3.9 Vector Properties

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.9 Vector Properties
Difficulty	Medium

Time allowed: 90

Score: /68

Percentage: /100

Question 1a

a)

Show that the vectors $a = 2i - 6j + k$ and $b = -i + 3j - k$ are not parallel.

[3 marks]

Question 1b

b)

Show that $|\mathbf{a} \cdot \mathbf{b}| < |\mathbf{a}||\mathbf{b}|$

[3 marks]

Question 1c

c)

Show that $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$

[2 marks]

Question 2

Consider the two vectors $\mathbf{s} = 3i + 4j - k$ and $\mathbf{t} = -2i + 2j - 3k$.

(i)
Find the cross product of $\mathbf{s}$ and $\mathbf{t}$.

(ii)
Hence, find the angle between $\mathbf{s}$ and $\mathbf{t}$. Give your answer in radians.

[5 marks]

Question 3

The vectors $\mathbf{a}$ and $\mathbf{b}$ are defined by $\mathbf{a} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 5 \\ 2 \\ -2 \end{pmatrix}$.

By finding the scalar product of $\mathbf{a}$ and $\mathbf{b}$, find the angle between them. Give your answer in degrees.

[4 marks]

Question 4a

Let $\mathbf{v} = \begin{pmatrix} t \\ -3 \\ t+2 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -6 \\ 7 \\ t \end{pmatrix}$.

a)

Given that $\mathbf{v}$ and $\mathbf{w}$ are perpendicular, find all possible values of t.

[4 marks]

Question 4b

b)

Show that the angle between $\mathbf{v}$ and $\mathbf{w}$ is acute for all $t > 7$.

[2 marks]

Question 5

Consider the vectors $\mathbf{a} = 3i - j + 4k$ and $\mathbf{b} = (2 + t)i - 2j + 2tk$.

By finding the vector product, determine the value of t, given that $\mathbf{a}$ and $\mathbf{b}$ are parallel.

[4 marks]

Question 6a

Consider the vectors $\mathbf{a} = -2i - j + 3k$ and $\mathbf{b} = 3i + 5k$.

a)
Find a vector of length 7 that is parallel to $\mathbf{a}$.

[3 marks]

Question 6b

b)
Find the vector that is normal to both $\mathbf{a}$ and $\mathbf{b}$.

[3 marks]

Question 7a

a)

Given the vectors $\mathbf{r} = -i + 2j + k$, $\mathbf{s} = 5i + j - k$ and $\mathbf{t} = 2i + 2j + 4k$, show that

(i)

$\mathbf{r} \cdot (\mathbf{s} + \mathbf{t}) = \mathbf{r} \cdot \mathbf{s} + \mathbf{r} \cdot \mathbf{t}$

(ii)

$\mathbf{r} \times (\mathbf{s} + \mathbf{t}) = \mathbf{r} \times \mathbf{s} + \mathbf{r} \times \mathbf{t}$

[5 marks]

Question 7b

b)

Given any two non-zero vectors $\mathbf{a}$ and $\mathbf{b}$, show that $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}.$

[4 marks]

Question 8a

Consider the vectors $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$ and $\mathbf{t} = \begin{pmatrix} -3 \\ 5 \\ 3 \end{pmatrix}$.

a)
Show that $3\mathbf{r} \times \mathbf{t} = 3(\mathbf{r} \times \mathbf{t})$.

[3 marks]

Question 8b

b)
Find the area of a triangle which has vectors $3\mathbf{r}$ and $\mathbf{t}$ as two of its sides.

[3 marks]

Question 9a

On a calm day, a remote-controlled boat is being driven along a vector $\mathbf{u} = i + 3j$ from one side of a pond to the other.

The boat is retrieved and taken to the same starting point, to make the journey again but this time a steady wind causes the boat to travel in a direction represented by the vector $\mathbf{w} = 2i - j$.

a)
Calculate the angle, in degrees, between the direction of travel on its initial journey and the direction on its subsequent journey.

[3 marks]

Question 9b

During the first journey, the boat takes 6.3 seconds to travel the 7.56 m to the other side of the pond.

b)
Find the velocity vector of the boat.

[4 marks]

Question 9c

c)
Given that during the second journey the boat covers a distance of 5.1 m, find the distance between the end points for both journeys.

[4 marks]

Question 10a

ABCD is a parallelogram with vertices $A(2, 3, 0)$, $B(3, 9, 4)$, $C(7, 4, 2)$ and $D(6, -2, -2)$.

a)
Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$.

[2 marks]

Question 10b

b)
Find the area of the parallelogram.

[3 marks]

Question 10c

c)
By finding the scalar product of $\overrightarrow{BA}$ and $\overrightarrow{BC}$, determine if the angle $A\widehat{B}C$ is acute or obtuse.

[4 marks]