# 3.5 Trigonometric Functions & Graphs

## Question Paper

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| Course | DP IB Maths |
| Section | 3. Geometry & Trigonometry |
| Topic | 3.5 Trigonometric Functions & Graphs |
| Difficulty | Medium |

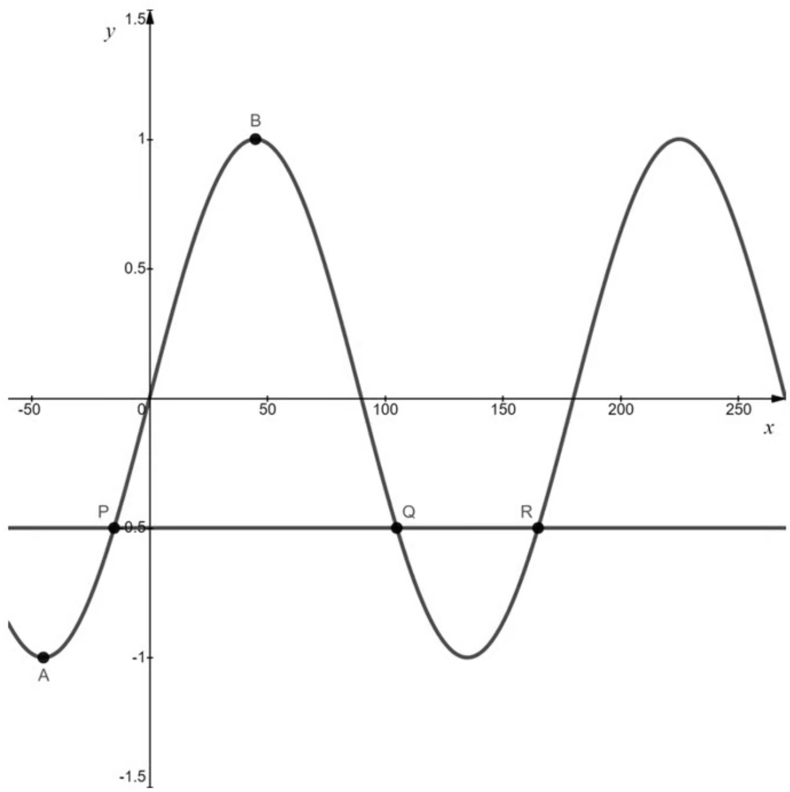
**Time allowed:** 60

**Score:** /50

**Percentage:** /100

## Question 1a

The graph below shows the curve with equation $y = \sin 2x$ in the interval $-60° \leq x \leq 270°$.




(a) Point $A$ has coordinates $(-45°, -1)$ and is the minimum point closest to the origin. Point $B$ is the maximum point closest to the origin. State the coordinates of $B$.

**[1 mark]**

## Question 1b

(b) A straight line with equation $y = -\frac{1}{2}$ meets the graph of $y = \sin 2x$ at the three points $P$, $Q$ and $R$, as shown in the diagram.

Given that point $P$ has coordinates $(-15°, -\frac{1}{2})$, use graph symmetries to determine the coordinates of $Q$ and $R$.

**[2 marks]**

## Question 2

(i) Sketch the graph of $y = \cos(\theta + 30°)$ in the interval $-180° \leq \theta \leq 360°$.

(ii) Write down all the values where $\cos(\theta + 30°) = 0$ in the given interval.

**[4 marks]**

## Question 3

A dolphin is swimming such that it is diving in and out of the water at a constant speed. On each jump and dive the dolphin reaches a height of 2 m above sea level and a depth of 2 m below sea level.

Starting at sea level, the dolphin takes $\frac{2\pi}{3}$ seconds to jump out of the water, dive back in and return to sea level.

Write down a model for the height, $h$ m, of the dolphin, relative to sea level, at time $t$ seconds, in the form $h = A\sin(Bt)$ where $A$ and $B$ are constants to be found.

**[3 marks]**

## Question 4

A section of a new rollercoaster has a series of rises and falls. The vertical displacement of the rollercoaster carriage, $y$, measured in metres relative to a fixed reference height, can be modelled using the function $y = 30\cos(24t)°$, where $t$ is the time in seconds.

(i) Sketch the function for the interval $0 \le t \le 30$.

(ii) How many times will the rollercoaster carriage fall during the 30 seconds?

(iii) How long does the model suggest it will take for the rollercoaster carriage to reach the bottom of the first fall?

**[6 marks]**

## Question 5a

The height, $h$ m, of water in a reservoir is modelled by the function

$$h(t) = A + B\sin\left(\frac{\pi}{6}t\right), \ \ t \geq 0,$$

where $t$ is the time in hours after midnight. $A$ and $B$ are positive constants.

(a) In terms of $A$ and $B$, write down the natural height of the water in the reservoir, as well as its maximum and minimum heights.

**[3 marks]**

## Question 5b

(b) The maximum level of water is 3 m higher than its natural level.
The level of water is three times higher at its maximum than at its minimum.

Find the maximum, minimum and natural water levels.

**[3 marks]**

## Question 5c

(c) (i) How many times per day does the water reach its maximum level?
(ii) Find the times of day when the water level is at its minimum?

**[3 marks]**

## Question 6a

A lifejacket falls over the side of a boat from a height of 3 m.
The height, $h$ m, of the lifejacket above or below sea level ($h = 0$), at time $t$ seconds after falling, is modelled by the equation $h = 3e^{-0.7t}\cos 4t$.

(a) The lifejacket reaches its furthest point below sea level after 0.742 seconds.
Find the total distance it has fallen, giving your answer to three significant figures.

**[2 marks]**

## Question 6b

(b) Write down the value of $t$ for the first three times the lifejacket is at sea level.

**[2 marks]**

## Question 6c

(c) (i) Find the value of $3e^{-0.7t}$ when $t = 6.2$.

(ii) Hence justify why, from 6.2 seconds on, the lifejacket will always be within 4 centimetres of sea level.

**[3 marks]**

## Question 7a

The number of daylight hours, $h$, in the UK, during a day $d$ days after the spring equinox (the day in spring when the number of daylight hours is 12), is modelled using the function

$$h = 12 + \frac{9}{2}\sin\left(\frac{2\pi}{365}d\right)$$

(a) (i) Find the number of daylight hours during the day that is 100 days after the spring equinox.

(ii) Find the number of days after the spring equinox that the two days occur during which the number of daylight hours is closest to 9.

**[5 marks]**

## Question 7b

(b) For how many days of the year does the model suggest that the number of daylight hours exceeds 15 hours? Give your answer as a whole number of days.

**[3 marks]**

## Question 8a

Felicity is a keen ice skater and has entered a competition that requires her to skate in a circular pathway in front of three judges. Her distance, $d$ meters, away from the judges table, $t$ seconds after commencing her routine can be modelled by the function

$$d = 12\cos\frac{\pi}{30}t + 15$$

(a) (i) State the distance Felicity is away from the judges table at the start of her routine.

(ii) State the distance Felicity is away from the judges table after 15 seconds.

**[3 marks]**

## Question 8b

(b) Find, in terms of $\pi$, the circumference of Felicity's circular pathway on the ice rink.

**[2 marks]**

## Question 8c

(c) Find, in terms of $\pi$, Felicity's average speed for each lap on the ice rink.

**[2 marks]**

## Question 8d

Felicity's routine took three laps in total around the ice rink.

(d) Find the times during Felicity's routine where she was at a distance of 21 metres from the judges table.

**[3 marks]**