5.5 Kinematics

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.5 Kinematics
Difficulty	Very Hard

Time allowed: 130

Score: /103

Percentage: /100

Question 1a

A toy car starts from a fixed point P and moves along a horizontal race track. The horizontal displacement, s cm, of the toy car from point P can be modelled by the function

$$s(t) = \frac{1}{1000}t(t^2 - 190t + 8400), \qquad 0 \leq t \leq 140$$

where t is the time in seconds since leaving point P.

(a) Sketch a graph of $s(t)$ against t.

[3 marks]

Question 1b

(b) Find an expression for the acceleration of the toy car at time t and hence find the acceleration of the toy car at $t = 85$ seconds.

[4 marks]

Question 1c

(c) Find the greatest speed that the toy car reaches when travelling such that its displacement is decreasing.

[3 marks]

Question 2a

The level of water, h m, in an estuary relative to the mean sea level, is observed over a 24-hour period starting from midnight and is modelled by the function

$$h(t) = \sin(0.262t - 0.5) - 3\cos(0.524t) + 1$$

where t is the time in hours after midnight.

(a) Find the rate of change of the height of the water level at 7 am.

[2 marks]

Question 2b

(b) Find the percentage of time within the 24-hour period that the water level remains above the mean sea level.

[4 marks]

Question 2c

The scientists observing the water level in the estuary anchor a buoy in place such that its horizontal position is fixed, but it is able to move up and down vertically with the changing water level.

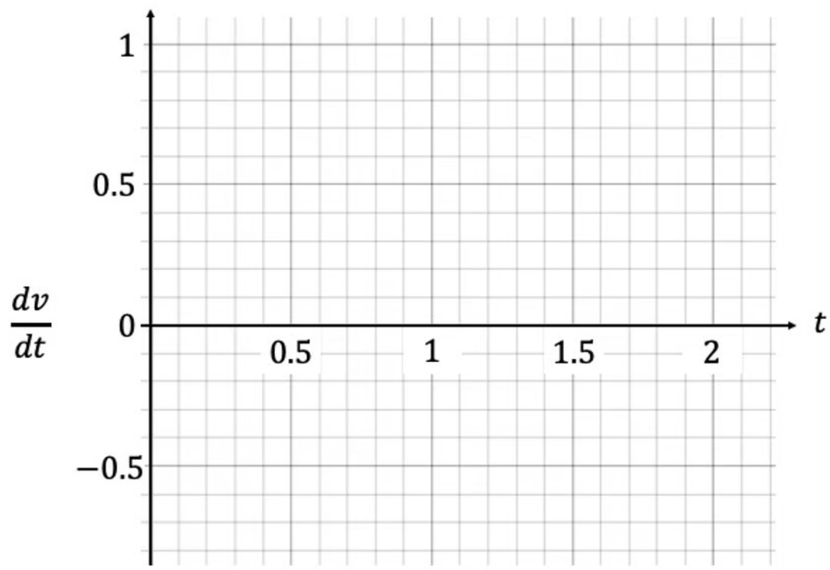
(c) Find the total vertical distance that the buoy moves during the 24-hour time period.

[4 marks]

Question 3a

For a particle P moving in a straight line let $\frac{dv}{dt} = (\sin(t))(\cos(4t))$ for $0 \leq t \leq 2$, where v is the velocity of the particle and t is the elapsed time in seconds.

(a) Sketch the graph of $\frac{dv}{dt}$ on the grid below.



[3 marks]

Question 3b

(b) Find the times at which the points of inflection would occur on a displacement-time graph representing the particle's movement, and explain the significance of these points in the context of this question.

[5 marks]

Question 3c

(c) Hence find the values of t for which the displacement-time graph would be concave down.

[2 marks]

Question 4a

Two particles, P_1 and P_2, are observed moving along a straight line. The displacements of the particles, respectively s_1 and s_2, in metres relative to a fixed point O can be modelled for $0 \leq t \leq 3$ by the following functions

$$s_1(t) = \frac{1}{2}\sin(t - 0.9) - \cos(2t - 1.8) - 1$$

$$s_2(t) = \cos(6t - 5.4) - \sin(t - 0.9) + 2.5$$

where t is the time in seconds from the start of the observation.

(a) Find an expression for the distance between the two particles at time t.

[2 marks]

Question 4b

(b) Hence find

(i) the maximum distance of the particles from one another

(ii) the time at which the maximum distance between the particles occurs.

[3 marks]

Question 4c

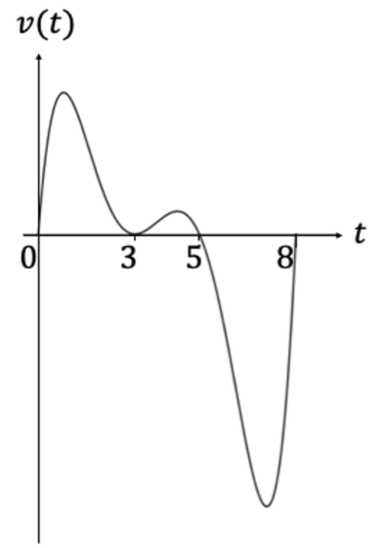
A collision occurs between the particles during the time of observation.

(c) Find the velocity of each of the particles 0.5 seconds before the time that they collide.

[3 marks]

Question 5a

A particle starts from point X and moves in a straight line. The graph below shows its velocity, v ms^{-1} after t seconds for $0 \leq t \leq 8$.



