11.3 Capacitance

Question Paper

Course	DP IB Physics
Section	11. Electromagnetic Induction (HL only)
Topic	11.3 Capacitance
Difficulty	Hard

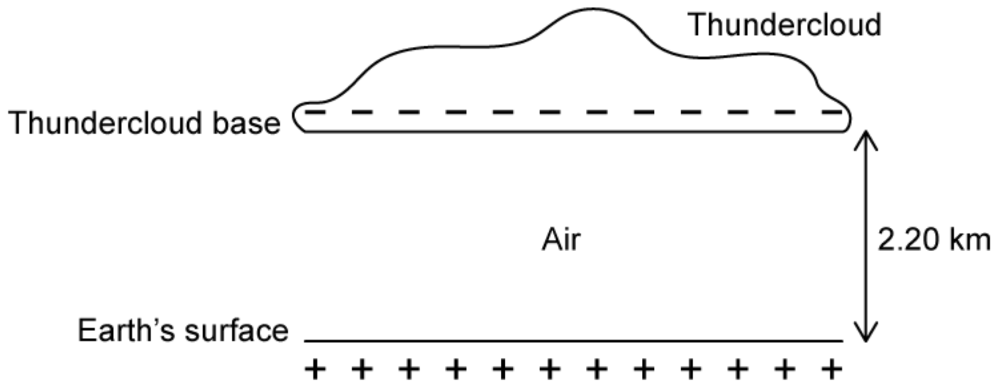
Time allowed: 60

Score: /46

Percentage: /100

Question 1a

A thundercloud is 2.20 km above the surface of Earth. The charge on the base of the cloud is –30.0 C. The air between the cloud and the Earth is humid and rainy, making it 4.35 % water by mass.



The relative permittivity for a homogenous mixed medium, $\varepsilon_{r(m)}$, is given by:

$$\varepsilon_{r(m)} = \phi_1 \varepsilon_{r(1)} + \phi_2 \varepsilon_{r(2)}$$

Where $\varepsilon_{r(1)}$ & $\varepsilon_{r(2)}$ represent the relative permittivities of each material in the mixture and Φ_1 & Φ_2 represent the fraction by mass of each material.

The permittivity of water is 7.08×10^{-10} F m^{-1} and the permittivity of air is 8.85×10^{-12} F m^{-1}.

(a)
Show that the dielectric constant of the rainy air is about 4.

[4]

[4 marks]

Question 1b

The cloud has a roughly rectangular base of length 5.00 km and the potential difference from the base of the cloud to Earth is -8.00×10^{8} V.

(b)
Calculate the width of the cloud.

[3]

[3 marks]

Question 1c

Lightning strikes a tree after strong winds increase the potential difference of the system to -9.0×10^{9} V. Air conducts electricity once there is a potential difference of 3.00×10^{6} V per metre.

(c)
Given that, in a storm, the rainy air has a resistance of 136 $\Omega\ m^{-1}$, determine the time period of the lightning strike.

Assume the cloud's area and distance from the ground are unchanged by the wind.

[4]

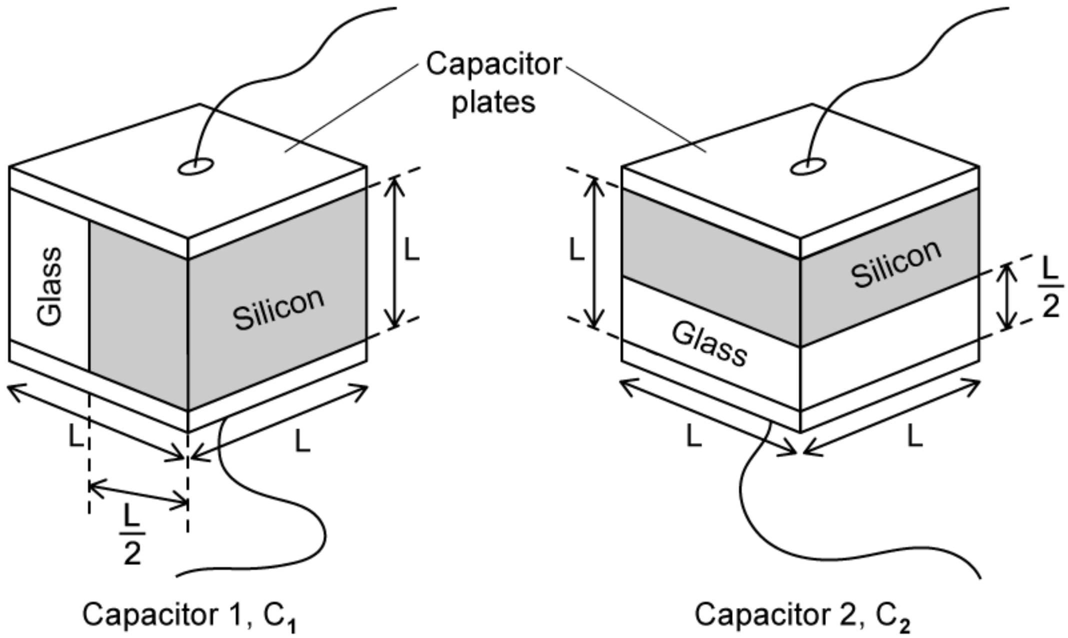
[4 marks]

Question 2a

A tattoo removal company uses a pulsed Nd:YAG laser used a capacitor to store energy and produce a short laser pulse. When pulses in the nanosecond range are discharged, tattoo pigments are removed without damaging skin cells.

