1.3 Sequences & Series

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.3 Sequences & Series
Difficulty	Medium

Time allowed: 130

Score: /105

Percentage: /100

Question 1a

The second term, u_2, of a geometric sequence is 44 and the third term, u_3, is 55.

(a) Find the common ratio, r, of the sequence.

[2 marks]

Question 1b

(b) Find the first term of the sequence, u_1.

[2 marks]

Question 1c

(c) Find S_5, the sum of the first 5 terms of the sequence.

[2 marks]

Question 2a

The sum of the first 16 terms of an arithmetic sequence is 920.

(a) Find the common difference, d, of the sequence if the first term is 27.5.

[3 marks]

Question 2b

(b) Find the first term of the sequence if the common difference, d, is 11.

[3 marks]

Question 3a

The sum of the first 5 terms of a geometric sequence is 461.12.

(a) Find the common ratio, r, of the sequence if the first term is 200, given that $r > 0$.

[3 marks]

Question 3b

(b) Find the first term of the sequence if the common ratio, r, is -2.
Give your answer correct to 2 decimal places.

[3 marks]

Question 4a

The table below shows information about the terms of four different sequences a_n, b_n, c_n and d_n.

	$n = 1$	$n = 2$	$n = 3$	$n = 4$
a_n		12	30	
b_n		12	30	
c_n	80			10
d_n	80			10

(a) Calculate a_1, a_4 and the common difference, d, given that a_n is an arithmetic sequence.

[2 marks]

Question 4b

(b) Calculate b_1, b_4 and the common ratio, r, given that b_n is a geometric sequence.

[2 marks]

Question 4c

(c) Calculate c_2, c_3 and the common difference, d, given that c_n is an arithmetic sequence.

[2 marks]

Question 4d

(d) Calculate d_2, d_3 and the common ratio, r, given that d_n is a geometric sequence.

[2 marks]

Question 5a

Students are arranged for a graduation photograph in rows which follows an arithmetic sequence. There are 20 students in the fourth row and 44 in the 10th row.

(a) (i) Find the common difference, d, of the arithmetic sequence.

(ii) Find the first term of the arithmetic sequence.

[3 marks]

Question 5b

(b) Given there are 20 rows of students in the photograph, calculate how many students there are altogether

[3 marks]

Question 6a

Marie is an athlete returning to running after an injury and wants to manage the number of kilometres she runs per week. She decides to run 4 km the first week and increase this by 1.5 km each week.

(a) Find the distance Marie ran in the 10th week.

[2 marks]

Question 6b

(b) Find the week in which Marie runs 26.5 km.

[3 marks]

Question 6c

Marie's coach says she can start preparing for her next race once she has run a total of 220 km.

(c) Find the week in which Marie will complete this.

[3 marks]

Question 7a

The eighth term, u_8, of an arithmetic sequence is 18 and the common difference, d, is 2.

(a) (i) Find the first term of the arithmetic sequence.

(ii) Find the value of u_{17}.

[4 marks]

Question 7b

The first and 17th terms of the arithmetic sequence are the third and fifth terms respectively of a geometric sequence.

(b) (i) Find the possible values for the common ratio, r, of the geometric sequence.

(ii) Find the first term of the geometric sequence.

[4 marks]

Question 8a

In a geometric sequence, $u_3 = 160$ and the common ratio, r, is $\frac{1}{4}$.

(a) (i) Find the first term, u_1.

(ii) Find u_6.

[4 marks]

Question 8b

(b) Find the value of the infinite sum of the sequence.

[2 marks]

Question 8c

The first and third terms of the geometric sequence are the seventh and ninth terms respectively of an arithmetic sequence.

(c) (i) Find the common difference, d, of the arithmetic sequence.

(ii) Find the first term of the arithmetic sequence.

[4 marks]

Question 9a

A sequence can be defined by $a_n = 32 - 7n$, for $n \in \mathbb{Z}^+$.

(a) Write an expression for $a_1 + a_2 + a_3 + \cdots + a_{12}$ using sigma notation and find the value of the sum.

[3 marks]

Question 9b

(b) Write an expression for $a_4 + a_5 + a_6 + \cdots + a_{15}$ using sigma notation and find the value of the sum.

[3 marks]

Question 10a

A sequence can be defined by $g_n = 4 \times 3^{n-1}$, for $n \in \mathbb{Z}^+$.

(a) Write an expression for $g_1 + g_2 + g_3 + \cdots + g_{10}$ using sigma notation and find the value of the sum.

[3 marks]

Question 10b

(b) Write an expression for $g_8 + g_9 + g_{10} + \cdots + g_{18}$ using sigma notation and find the value of the sum.

[3 marks]

Question 11a

The kiwi is a flightless bird and is a national treasure in New Zealand. At the start of 2021 there were approximately 68 000 kiwi left, with the population decreasing by 2% every year.

(a) Find the expected population size of kiwis in 2030 assuming the rate of decrease in kiwi population remains the same.

[3 marks]

Question 11b

(b) Find the year in which the population of kiwis falls below 50 000 assuming the rate of decrease in kiwi population remains the same.

[3 marks]

Question 12a

Aaron is working on his cycling in preparation for a triathlon event in 10 months. He cycles a total of 240 km in the first month and plans to increase this by 12.5% each month.

(a) Find the distance Aaron cycles in the fifth month of preparation.

[3 marks]

Question 12b

(b) Calculate the total distance Aaron cycles until the triathlon.

[3 marks]

Question 13a

A geometric sequence has $u_1 = 0.5$ and $r = 3$.

(a) Find

(i) u_4

(ii) S_5.

[2 marks]

Question 13b

An arithmetic sequence has the same u_4 and S_5 as the geometric sequence above.

(b) Find u_1 and d for the arithmetic sequence.

[4 marks]

Question 14a

Daniel and Jonah have each been given $5000 to save for university.

Daniel invests his money in an account that pays a nominal annual interest rate of 2.24%,**compounded quarterly.**

(a) Calculate the amount Daniel will have in his account after 8 years.
Give your answer to 2 decimal places.

[3 marks]

Question 14b

Jonah wants to invest his money in an account such that his investment will double in 10 years. Assume the account pays a nominal annual interest of $r\%$, **compounded half-yearly.**

(b) Determine the value of r.

[3 marks]

Question 15a

On his 40th birthday, Robert invests \$15 000 into a savings account that pays a nominal annual interest rate of 4.78%, **compounded monthly**.

(a) (i) Write an expression for the total value of the investment after n years. Give your answer to 2 decimal places.

(ii) Find the total amount in the savings account after 3 and 5 years.

[3 marks]

Question 15b

(b) Find the age Robert will be when the amount of money in his account will be 1.5 times the initial amount.

[2 marks]

Question 16

The sum of the first two terms of a geometric sequence is 15.3 and the sum of the infinite geometric sequence is 30. Find the positive value of the common ratio, r.

[6 marks]