4.3 Further Correlation & Regression

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.3 Further Correlation & Regression
Difficulty	Medium

Time allowed: 140

Score: /109

Percentage: /100

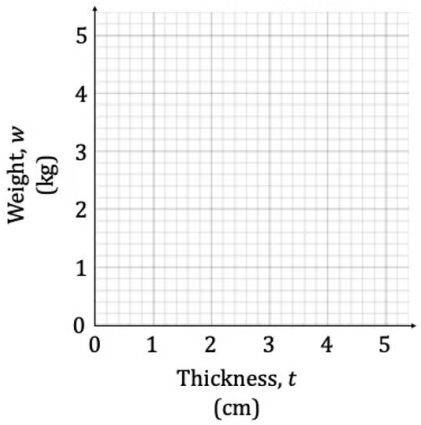
Question 1a

Taro discovers a group of rocks and decides to measure their thickness, t, and weight, w. The results are shown below.

Thickness, t (cm)	2.2	4.5	3.2	2.8	3.7
Weight, w (kg)	2.6	3.6	2.7	3.0	2.9

Taro draws a line of best fit through the data points using the equation $w = \frac{2}{5}t + \frac{8}{5}$.

(a)
Draw a scatter diagram of the data and sketch the given line of best fit on the same axes.



[3 marks]

Question 1b

(b)
Calculate the residual of each point.

[3 marks]

Question 1c

(c)
Hence find the sum of the squared residuals, SS_{res}.

[1 mark]

Question 1d

(d)
Comment on the fit of the model to the data.

[2 marks]

Question 2a

Syafiqah wants to model the path of the water from a water fountain. She measures the horizontal and vertical distances of specific points on the path that the water takes with respect to the base of the water spout. These measurements are recorded in the table below.

Horizontal Distance, x (m)	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
Vertical Distance, y (m)	1.32	1.47	1.50	1.56	1.39	1.33	0.98	0.63	0.00

(a)
State an appropriate type of function to model the path way of the water.

[1 mark]

Question 2b

(b)
Use your graphic display calculator to find the best fit function for the data points.

[2 marks]

Question 2c

(c)
Find the coefficient of determination and comment on the closeness of fit to the original data.

[2 marks]

Question 2d

(d)
State the maximum height that the water reaches according to the model and the horizontal distance of this point from the base of the water spout.

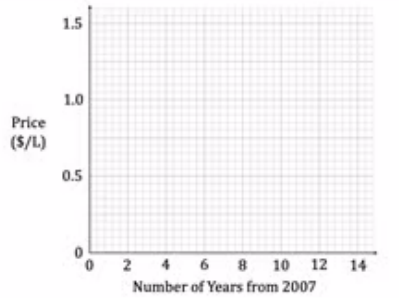
[3 marks]

Question 3a

The average price for cooking oil in USD per litre was recorded each year from 2007 until 2021 and the results put in the table below. Number of years from 2007.

Year	2007	2009	2011	2013	2015	2017	2019	2021
Price (USD/L)	0.78	1.32	1.31	1.22	0.99	0.85	0.84	1.24

(a)
Draw a scatter graph of the data points on the axes provided below, with the price in USD per litre against the number of years from 2007.



[3 marks]

Question 3b

(b)
State the type of function that would be appropriate to model the data.

[1 mark]

Question 3c

(c)
Using your graphic display calculator find the function of best fit for the data.

[2 marks]

Question 3d

(d)
Find the coefficient of determination and interpret the result in context.

[2 marks]

Question 3e

(e)
Sketch the model function on top of the scatter graph in part (a) and comment on the closeness of fit to the original data.

[3 marks]

Question 4a

A specialist in infectious diseases is investigating the spread of a new disease. She is looking at the number of infections in a community based on the time, in days, since the first recorded case in the area.

She has focused on three sites and recorded the following information.

Site	1	2	3
Time since first recorded case, t	35	180	166
Number of infections, N	9	411	247

The scientist believes that the number of infections in a population can be modelled by an exponential function of the form

$N = A(1.2)^{bt}$, where A and b are constants.

(a)
Given that $A = 0.6$ and $b = 0.2$, find the predicted number of infections and hence the residuals for each site.

[3 marks]

Question 4b

A second model proposes that $A = 2.5$ and $b = 0.15$.

(b)
Find the residuals for each site for the second model.

[2 marks]

Question 4c

The scientist will use the model that has the lowest value for the sum of the squares of the residuals.

(c)
Determine which model the scientist should use.

[3 marks]

Question 4d

(d)
State one concern about the reliability of the model.

[1 mark]

Question 5a

A unicorn leaves a magical sparkling trail behind him when he jumps off a rock that is 3 metres above ground level and flies away into the sky. The horizontal and vertical distances in metres from the base of the rock that the unicorn started from have been measured for several points in the sparkling trail and recorded in the table below.

x	0	2	4	6	7	10
y	3	3	4.2	9.5	15	53.3

(a)
Find an appropriate quadratic model for the data.

[2 marks]

Question 5b

(b)
Find the coefficient of determination for the quadratic model and comment on the closeness of fit to the data.

[3 marks]

Question 5c

(c)
State an alternative type of model that may be more suitable for the data.

[1 mark]

Question 5d

(d)
Find an equation for the alternative model.

[2 marks]

Question 5e

(e)
Comment on why this model is a better fit for the sparkly trail than the initial quadratic.

