5.7 Basic Limits & Continuity

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.7 Basic Limits & Continuity
Difficulty	Very Hard

Time allowed: 70

Score: /51

Percentage: /100

Question 1a

For each of the following, either show that the limit converges and find its value, or else explain why the limit diverges:

(a)

$$\lim_{x \to 0} \tan\left(x - \frac{\pi}{4}\right)$$

[2 marks]

Question 1b

(b)

$$\lim_{x \to \frac{3\pi}{4}} \tan\left(x - \frac{\pi}{4}\right)$$

[2 marks]

Question 1c

(c)

$$\lim_{x \to \frac{3\pi}{4}} \frac{\tan\left(x - \frac{\pi}{4}\right)}{\sec\left(x - \frac{\pi}{4}\right)}$$

[3 marks]

Question 2a

(a)

Evaluate the limit

$$\lim_{x \to +\infty} \cos\left(\frac{3}{x^2}\right)$$

justifying your answer by clear mathematical reasoning.

[3 marks]

Question 2b

(b)

(i)

Show that the limit

$$\lim_{x \to -\infty}\left(\frac{2x^3+6x^2+1}{2x^2}+\tan\left(\frac{\pi x^3-2x^2+3}{7-2x-4x^3}\right)\right)$$

diverges. Be sure to show clear algebraic working.

(ii)

Determine the asymptotic behaviour of the curve with equation

$$y=\frac{2x^3+6x^2+1}{2x^2}+\tan\left(\frac{\pi x^3-2x^2+3}{7-2x-4x^3}\right)$$

as $x \to \pm\infty$.

[5 marks]

Question 3a

A student has attempted to evaluate the limit

$$\lim_{x \to -\infty}\left(\frac{x^2 + x + 14}{x^2 + x - 2}\right)$$

as follows:

$$\lim_{x \to -\infty}\left(\frac{x^2 + x + 14}{x^2 + x - 2}\right) = \frac{(-\infty)^2 + (-\infty) + 14}{(-\infty)^2 + (-\infty) - 2} = \frac{(+\infty) + (-\infty) + 14}{(+\infty) + (-\infty) - 2} = \frac{0 + 14}{0 - 2} = -7$$

(a)
Explain what is wrong with the student's work.

[2 marks]

Question 3b

(b)
Determine the correct evaluation of the limit, justifying your answer by clear mathematical reasoning.

[2 marks]

Question 3c

(c)
Use technology to help you sketch the graph of $y = \frac{x^2 - x + 14}{x^2 - x - 2}$, and show that the graph confirms your answer to part (b).

[2 marks]

Question 4a

Consider the function f defined by

$$f(x) = \frac{1}{(\arctan x)^2}$$

(a)

Evaluate the limits

(i)

$$\lim_{x \to 0^-} f(x)$$

(ii)

$$\lim_{x \to 0^+} f(x)$$

[3 marks]

Question 4b

(b)

Evaluate the limits

(i)

$$\lim_{x \to -\infty} f(x)$$

(ii)

$$\lim_{x \to +\infty} f(x)$$

[4 marks]

Question 4c

(c)
Use your results from parts (a) and (b) to write down the equations of any asymptotes on the graph of $y = f(x)$.

[2 marks]

Question 4d

(d)
Use technology to help you sketch the graph of $y = f(x)$, and show that this confirms your results from parts (a), (b) and (c).

[2 marks]

Question 5a

Consider the function g defined by

$$g(x) = \frac{2x-3}{x} - \frac{1}{x^2+1}$$

(a)
Evaluate the limits
(i)
$\lim_{x \to -1^-} g(x)$
(ii)
$\lim_{x \to -1^+} g(x)$

[3 marks]

Question 5b

(b)
Evaluate the limits
(i)
$$\lim_{x \to -\infty} g(x)$$
(ii)
$$\lim_{x \to +\infty} g(x)$$

[3 marks]

Question 5c

Write down the equations of any asymptotes on the graph of $y = g(x)$.

[3 marks]

Question 5d

(d)
Use technology to help you sketch the graph of $y = g(x)$, and show that this confirms your results from parts (a), (b) and (c).

[2 marks]

Question 6a

(a)
The function f is a piecewise function defined by

$$f(x) = \begin{cases} 10 - 3x, & x < 2 \\ \dfrac{x^2 - 4}{x - 2}, & x = 2 \\ |x^2 - 2x - 4|, & x > 2 \end{cases}$$

Explain why f is not continuous at $x = 2$.

[3 marks]

Question 6b

(b)
Give an example of a function g that is continuous for all $x \in \mathbb{R}$, but which is not differentiable at $x = 3$. Include a sketch of the graph of the function, identifying all points where the function is not differentiable.

[3 marks]

Question 6c

(c)
Write down a continuous function h for which $\lim_{x \to -\infty} h(x)$ and $\lim_{x \to +\infty} h(x)$ both exist and are finite, but for which $\lim_{x \to -\infty} h(x) \neq \lim_{x \to +\infty} h(x)$.

[2 marks]