

3.8 Further Trigonometry

Question Paper

Course	DPIB Maths
Section	3. Geometry & Trigonometry
Topic	3.8 Further Trigonometry
Difficulty	Medium

Time allowed: 80
Score: /62
Percentage: /100

Question 1

Show that

(i)

$$\cos\left(\theta + \frac{\pi}{2}\right) = -\sin \theta$$

(ii)

$$\tan(\theta - \pi) = \tan \theta$$

(iii)

$$\sin\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}(\sin \theta - \cos \theta)$$

[6 marks]

Question 2

Let $f(x) = \tan(x + \pi) \sin\left(x + \frac{\pi}{2}\right)$ where $0 < x < \frac{\pi}{2}$.

By using the compound angle formulae, express $f(x)$ in terms of $\sin x$.

[4 marks]

Question 3

Consider the equation $\cos(x - 45) = 2 \sin x$ in the interval $0 \leq x \leq 360^\circ$.

Find an exact value for $\tan x$.

[5 marks]

Question 4a

a)

Express $\cos 4\theta$ in terms of $\cos 2\theta$.

[1 mark]

Question 4b

b)

Hence, show that $\cos 4\theta = 8\cos^2 \theta (\cos^2 \theta - 1) + 1$.

[5 marks]

Question 5

Given that $\tan A = \frac{\sqrt{3}}{2}$, solve the equation $\tan(A + x) = \frac{4}{5}$ in the interval $0 \leq x \leq 360^\circ$.

[6 marks]

Question 6

Prove that $\cos 3x \equiv 4 \cos^3 x - 3 \cos x$.

[6 marks]

Question 7

Solve the equation $\sin 2x - \cos 2x = \frac{\sin x + \cos x}{2} - 1$ for the interval $-\pi < x < 0$.

[7 marks]

Question 8a

a)

Show that $1 - \cos 2x = 2 - 2\cos^2 x$ **[2 marks]****Question 8b**

b)

Show that $\frac{1}{\cos 2x} - \tan 2x = \frac{\cos x - \sin x}{\cos x + \sin x}$ **[5 marks]****Question 9a**

a)

Find the exact values for $\tan x$ given that $\tan^2 x + 4 \tan x + 1 = 0$ **[3 marks]**

Question 9b

b)

Hence, solve the equation $\frac{\tan x}{2 \tan x + 1} = \tan 2x$ algebraically for the interval $0 \leq x \leq 2\pi$.

[5 marks]

Question 10a

The following diagram shows the triangle ABC where $AB = \sqrt{2}$, $AC = \sqrt{3}$ and $\widehat{BAC} = 75^\circ$.



a)

By writing 75° as $30^\circ + 45^\circ$ find the value of $\sin(75^\circ)$.

[3 marks]

Question 10b

b)

Find the area of the triangle, giving your answer in the form $\frac{a+\sqrt{b}}{c}$, where $a, b, c \in \mathbb{Z}$.

[4 marks]