1.1 Number & Algebra Toolkit

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.1 Number & Algebra Toolkit
Difficulty	Very Hard

Time allowed: 70

Score: /57

Percentage: /100

Question 1a

Consider the numbers $a = 11\sqrt{2}, b = (5 + 6\pi), c = \sqrt{2}, d = 6(\pi - 1)$.

(a) Giving your answer to 1 decimal place, calculate the value of

(i) a.

(ii) b.

(iii) c.

(iv) d.

[2 marks]

Question 1b

Points P and Q have coordinates (a, b) and (c, d) respectively.

The formula for the distance, d, between two points with coordinates (x_1, y_1) and (x_2, y_2) is given in your formula booklet.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

(b) Using your answers from part (a), calculate the distance, d, between points P and Q. Give your answer correct to 1 decimal place.

[2 marks]

Question 2

Let $Y = (pq)^{-2}r^3$ and $T = pqr^{-1}$, where $p = \sin\frac{\pi}{3}$, $q = \sqrt{3}$, $r = 2$.

Find the exact value of YT.

[5 marks]

Question 3a

Point A has coordinates $(-1, 7)$ and point B has coordinates $(11, 12)$.

The formula for the distance, d, between two points with coordinates (x_1, y_1) and (x_2, y_2) is given in your formula booklet.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

(a) Calculate the distance between points A and B.

[3 marks]

Question 3b

The formula for the coordinates of the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2) is given in your formula booklet.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

(b) Calculate the midpoint of the line segment with endpoints A and B.

[2 marks]

Question 4a

Let $S = (a \sin^2 4b)(c^2 \tan^2 12d)^{-1}\left(\sqrt{a} + c - \cos 48b\right)$, where $a = 16, b = 7.5°, c = 3$ and $d = 5°$.

Note: $\sin^2\theta = (\sin\theta)^2$

(a) Find the value of S, giving your answer as a fraction.

[2 marks]

Question 4b

Let $X = \frac{\sqrt{a}+c^2-2\sin 54d}{\sqrt{a^3}-a-c}$

(b) Find the value of X, giving your answer as a fraction.

[2 marks]

Question 4c

(c) Calculate the value of SX, giving your answer as a fraction.

[2 marks]

Question 5a

Consider the numbers $p = 2.41 \times 10^4$ and $q = 4.12 \times 10^5$.

(a) Giving your answers in the form $a \times 10^k$, where $1 \leq a < 10, k \in \mathbb{Z}$, calculate

(i) $p + q$

(ii) $p - q$

(iii) $q - p$

(iv) $\frac{p}{q}$.

[4 marks]

Question 5b

The formula for the distance, d, between two points with coordinates (x_1, y_1) and (x_2, y_2) is given in your formula booklet.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

(b) Using your answers to part (a), estimate the distance between points A$(p + q, p - q)$ and B$\left(q - p, \frac{p}{q}\right)$.

[2 marks]

Question 6

The mean height of the four tallest students in a classroom is 176 cm and the mean height of the six tallest students is 165 cm. The fifth tallest student is 4 cm taller than the sixth tallest student.

Find the heights of the fifth and sixth tallest students.

[6 marks]

Question 7a

In this question give your answers in the form $a \times 10^k$, where $1 \leq a < 10, k \in \mathbb{Z}$.

The surface area of the Earth is approximately 5.1×10^8 km^2.

(a) Given that the surface area of the Earth is approximately 13 times larger than the surface area of the moon, find an approximation for the surface area of the moon.

[3 marks]

Question 7b

(b) Given that the surface area of Jupiter is approximately 6.1×10^9 km^2, find how many times larger the surface area of Jupiter is than the surface area of the Earth.

[4 marks]

Question 8

Solve the following systems of linear equations using technology.

(i)

$$2x - 5y - 7z = -21$$
$$3z + x - 4y = 44$$
$$x + z - y = 12$$

(ii)

$$z - x - y = -11$$
$$5x + 11z - 2y = -28$$
$$3y - 4z + x = 30$$

[6 marks]

Question 9a

(a) Write $\frac{4x+5}{2x+1}$ in the form $A+\frac{B}{2x+1}$, where A and B are integers to be found.

[2 marks]

Question 9b

(b) The expression $\frac{6ax+b}{a+bx}$ can be written as the sum of partial fractions in the form $4-\frac{5}{a+bx}$.

Find the value of a and b.

[4 marks]

Question 10a

The expression $\frac{-3x^2-x+34}{x^3-5x^2+3x+9}$ can be written as the sum of partial fractions in the form $\frac{A}{x+1}+\frac{B}{x-a}+\frac{C}{(x-a)^2}$, where $a, A, B, C \in \mathbb{Z}$.

(a) Write down the value of a.

[1 mark]

Question 10b

(b) Find the value of A, B, and C.

[5 marks]