

3.3 Trigonometry

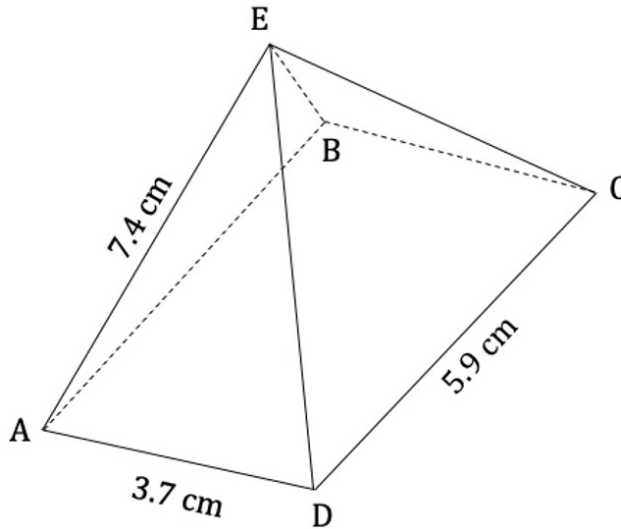
Question Paper

Course	DPIB Maths
Section	3. Geometry & Trigonometry
Topic	3.3 Trigonometry
Difficulty	Very Hard

Time allowed: 110
Score: /86
Percentage: /100

Question 1a

The diagram below shows a rectangular based pyramid $ABCDE$. $DC = 5.9$ cm, $AD = 3.7$ cm and $AE = 7.4$ cm. The vertex, E , is positioned directly above the midpoint of the base $ABCD$.



(a) Find the size of angle \widehat{AEC} .

[3 marks]

Question 1b

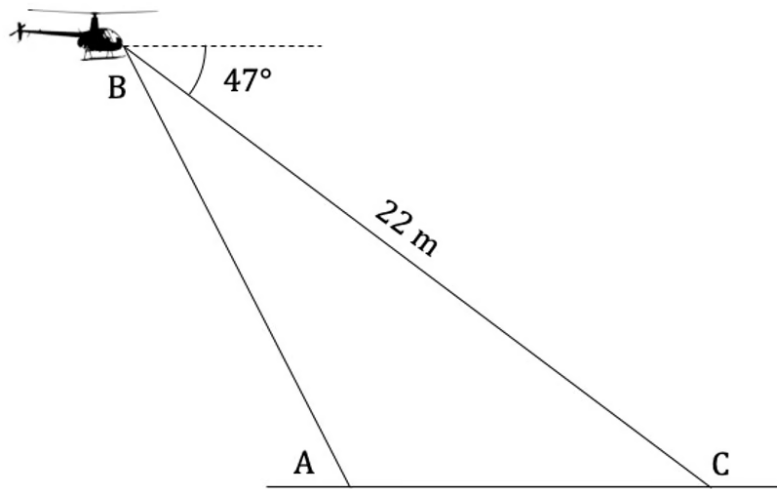
P is a point located on the edge EB , such that $EP : PB = 1 : 4$.

(b) Find the area of the triangle EPD .

[3 marks]

Question 2a

The diagram below shows a police helicopter using a high beam light at point B to search an area on the ground between A and C. The length of the edge of the light beam that is furthest from the helicopter is 22 m and the angle of depression from the helicopter to the same edge is 47° .



The area of the cross section of the search beam, ABC, is 23 m^2 .

(a) Calculate the horizontal length of ground, AC, that is lit by the beam.

[3 marks]

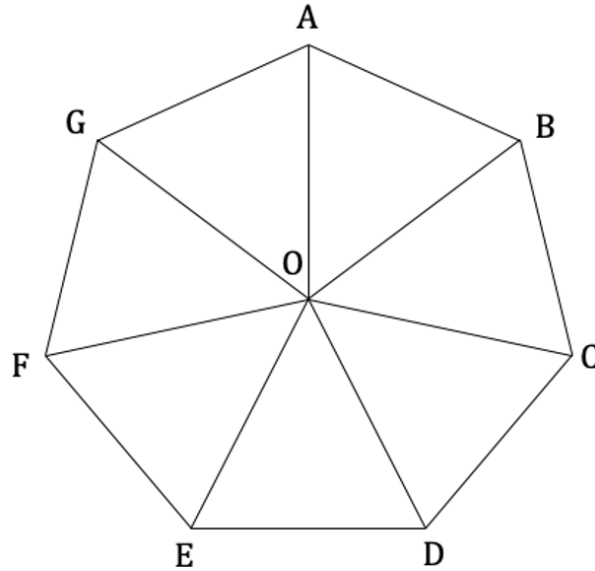
Question 2b

(b) Find the size of the angle of the beam \widehat{ABC} .