[2 marks]

Question 6a

Grainne records the displacement, in centimetres, of a particle from a fixed point O over the course of 9 seconds. She wishes to find an equation that will model the movement of the particle.

The coordinates of some of the data points of the particle are shown in the table below.

Time, t	0.9	1.2	2.7	3.9	4.2	5.1	5.7	7.7	8.3
Displacement, s	-5.9	-7.0	-8.2	-4.5	-3.0	-3.2	-5.8	-7.9	-6.0

(a)
State why a quadratic would not be a good fit for this model. Give a reason for your answer.

[2 marks]

Question 6b

(b)
State a type of function that would be appropriate to model the particle's movement.

[1 mark]

Question 6c

(c)
Find an equation of the least squares regression for the type of function stated in part (b).

[2 marks]

Question 6d

(d)
Given that the displacement of the particle from point O is -4.7 cm, find the time(s) at which this occurs, for $0 \leq t \leq 9$.

[3 marks]

Question 7a

A professional skydiver is attempting to perform the highest skydive without the use of specialist breathing equipment. A sensor has been released from a height of 12,000 m to record the atmospheric pressure at different heights above sea level. The data from the sensor is recorded in the table below.

Height above sea level, h (m)	0	2885	5476	7732	9640	12000
Atmospheric pressure, p (Pa)	1013	766	483	350	301	215

An exponential model of the form $p = a(b)^h$ is proposed to model the data, where a, b are constants to be found.

(a)
Find the values of a and b.

[3 marks]

Question 7b

(b)
Comment on any limits there might be for the domain of this model.

[2 marks]

Question 7c

It is agreed that an oxygen supply should be used when the atmospheric pressure is less than 365 Pa.

(c)
State whether a jump from a height of 8000 m could be attempted safely without an oxygen supply.

[3 marks]

Question 7d

(d)
Explain what action could be taken to increase the reliability of the model.

[1 mark]

Question 8a

A digital artist has taken a photograph of a building and wishes to render it digitally. The photograph is imported to graphing software and the following points that follow the line of the roof are plotted.

x	0	0.7	1.8	2.3	3.3	4.0	4.5
y	1.2	1.5	1.7	2.9	6.5	8.0	13.8

One possible model for the roof is a cubic.

Another possible model for the roof is an exponential function, $y = ab^x$, where $a, b \in \mathbb{R}$.

(a)
Find the equation of the least squares regression curve for

(i)
the cubic model

(ii)
the exponential model.

[4 marks]

Question 8b

(b)
Find the coefficient of determination for

(i)
the cubic model

(ii)
the exponential model.

[3 marks]

Question 8c

(c)
Hence state which function is more appropriate to model the roof line of the building. Give a reason for your answer.

[2 marks]

Question 8d

(d)
Using the model that was chosen in part (b), find the height of the roof when $x = 9.3$.

[2 marks]

Question 9a

Mo is looking at the growth rate of a particular type of bacteria. She records the number of cells of the bacteria and the time in minutes that has elapsed since the start of her observation. The results are recorded in the table below.

Time (minutes)	0	2	4	6	8	10	12	14
Number of cells	1	3	16	112	2667	6310	210863	914113

(a)
Express the number of cells as a logarithm. Give each value to 3 dp.

[3 marks]

Question 9b

Mo is not sure if a linear model or a quadratic model will fit the data best.

(b)
Find the coefficient of determination for

(i)
a linear model

(ii)
a quadratic model

[3 marks]

Question 9c

(c)
Comment on which model from part (c) best fits the graph.

[1 mark]

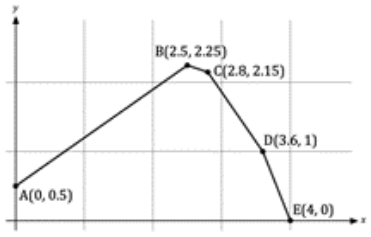
Question 9d

(d)
Find the equation of the least squares regression curve for the model specified in part (c).

[2 marks]

Question 10a

Amelie wants to create a scale model of the cross section of a hill. She has measured the horizontal and vertical distances of several points on the hill from a fixed point O. These points can be seen in the diagram below.



Amelie decides to model the first section as a straight line and the second section as a quadratic with equation $y = -0.9x^2 + 4.5x - 3.3$.

Find the equation of the straight line passing through points A and B.

[2 marks]

Question 10b

(b)
Find the residuals for each point B, C, D and E for the quadratic model.

[4 marks]

Question 10c

(c)
Find the equation of the least squares quadratic curve for points B, C, D and E.

[3 marks]

Question 10d

(d)
By examining the sum of the squares of the residuals, show that the model found in part (c) is a better fit for the data.

[4 marks]

Question 11a

The number of active profiles, N, a new social media platform has t months after launching are shown in the table below.

t	3	5	8	12	15	24
N	542	1122	2345	4254	8621	22 324
log N						

(a) Complete the third row of the table above.

[2 marks]

Question 11b

b) Draw a scatter diagram of log N versus t.

[3 marks]

Question 11c

The regression line of log N on t can be written in the form $\log N = a + bt$.

(c) Find the values of a and b.

[3 marks]

Question 11d

d)
Calculate the Pearson's product-moment correlation coefficient, r, between log N and t and comment on the validity of the regression line found in part (c).

[3 marks]

Question 11e

e) Based on part (d) suggest a suitable type of regression model for N on t.

[1 mark]