The research and design team of a company that produces these lasers are experimenting with different dielectrics in the capacitor. One suggestion is to use a cubic dielectric material that is half glass, with permittivity ε_G, and half silicon, with permittivity ε_{Si}, split down the middle.

This mixed medium is used in two orientations during tests.



The capacitor with the dielectric split vertically has a capacitance of C_1 and the capacitor with the dielectric split down the middle has a capacitance of C_2.

(a)
Write an expression for capacitance, C_1, in terms of L, ε_G and ε_{Si}.

[3]

[3 marks]

Question 2b

(b)
Write an expression for capacitance, C_2, in terms of L, ε_G and ε_{Si}.

[4]

[4 marks]

Question 2c

Capacitor 1 and capacitor two are connected in two different circuits. The charge on capacitor 1 is twice that of capacitor 2.

The dielectric constant of the silicon is 4.3 and the relative permittivity of the glass is 6.5

(c)
Show that the energy stored in capacitor 2 is 26 % of the energy stored in capacitor 1.

(i)
Show that $\frac{E_2}{E_1} = \frac{(\varepsilon_{Si} + \varepsilon_G)^2}{16\varepsilon_G\varepsilon_{Si}}$, where E_n is the energy stored in capacitor n and ε_z is the permittivity of material z.

[3]

(ii)
Calculate E_2 as a percentage of E_1.

[2]

[6 marks]

Question 3a

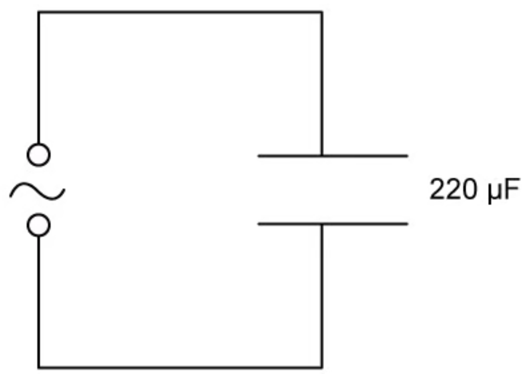
A capacitor with capacitance 220 μF is attached to an ac voltage source which provides a voltage which varies according to the following equation:

$$V = V_0 \sin(\omega t)$$

The rate of change of the voltage is given by:

$$\frac{\Delta V}{\Delta t} = \omega V_0 \cos(\omega t)$$

This circuit is called 'purely capacitive', meaning that resistance can be assumed to be zero.



The power supply is adjusted such that the initial voltage is 6.0 V and the alternating voltage frequency is 4000 rad s^{-1}.

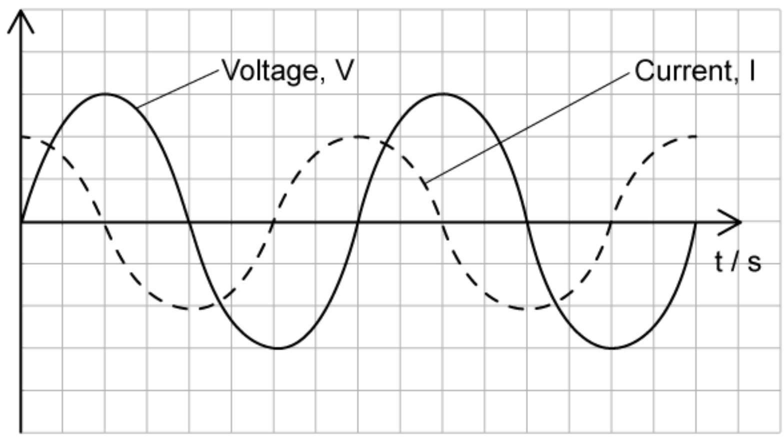
(a)
Calculate the magnitude of the current flowing in the circuit at time *t* = 3.14 s.

[4]

[4 marks]

Question 3b

The variation of voltage *V* and current *I* in the circuit is shown.



(b)
Discuss the phase difference between the variation of *V* and *I* in the circuit.

[3]

[3 marks]

Question 3c

'Capacitive reactance' *X* is the opposition to current flow in a purely capacitive circuit as described in part (a). Capacitive reactance is comparative to resistance, in that it is measured by the same units Ω.