[4 marks]

Question 3a

A spider starts to weave a web, ABCDEFGO, with threads of equal length (AB, BC, etc.) linking the 7 vertices that are equally spaced around the centre point, O. Threads connecting each vertex to the centre (OA, OB, etc.) are also created by the spider. Each line from the centre has a length of 12.6 cm, and the points O, A, B, C, D, E, F and G all lie in a single plane. This is represented in the diagram below.



The spider is located at point G and a fly lands at point D.

(a) Calculate the angle \widehat{OGD} .

[2 marks]

Question 3b

The spider decides to add more silk to its web by connecting each vertex to the midpoint of the adjacent line when moving clockwise around the web, for example from G to the midpoint of AO.

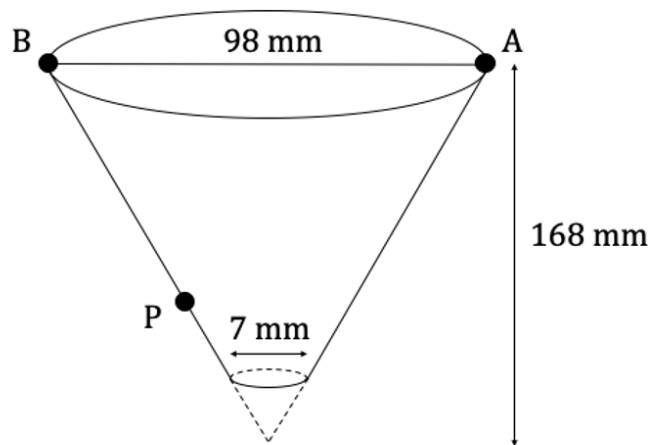
(b) Given that the spider can only produce 220 cm of silk a day, show that the spider is unable to complete the web on the same day that he started it.

[5 marks]

Question 4a

The diagram below shows a funnel in the shape of a right cone with a smaller cone removed from the end. The circular planes at both ends of the object are parallel to one another. The perpendicular height of the complete cone is 168 mm. The initial diameter of the funnel is 98 mm at the upper end and narrows to a diameter of 7 mm at the bottom.

The funnel has been used to pour some sugar into a bottle and a grain of sugar remains at point P, $\frac{1}{3}$ of the way up the slanted height of the funnel. An ant sits on the edge at the top of the funnel at point A. [AB] is a diameter of the large circular face. Points A, B, P and the axis of the cone all lie on a single plane.



(a) Calculate the direct distance between the ant and the grain of sugar, AP.

[6 marks]

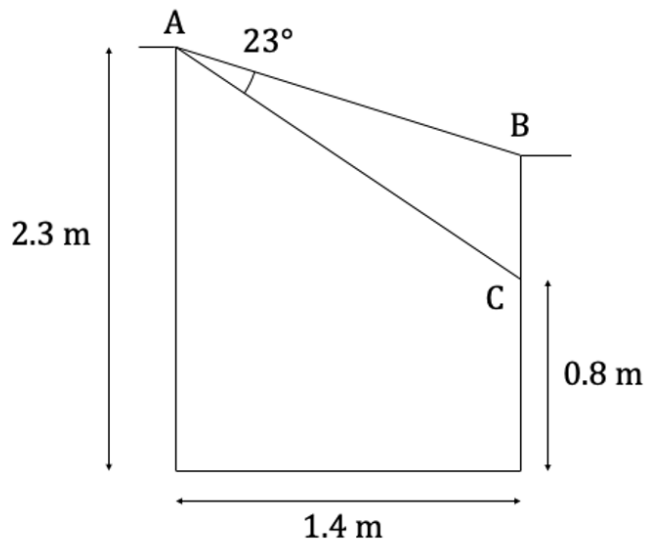
Question 4b

(b) Find the size of the angle of depression from the ant to the grain of sugar.

[3 marks]

Question 5a

A piece of equipment in a playground consists of two ropes fixed to a hook at point A at the edge of a gap and pulled taut to the other side and fixed at points B and C. The height of the taller left embankment above the ground is 2.3 m. Point B is located at the top of the right embankment. Point C is vertically beneath point B, and the height from point C to the ground is 0.8 m. The angle between the ropes is 23° and the horizontal distance of the gap that is being bridged is 1.4 m. The information is shown on the diagram below.



(a) Calculate the length BC.

