3.9 Vector Properties

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.9 Vector Properties
Difficulty	Hard

Time allowed: 90

Score: /71

Percentage: /100

Question 1a

(a)

Given $|a| = 5$, $b = \begin{pmatrix} 12 \\ -5 \end{pmatrix}$ and $a \cdot b = 16$, find the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

[3 marks]

Question 1b

Consider a third vector $\mathbf{c}$, where $|c| = 8$.

(b)

When the angle between a and c is $\frac{\pi}{4}$, show that $|a \cdot (b + c)| = 16 + 20\sqrt{2}$.

[3 marks]

Question 2a

The points A and B have position vectors a and b respectively.

$$|\boldsymbol{a}| = 9,\ \boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix},\ \boldsymbol{a} \cdot \boldsymbol{b} = 16$$

(a)

Find the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

[4 marks]

Question 2b

The points A and B form a triangle with the origin O.

(b)
Find $|\boldsymbol{b}|$.

[2 marks]

Question 3

ABCD is a parallelogram with vertices $\mathrm{A}(-2,5,3)$, B, C and $\mathrm{D}(3-1,t)$ where $t>0$.

$$\overrightarrow{\mathrm{AB}}=\overrightarrow{\mathrm{DC}}=\begin{pmatrix}2\\4\\1\end{pmatrix}$$

Given that the area of the parallelogram is $\sqrt{1221}$ units, find the value of t.

[6 marks]

Question 4

Two points A and B have position vectors $a = \begin{pmatrix} 6 \\ 2 \\ -5 \end{pmatrix}$ and $b = \lambda \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ respectively.

A third point C is located such that $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix}$.

Given that the angle between the vectors $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$ is obtuse, find the range of possible values for λ.

[4 marks]

Question 5a

Consider the parallel sides of a trapezium with vectors $u = \begin{pmatrix} t-4 \\ 3t \\ 2 \end{pmatrix}$ and $v = \begin{pmatrix} 6 \\ t \\ -4 \end{pmatrix}$.

(a)
Given that $|\mathrm{u}||\mathrm{v}| = 13$, and $t > 0$, find t.

[3 marks]

Question 5b

A third side of the trapezium, with vector $w = \begin{pmatrix} x \\ -2 \\ z \end{pmatrix}$, is perpendicular to both u and v.

(b)
Find $|w|$.

[4 marks]

Question 6a

The points $A(-2,5,3), B, C$ and D form a parallelogram.

$$\overrightarrow{AB} = \begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} -4 \\ 3 \\ 4 \end{pmatrix}$$

(a)
Find the area of the parallelogram.

[2 marks]

Question 6b

(b)

Show that the diagonals of the parallelogram are perpendicular to each another.

[3 marks]

Question 6c

(c)

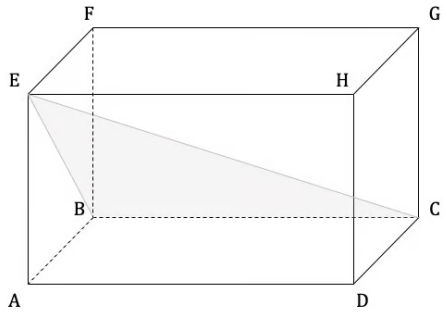
Determine the nature of angle $\widehat{CDA}$.

[3 marks]

Question 7a

$ABCDEFGH$ is a cuboid as shown in the diagram below.

Point A is located at $(5, -3, 4)$, $\overrightarrow{AB} = 2j - k$ and $\overrightarrow{BC} = 4i + j + 2k$.



The perpendicular distance between the faces ABCD and EFGH of the cuboid is $2\sqrt{105}$ units.

(a)
Find the coordinates of the point $E(x, y, z)$, where $x, y, z \in \mathbb{Z}^+$.

[5 marks]

Question 7b

A triangle is formed inside the cuboid by connecting the vertices B, C and E, where $B\hat{C}E = \theta$.

(b)
Using vector methods, find $\cos\theta$.

[3 marks]

Question 8a

The points $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D form the vertices of a parallelogram with position vectors a, b, c and d respectively.

(a)
Show that the area of the parallelogram is $|a \times b + b \times d + d \times a|$.

[4 marks]

Question 8b

(b)
Hence show that the shortest distance from B to AD is $\frac{|a \times b + b \times d + d \times a|}{|d - a|}$.

[3 marks]

Question 9a

Point A has position vector $a = 3\mathrm{i} + 2\mathrm{j} - \mathrm{k}$ and point B has position vector $b = \mathrm{i} - \mathrm{j} + 4\mathrm{k}$ relative to the origin O.

(a)

Find the area of the triangle $\mathrm{A\widehat{O}B}$.

[3 marks]

Question 9b

Point X is located a distance of 8 units from the origin in the direction perpendicular to the plane formed by AOB.

(b)

Find all possible vectors $\overrightarrow{\mathrm{OX}}$.

[4 marks]

Question 9c

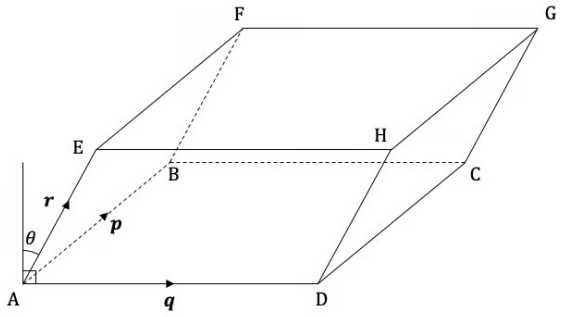
(c)

Find the volume of the tetrahedron AOBX. Give your answer in the form $c\sqrt{d}$, where $c, d \in \mathbb{Z}$.

[2 marks]

Question 10a

ABCDEFGH are vertices of a parallelepiped with the vectors $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AE}$ defined as p, q and r respectively. θ is the angle between $\overrightarrow{AE}$ and the normal to the base ABCD. This information can be seen in the diagram below.



(a)
Find an expression for
(i)
the area of the base ABCD,
(ii)
the perpendicular height of the parallelepiped.

[2 marks]

Question 10b

(b)
Hence, show that the volume of a parallelepiped is given by $|(p \times q) \cdot r|$ units^3.

[2 marks]

Question 10c

(c)

Find the volume of a parallelepiped with vertices $A(5,7,-3)$, $B(6,10,-2)$, $C(9,11,0)$ and $E(4,5,4)$.

[6 marks]