

3.6 Matrix Transformations

Question Paper

Course	DPIB Maths
Section	3. Geometry & Trigonometry
Topic	3.6 Matrix Transformations
Difficulty	Hard

Time allowed: 90
Score: /67
Percentage: /100

Question 1a

a)
Find a single matrix to represent the composite transformation comprising a reflection in the x -axis followed by a rotation of 60° counter-clockwise about the origin.

[3 marks]

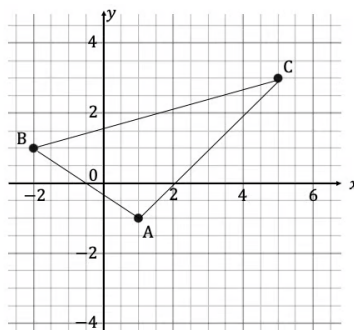
Question 1b

b)
Hence find the coordinates of the image of the point $(-7, 5)$, which undergoes the composite transformation stated in part (a).

[2 marks]

Question 2a

The diagram below shows the triangle ABC.



a)
Write down the matrix that will rotate the triangle about the origin so that $[AC]$ is parallel to the x -axis.

[3 marks]

Question 2b

b)

Find the position matrix of the mapped image.

[3 marks]**Question 2c**

c)

Hence find the area of the triangle.

[2 marks]

Question 3a

Points in a plane are subjected to a transformation $T: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x' \\ y' \end{pmatrix}$, where T is defined by:

$$T: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} p \\ 6 \end{pmatrix}$$

a)

Given that a point $A(4, q)$ is mapped to $A'(7 - \sqrt{3}, 2\sqrt{3} + 7)$, find p and q , where $p, q \in \mathbb{R}$.

[2 marks]

Question 3b

b)

Given that T comprises two individual transformations describe in full the composite transformation T .

[4 marks]

Question 4a

A parallelogram with a base length of cm undergoes a transformation represented by the matrix $\begin{pmatrix} -8 & 3 \\ 7 & -3 \end{pmatrix}$

a)

Given that the area of the image after the transformation is 48 cm^2 , find the area of the original parallelogram.

[3 marks]

Question 4b

b)

Hence find the perpendicular height of the original parallelogram.

[2 marks]**Question 5a**A function f is defined by $f(x) = x^2$.

a)

Given that $g(x) = \frac{1}{2}f(4x)$, describe fully the transformation that maps $f(x) \mapsto g(x)$.**[3 marks]****Question 5b**

b)

Hence find the single matrix that represents this transformation.

[3 marks]

Question 6

The points $A(4, 9)$ and $B(-3, -11)$ are transformed by T to become the points $A'(3, 8)$ and $B'(2, -6)$ respectively.

Given that the point $C(-5, -7)$ is transformed by T^3 , find the coordinates of the image point C' .

[6 marks]

Question 7a

Consider matrix A , where $A = \begin{pmatrix} 4 & 2 \\ -3 & 6 \end{pmatrix}$, which represents a series of transformations in the following order:

- A transformation represented by the matrix $\begin{pmatrix} 4 & 2 \\ 3 & -6 \end{pmatrix}$
- A counter-clockwise rotation of 270°
- A single transformation represented by the matrix B

a)

Find matrix B and describe the effect of the transformation it represents in full.

[5 marks]

Question 7b

b)
Find B^{-1} .

[2 marks]

Question 7c

c)
Compare the matrix B with its inverse and explain any similarities that can be observed.

[2 marks]

Question 8a

The triangle PQR with vertices $P(4, -1)$, $Q(-4, 2)$ and $R(-4, 2)$ is enlarged by a scale factor of 5 with the centre of enlargement at $X(-3, -1)$. The enlarged shape has coordinates $P'Q'R'$.

a)

Write down the column vector required to translate the centre of enlargement to the origin.

[1 mark]**Question 8b**

b)

Write down the position matrix of the vertices of the triangle after undergoing the translation stated in part (a).

[2 marks]**Question 8c**

c)

Find a single transformation in the form $AX + b$ that will map the coordinates of the vertices of the triangle after they have been translated to their final position $P'Q'R'$.

[2 marks]**Question 8d**

d)

Hence determine the coordinates of $P'Q'R'$.

[2 marks]

Question 9a

The matrices R , S and T are defined by $R = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$, $S = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ and $T = \begin{pmatrix} 1 & 0 \\ 0 & \frac{4}{9} \end{pmatrix}$.

Triangle X is mapped onto triangle Y by the transformation represented by $R^3 STS$.

a)

Describe in full the two geometric transformations, A and B , that are equivalent to $R^3 STS$ and map triangle X onto triangle Y .

[4 marks]

Question 9b

The coordinates of triangle Y are $(2, -8)$, $(2, 12)$ and $(14, 3)$.

b)

Find the area of triangle X .

[3 marks]

Question 10a

The quadrilateral PQRS with position matrix T_0 has vertices $P(0,0)$, $Q(0,9)$, $R(6,9)$, and $S(6,0)$.

T_n denotes the position matrix of the image quadrilateral after PQRS has been transformed n times by matrix M .

After a transformation, represented by the matrix M , where $M = AX + b$, the position matrix of the image quadrilateral T_1 has vertices $P'(6, -1)$, $Q'(6, 5)$, $R'(10, 5)$ and $S'(10, -1)$.

a)
Find M .

[3 marks]

Question 10b

b)
Find the position matrix of the image quadrilateral T_2 .

[2 marks]

Question 10c

c)
Find the perimeter of the shape formed by the quadrilaterals T_0 , T_1 and T_2 .

[3 marks]