The particle has an instantaneous velocity of 0 ms^{-1} at $t = 0, t = 3, t = 5$ and $t = 8$.

The function $s(t)$ represents the displacement of the particle from point X after t seconds.

It is known that the particle travels 22 metres in the first 3 seconds.

It is also known that $s(3) = s(7)$ and $\int_3^5 v \, \mathrm{d}t = 9$.

(a) Find the value of $s(5) - s(3)$.

[2 marks]

Question 5b

(b) Find the total distance travelled by the particle in the first 7 seconds.

[7 marks]

Question 6a

A particle P moves along a straight line. The velocity of the particle after t seconds, v_{P} ms^{-1}, is given by

$$v_{\mathrm{P}} = t^2 \cos\left(\frac{\pi}{4}t\right), \qquad 0 \leq t \leq 10$$

(a) Write down the first value of t at which P changes its direction of motion.

[1 mark]

Question 6b

(b) Find the total distance travelled by P during the periods when its speed is increasing.

[3 marks]

Question 6c

A second particle, Q, also moves along a straight line. Its velocity after t seconds, v_Q ms^{-1}, is given by

$$v_Q = 6t + 2, \qquad 0 \leq t \leq 8$$

After k seconds, the total distance that Q has travelled is the same as the distance that P travels during its periods of increasing speed.

(c) Find the value of k.

[4 marks]

Question 7a

A particle moves in a straight line starting from point P. The particle is found to have a velocity, v ms^{-1}, given by the piecewise function

$$v(t) = \begin{cases} 9t - 3t^2 & 0 \leq t \leq 4 \\ -3t + \dfrac{16}{t^3} - \dfrac{1}{4} & 4 < t \leq 10 \end{cases}$$

(a) Find the maximum velocity reached by the particle and the time at which that maximum velocity is reached.

[2 marks]

Question 7b

(b) Find an expression for the displacement of the particle from the starting point P at time t, given that the displacement of the particle from point P at the end of the time period is -119.08 m.

[5 marks]

Question 7c

(c) Find the total distance travelled by the particle.

[4 marks]

Question 8a

A particle A moves along a horizontal straight line L_1. The displacement, s_A m, of particle A from a fixed point P on L_1 is given by the function

$$s_{\mathrm{A}}(t) = \frac{1}{2}t - 2t^3\mathrm{e}^{-0.3t} + 24, \qquad 0 \leq t \leq 22$$

where t is the time in seconds from the start of the motion.

Starting at the same time, another particle, B, moves along a horizontal straight line L_2 which is parallel to L_1.

The velocity of particle B, v_B m s^{-1}, at time t seconds is given by

$$v_{\mathrm{B}}(t) = \frac{1}{2}t + 12, \qquad 0 \leq t \leq 22$$

(a) Find the value(s) of t for which particle A is at point P.

[2 marks]

Question 8b

(b) Find the value of t at which particle A first changes direction.

[2 marks]

Question 8c

(c) Find the total distance travelled by particle A in the first 8 seconds of its motion.

[3 marks]

Question 8d

The displacement, s_B m, of particle B is measured relative to a fixed point Q on L_2.

(d) Given that $s_{\mathrm{A}}(0) = s_{\mathrm{B}}(5)$, find:

(i) the displacement function s_{B} for particle B

(ii) the displacement of each particle at the time when the displacement of particle A from point P is the same as the displacement of particle B from point Q.

[7 marks]

Question 9a

A particle is moving along a straight line. The position of the particle at time t seconds, measured in metres relative to a fixed origin point, is denoted by $x(t)$.

The particle starts at the origin at time $t = 0$ with a velocity of $-3\ \mathrm{ms}^{-1}$, and its motion over the next ten seconds is described by the equation

$$\ddot{x}(t) = \frac{3}{32}\mathrm{e}^{\frac{3t}{8}} - \frac{9\pi^2}{25}\cos\left(\frac{3\pi t}{5}\right), \quad 0 \leq t \leq 10$$

(a) Considering the total distance travelled by the particle over the ten seconds, calculate the percentage of that total distance that the particle travels in the first five seconds of its movement.

[7 marks]

Question 9b

(b) Find the greatest distance from the origin point that the particle reaches, and the time t at which that greatest distance is reached.

[5 marks]

Question 10a

Professor Goodwin Vundera, a kinematics researcher, has been studying a particular type of particle. The particle is known only to move along a straight line, with its acceleration, a ms^{-2}, and velocity, v ms^{-1}, both dependent on the particle's displacement, x m, with reference to a fixed origin point.

The professor has defined a new real-valued function, the 'Vundera function', which he believes captures all necessary information about the motion of the particle. This Vundera function, W, is defined by $W(x) = \frac{a(x)}{v(x)}$, where $a(x)$ and $v(x)$ are functions describing, respectively, the particle's acceleration and velocity in terms of x. For one of the particles studied by the professor, the associated Vundera function is found to be

$$W(x) = \frac{2x}{\sqrt{225 - 9x^2}}, \qquad a < x < b$$

where a and b are real constants.

(a) Given that W has the largest possible domain, write down the values of a and b.

[2 marks]

Question 10b

Additionally it is known that when $x = 4$, the velocity of the particle is -2 ms^{-1}.

(b) By using the above information and solving an appropriate indefinite integral, find expressions for the functions $v(x)$ and $a(x)$ associated with the particle.

[6 marks]

Question 10c

(c) Explain the relationship between the particle's speed and acceleration as x varies across all the values in the domain.

[3 marks]