

## 5.7 Basic Limits & Continuity

### **Question Paper**

Course	DP IB Maths
Section	5. Calculus
Торіс	5.7 Basic Limits & Continuity
Difficulty	Very Hard

Time allowed:	70
Score:	/51
Percentage:	/100

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#### **Question la**

For each of the following, either show that the limit converges and find its value, or else explain why the limit diverges:

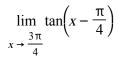
(a)

$$\lim_{x \to 0} \tan\left(x - \frac{\pi}{4}\right)$$

[2 marks]

#### **Question 1b**

(b)



[2 marks]

#### Question lc

(c)

$$\lim_{x \to \frac{3\pi}{4}} \frac{\tan\left(x - \frac{\pi}{4}\right)}{\sec\left(x - \frac{\pi}{4}\right)}$$

[3 marks]

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#### Question 2a

(a) Evaluate the limit

 $\lim_{x \to +\infty} \cos\left(\frac{3}{x^2}\right)$ 

justifying your answer by clear mathematical reasoning.

[3 marks]

#### **Question 2b**

(b) (i) Show that the limit

$$\lim_{x \to -\infty} \left( \frac{2x^3 + 6x^2 + 1}{2x^2} + \tan\left(\frac{\pi x^3 - 2x^2 + 3}{7 - 2x - 4x^3}\right) \right)$$

diverges. Be sure to show clear algebraic working.

(ii)

Determine the asymptotic behaviour of the curve with equation

$$y = \frac{2x^3 + 6x^2 + 1}{2x^2} + \tan\left(\frac{\pi x^3 - 2x^2 + 3}{7 - 2x - 4x^3}\right)$$

as  $x \to \pm \infty$ .

[5 marks]

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#### **Question 3a**

A student has attempted to evaluate the limit

$$\lim_{x \to -\infty} \left( \frac{x^2 + x + 14}{x^2 + x - 2} \right)$$

as follows:

$$\lim_{x \to -\infty} \left( \frac{x^2 + x + 14}{x^2 + x - 2} \right) = \frac{(-\infty)^2 + (-\infty) + 14}{(-\infty)^2 + (-\infty) - 2} = \frac{(+\infty) + (-\infty) + 14}{(+\infty) + (-\infty) - 2} = \frac{0 + 14}{0 - 2} = -7$$

(a) Explain what is wrong with the student's work.

[2 marks]

#### **Question 3b**

(b)

Determine the correct evaluation of the limit, justifying your answer by clear mathematical reasoning.

[2 marks]

#### **Question 3c**

(c) Use technology to help you sketch the graph of  $y = \frac{x^2 - x + 14}{x^2 - x - 2}$ , and show that the graph confirms your answer to part (b).

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[2 marks]

#### Question 4a

Consider the function f defined by

$$f(x) = \frac{1}{(\arctan x)^2}$$

(a) Evaluate the limits (i)  $\lim_{x \to 0^{-}} f(x)$ (ii)  $\lim_{x \to 0^{+}} f(x)$ 

[3 marks]

#### **Question 4b**

```
(b)

Evaluate the limits

(i)

\lim_{x \to -\infty} f(x)
(ii)

\lim_{x \to +\infty} f(x)
```

[4 marks]



#### **Question 4c**

(c)

Use your results from parts (a) and (b) to write down the equations of any asymptotes on the graph of y = f(x).

[2 marks]

#### Question 4d

(d)

Use technology to help you sketch the graph of y = f(x), and show that this confirms your results from parts (a), (b) and (c).

[2 marks]

#### **Question 5a**

Consider the function g defined by

$$g(x) = \frac{2x-3}{x} - \frac{1}{x^2+1}$$

(a) Evaluate the limits (i)  $\lim_{x \to -1^{-}} g(x)$ (ii)  $\lim_{x \to -1^{+}} g(x)$ 

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[3 marks]

#### **Question 5b**

(b) Evaluate the limits (i)  $\lim_{x \to -\infty} g(x)$ (ii)  $\lim_{x \to +\infty} g(x)$ 

[3 marks]

#### Question 5c

Write down the equations of any asymptotes on the graph of y = g(x).

[3 marks]

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#### **Question 5d**

(d)

Use technology to help you sketch the graph of y = g(x), and show that this confirms your results from parts (a), (b) and (c).

[2 marks]

#### Question 6a

(a)

The function f is a piecewise function defined by

$$f(x) = \begin{cases} 10 - 3x, & x < 2\\ \frac{x^2 - 4}{x - 2}, & x = 2\\ |x^2 - 2x - 4|, & x > 2 \end{cases}$$

Explain why f is not continuous at x = 2.

[3 marks]

#### **Question 6b**

(b)

Give an example of a function g that is continuous for all  $x \in \mathbb{R}$ , but which is not differentiable at x = 3. Include a sketch of the graph of the function, identifying all points where the function is not differentiable.

[3 marks]



#### Question 6c

(c) Write down a continuous function h for which  $\lim_{x \to -\infty} h(x)$  and  $\lim_{x \to +\infty} h(x)$  both exist and are finite, but for which

 $\lim_{x \to -\infty} h(x) \neq \lim_{x \to +\infty} h(x).$ 

[2 marks]