[4 marks]

Question 5b

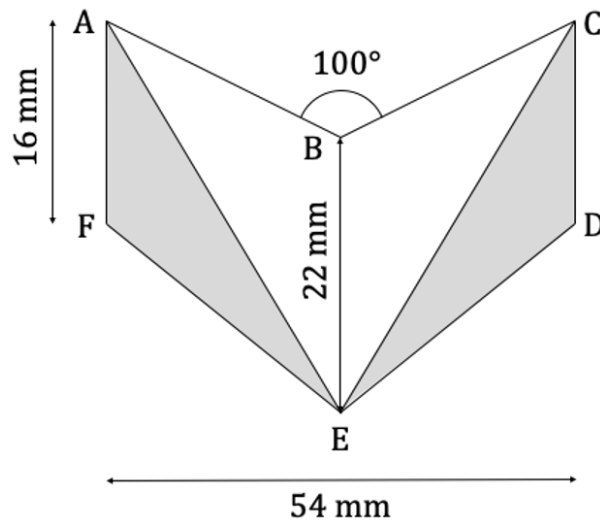
A third piece of rope of length 0.9 m is to be added to the structure at point B and fixed at a point P on the rope AC.

(b) Calculate the angle \widehat{BPC} and hence the distance PC.

[4 marks]

Question 6

A symmetrical pendant for a necklace is made in the shape of an irregular hexagon, ABCDEF, as can be seen in the diagram below. AF, BE and CD are parallel. $BE = 22\text{ mm}$, $AF = CD = 16\text{ mm}$ and $\widehat{ABC} = 100^\circ$. $AB = BC$ and $EF = DE$. The shaded area is silver and the white area is gold. The width of the entire pendant is 54 mm. This information is shown in the diagram below:

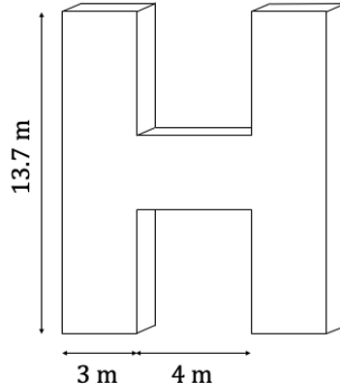


Calculate the percentage of the area of the pendant that is silver.

[4 marks]

Question 7a

The 'H' on the Hollywood sign is 13.7 m tall, with each leg being 3 m wide and the gap between the legs being 4 m. The width of the cross bar of the 'H' (measured from top to bottom in the diagram) is the same as the width of the legs, and the cross bar is situated at the centre of the height of the 'H'. This information can be seen in the diagram below.



The 'H' is situated on horizontal ground but is tilted backwards slightly such that the rear face of the 'H' makes an angle of 5° to the vertical. During repair works, a metal support bar is connected between a point A on the ground and point M, which is the midpoint of the rear of the cross bar of the 'H'. The plane defined by points A, M and B, where B is the midpoint of the gap between the legs of the 'H' where they touch the ground, is perpendicular to the plane defined by the rear face of the 'H'. The angle \widehat{AMB} between the support bar and the rear face of the 'H' is 30° .

(a) Calculate the length of the metal support bar.

[2 marks]

Question 7b

Additional support bars are required to reach from the ground to the midpoint at the top of the rear surface of each of the two legs. These additional supports are to contact the ground at the same point as the initial metal support bar.

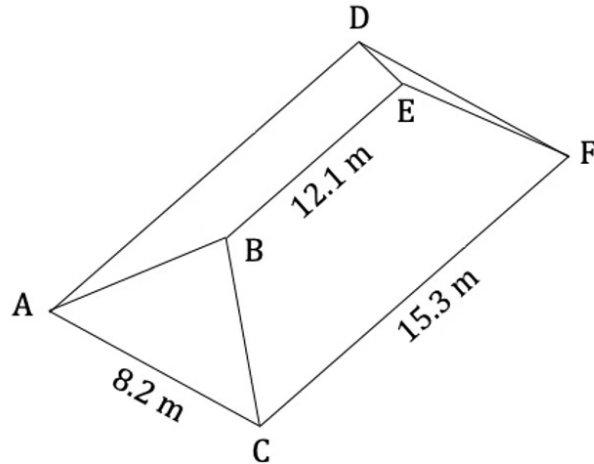
(b) Calculate:

- (i) the length of one of the additional supports, and
- (ii) the angle that it makes with the horizontal.

[8 marks]

Question 8a

A pitched roof is made up of a timber frame, ABCDEF, whose horizontal rectangular base ADFC covers an area of 15.3 m by 8.2 m. The raised central ridge BE is parallel to AD, and its midpoint is situated directly above the intersection of line segments AF and CD. BE is 12.1 m long and is at a height of 2.2 m above the plane defined by ADFC.



