, is defined as the number of tails when the two coins are tossed multiplied by the number the spinner lands on when it is spun.

For each coin the probability of getting tails is $\dfrac{1}{6}$.

(b) Complete the following probability distribution table for X:

x	0	1	2	3	4	6	8
$P(X = x)$							

[4 marks]

Question 7c

(c) Represent the probability distribution for X as a piecewise function in the form:

$$P(X = x) = f(x) = \begin{cases} & \\ & \\ & \end{cases}$$

[3 marks]

Question 8a

A discrete random variable X has the following probability distribution:

x	-5	-3	-1	0	1	3	5
$P(X = x)$	0.24	$2k^2$	0.04	0.12	$3k$	0.19	0.11

(a) Find the value of k.

[4 marks]

Question 8b

(b) Find E(X).

[3 marks]

Question 9a

A discrete random variable X has the probability distribution shown in the following table:

x	0	1	2	3	4	5	6
$\mathbf{P}(X = x)$	a	$2a$	$3a$	$4a$	$5a$	$6a$	$7a$

(a) Find the value of a.

[2 marks]

Question 9b

(b) Find:

     (i) $P(X \leq 4)$

     (ii) $P(X \geq 2)$

     (iii) $P(1 < X \leq 5)$

     (iv) $P(0 < X < 6)$.

[6 marks]

Question 10a

Tom has constructed a biased spinner with six sectors labelled 1 to 6.
The probability of the spinner landing on each of the six sectors is shown in the following table:

Number on sector	1	2	3	4	5	6
probability	$1.5p$	p	$\dfrac{3}{25}$	$\dfrac{1}{10}$	$\dfrac{3}{50}$	$\dfrac{1}{100}$

(a) Find the value of p.

[2 marks]

Question 10b

Tom is playing a game with his biased spinner. The score for the game, X, is the number which the spinner lands on after being spun.

(b) Tom plays the game twice and adds the two scores together. Find the probability that Tom has a *total* score of 9.

[3 marks]

Question 10c

(c) Find the probability that X is

    (i)   no more than 3

    (ii)   at least 5.

[2 marks]