4.6 Normal Distribution

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.6 Normal Distribution
Difficulty	Hard

Time allowed: 110

Score: /85

Percentage: /100

Question 1a

A scientist is studying the movement of snails and has observed that the distribution of their speeds, S, follows a normal distribution with a mean of 48 m/h and a standard deviation of 1.5 m/h.

(a) Sketch a diagram to represent this information.

[2 marks]

Question 1b

(b) Find the probability that a randomly selected snail has a speed of less than 46.5 m/h.

[2 marks]

Question 1c

(c) From a sample of 80 snails, calculate the expected number of snails that would have a speed of less than 46.5 m/h. Give your answer to the nearest integer.

[2 marks]

Question 2a

The height of the average female giraffe, from a population where the heights follow a normal distribution, is 4.57 m and has a standard deviation of 1.28 m.

(a) Find:

(i) Q_1

(ii) Q_3

(iii) IQR

[4 marks]

Question 2b

The heights, H, of male giraffes are also normally distributed, where $H \sim N(4.96, \sigma^2)$.

(b) If 68% of male giraffes are shorter than the upper quartile of the female giraffe population, find the standard deviation of the male giraffes.

[3 marks]

Question 3a

If $X \sim N(24,16)$

(a) Find x given that:

(i) $P(X < x) = 0.7$

(ii) $P(X > x) = 0.15$

[3 marks]

Question 3b

(b) Find a and b given that $P(a < X < b) = 0.95$ and a and b are equal distances from the mean.

[3 marks]

Question 4a

The weight, W, of pumpkins purchased are normally distributed with a mean of 11.3 kg and a standard deviation of 2.1 kg.

In one year, a farmer grows 350 pumpkins on his farm.

(a) Predict the number of pumpkins that weigh between 7.2 kg and 12.5 kg from the farm.

[3 marks]

Question 4b

The heaviest 7% of pumpkins are classified as large and are sold at a higher price.

(b) Find the range of weights of pumpkins that can be sold for a higher price.

[3 marks]

Question 5a

The distribution of birth weight of a newborn can be assumed to follow a normal distribution with a mean of 3369 g and a standard deviation of 567 g.

A baby is classified as being of low birth weight if its weight is less than 2500 g.

(a) Draw a diagram to represent the situation, labelling clearly the mean and the boundary for low birth weight.

[2 marks]

Question 5b

(b) Find the expected number of babies from a sample of 1000 that are born with a low birth weight. Give your answer to the nearest integer.

[2 marks]

Question 5c

14% of babies born at a particular hospital weigh more than 4 kg.

(c) Show that the babies born at this hospital are not representative of the population.

[2 marks]

Question 5d

(d) Given that that the standard deviation is the same as the population, find the mean birth weight of a newborn at this particular hospital. Give your answer to the nearest gram.

[3 marks]

Question 6a

A reaction time test has been used to collate data about how quickly a person can react to a given signal by pressing the space bar on a computer. It is found that the results of the test, X, are distributed normally with a mean of 273 milliseconds and a variance of 121 milliseconds.

(a) Use a sketch of the distribution to show that $\mathrm{P}(284 < X < 295) = 0.135$.

[3 marks]

Question 6b

A result that is lies outside of two and a half standard deviations from the mean is considered to be extreme. A group of 145 students decide to measure their reaction times using the test.

(b) Estimate the number of students that will receive a result that would be considered extreme. Give your answer to the nearest integer.

[4 marks]

Question 7a

A software firm has recorded the distance, D, that their users have been scrolled on a computer by a mouse over the course of one year. The data is distributed normally with a mean of 18.4 miles and a standard deviation of 5.8 miles.

(a) Calculate the interquartile range of the data.

[4 marks]

Question 7b

It is known that 8% of the users scroll more than d miles but less than 22 miles.

(b) Find the value of d.

[4 marks]

Question 8a

A company states that the lifespan in hours, H, of the bulbs that they manufacture follows a normal distribution with the parameters $\mathrm{H} \sim N(3000, 720^2)$.

The company advertises that their lightbulbs exceed a lifespan of h hours.

(a) Given that $\mathrm{P}(H < h) = 0.4$, find h.

[2 marks]

Question 8b

(b) From a batch of 4000 lightbulbs, calculate the expected number of lightbulbs that will have a lifespan greater than h found in part (a) but less than 3150 hours.

[3 marks]

Question 8c

The 5% of light bulbs with the shortest life span are considered to be defective.

(c) One of the lightbulbs that is tested has a lifespan of 2213 hours. Determine whether the lightbulb is considered to be defective. Give a reason for your answer.

[3 marks]

Question 8d

A second company also manufactures lightbulbs, whose lifespan again follows a normal distribution, $H \sim (\mu, \sigma^2)$.

(d) Given that $\mathrm{P}(H < 2600) = 0.404$ and $\mathrm{P}(H > 3050) = 0.358$, find the values of μ and σ.

[4 marks]

Question 9a

From a given population it is found that the average person spends on average 122 minutes exercising each week. The data follows a normal distribution and has a standard deviation of σ minutes.

It is known that approximately 81.5% of people spend between 50 and 158 minutes exercising each week.

(a) Using a sketch of the distribution or otherwise, explain why 36 minutes would provide a good approximation for the value of σ.

[4 marks]

Question 9b

From a sample of people within the population, it is known that 15 of them spent less than 65 minutes exercising each week.

(b) Using the value above, of 36 minutes for the standard deviation, find the total number of people within the sample.

[3 marks]

Question 10a

The running time of films is normally distributed with a mean time of 102 minutes and a standard deviation of 13 minutes.

(a) Find the probability that, on a randomly selected day, the feature film playing at the cinema has a running time of between 97 and 108 minutes.

[2 marks]

Question 10b

Jonah watches a film on 18 different occasions.

(b) Find the expected number of occasions on which the film he watches will last less than 95 minutes.

[3 marks]

Question 10c

(c) Find the probability that on at least 6 out of the 18 occasions, the film will last for longer than 99 minutes.

[3 marks]

Question 11a

The length L of carrots in centimetres is normally distributed with mean μ. The following table shows the probabilities for values of L.

Values of L	$L < 13.9$	$13.9 \leq L \leq 21.7$	$21.7 < L$
$\mathrm{P}(L)$	k	0.94	0.03

Any carrots that have a length longer than 20.3 cm are classed as oversized carrots.

(a) Write down the value of k.

[2 marks]

Question 11b

(b) Show that the value of μ is 17.8 cm.

[2 marks]

Question 11c

(c) Find the probability that when picking a carrot at random, an oversized carrot is chosen.

[5 marks]