(a) Calculate the total length of timber required for the frame.

[3 marks]

Question 8b

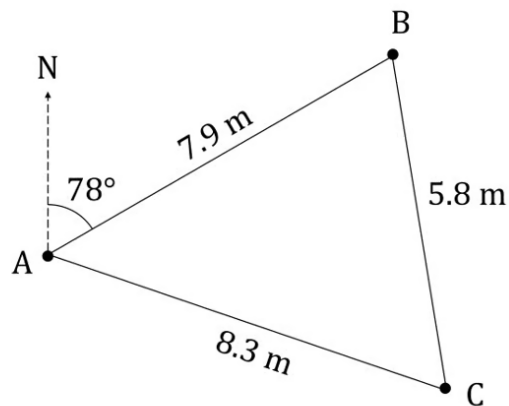
An internal beam runs from the midpoint of AC, M, to point E.

(b) Calculate the length ME.

[3 marks]

Question 9a

In the diagram below, AB, BC and AC are steel beams at the first floor level of a building under construction. ABC lies in a horizontal plane 5 m above ground level, with $AB = 7.9$ m, $BC = 5.8$ m, and $AC = 8.3$ m. The line of beam AB makes an angle of 78° with due north when viewed from above.



A pot of paint has been left on beam BC, halfway between points B and C.

(a) Find the bearing from point A to the pot of paint.

[5 marks]

Question 9b

A workman is standing on the second floor of the building directly above point A, with his eyes at a height of 12 m above ground level.

(b) Find

(i) the angle of depression, and

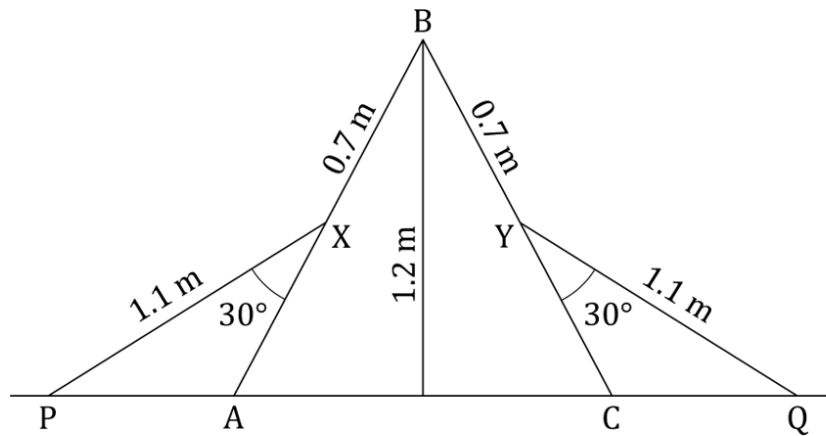
(ii) the distance

from the workman's eyes to the pot of paint.

[3 marks]

Question 10

A tent with a symmetrical triangular cross-section, ABC , is fixed to a horizontal surface with guy ropes. The tent has a perpendicular height of 1.2 m, and the guy ropes are attached to points X and Y on AB and BC , respectively, such that $BX = BY = 0.7$ m. The guy ropes are each 1.1 m long, and they are anchored to the ground at points P and Q such that the angles \widehat{PXA} and \widehat{QYC} are both 30° . Points A, B, C, P, Q, X and Y all lie in a single plane. A diagram to illustrate this is provided below.



Calculate the distance, PQ , between the anchor points of the two guy ropes.

[6 marks]

Question 11a

Triangle ABC is such that the length of side AB is x units, the length of side BC is y units, and $\widehat{BAC} = \theta$ is an acute angle.

- (a) Use a diagram to show that if (and only if) $x \sin \theta < y < x$ then there are two triangles, ABC_1 and ABC_2 , which satisfy the conditions above, where C_1 and C_2 are points such that $AC_1 > AC_2$.

[5 marks]

Question 11b

Given that $x \sin \theta < y < x$, let angle $\widehat{AC_1B}$ be denoted by ϕ .

- (b) (i) Write down an expression for ϕ in terms of x , y and θ .
- (ii) Write down expressions for angles $\widehat{ABC_1}$, $\widehat{AC_2B}$ and $\widehat{ABC_2}$ in terms of θ and ϕ .

[5 marks]

Question 11c

(c) Show that the difference between the areas of triangles ABC_1 and ABC_2 is equal to

$$\frac{1}{2}y^2 \sin 2\phi$$

[5 marks]