(c)
By considering the ratio of the maximum voltage and current in the circuit, show that the capacitive reactance *X* is given by

$$X = \frac{1}{\omega C}$$

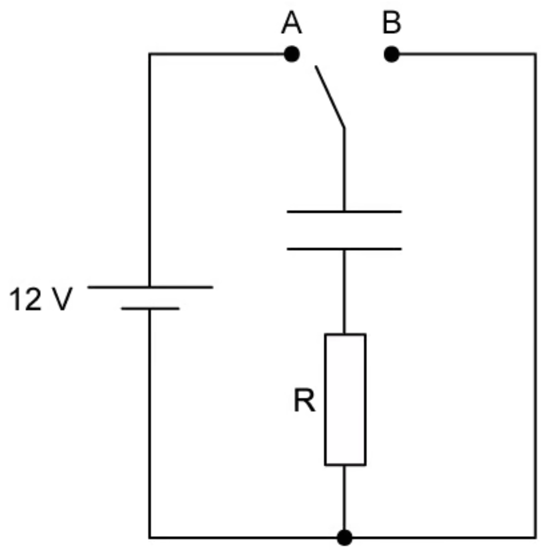
and verify that *X* has the same units as resistance.

[4]

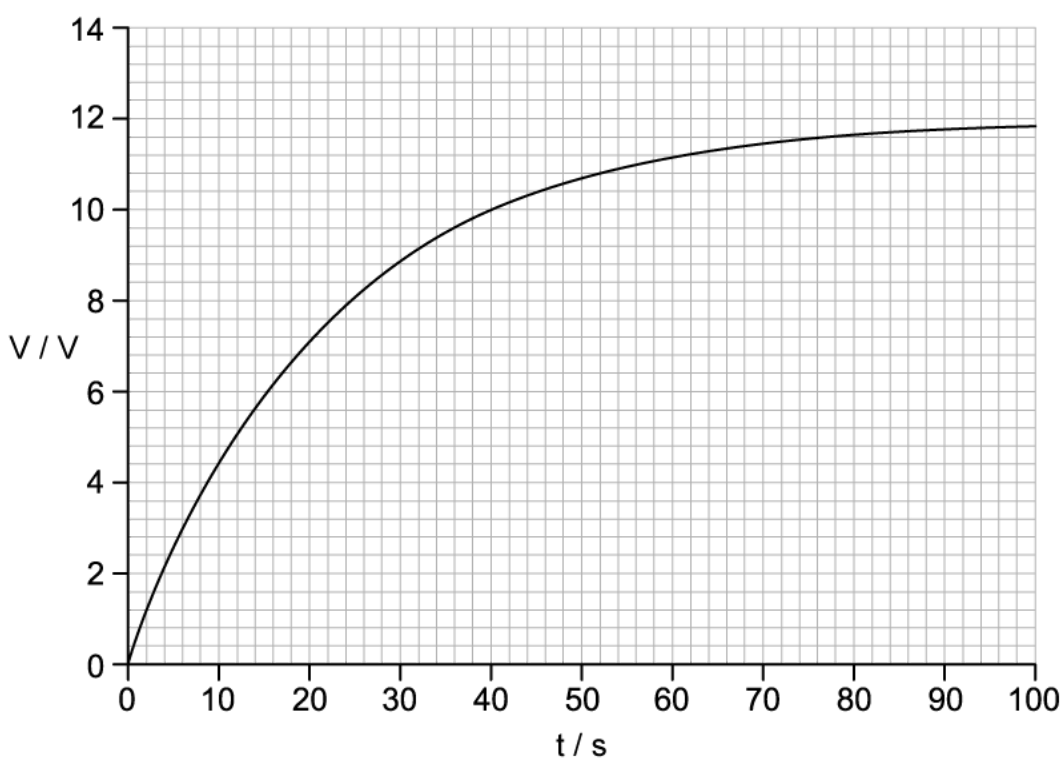
[4 marks]

Question 4a

A vacuum capacitor is connected, along with a resistor, to a cell with an emf of 12 V in the configuration below. In a vacuum, the capacitor has a capacitance of 4.5 μF.



Initially, the vacuum capacitor is uncharged. At a time of t = 0 s, the switch is placed at position A. The voltage across the capacitor is recorded over time and plotted in the graph below.



(a)
Sketch a second line on the axes, showing the variation in the voltage across the resistor over time.

[2]

[2 marks]

Question 4b

(b)
Using the graph, calculate the resistance, *R*, of the resistor.

[3]

[3 marks]

Question 4c

The vacuum chamber is now filled with acetone, which has a dielectric constant of 19.5.

(c)
Calculate the new charge stored in the capacitor when the voltage across the capacitor is half its maximum value.

[3]

[3 marks]

Question 4d

(d)
Once the capacitor is fully charged, the switch in the diagram in part (a) changes to position B.

(i)
Describe the energy changes in the capacitor.

[1]

(ii)
Explain why these energy changes occur.

[2]

[3 marks]