6.1 Extended Questions (Section B, SL)
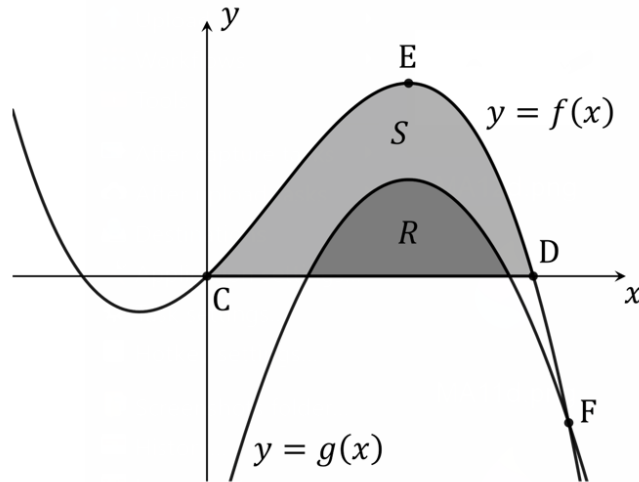
Question Paper

Course	DP IB Maths
Section	6. Extended Questions
Topic	6.1 Extended Questions (Section B, SL)
Difficulty	Hard

Time allowed: 120

Score: /98

Percentage: /100

Question 1a

The following diagram shows the graph of the function f defined by $f(x) = 4x^2 + 4x - 4x^3$ as well as the graph of $y = g(x)$ for a function g, with both functions defined for all $x \in \mathbb{R}$.

Points C and D are the non-negative x-axis intercepts of f, while E is the local maximum on the graph of f. Point F is the point of intersection between the graphs of f and g. The shaded region R represents the area between the graph of g and the x-axis. The shaded region S represents the area above the positive x-axis between the graphs of f and g.

(a) Find the coordinates of C, D and E, giving your answers correct to three significant figures.

[3 marks]

Question 1b

The graph of g is a parabola with x-intercepts $\left(\frac{1}{2}, 0\right)$ and $\left(\frac{3}{2}, 0\right)$, and vertex $(1, 2)$.
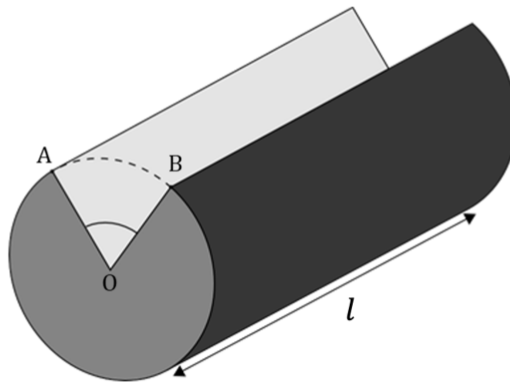
(b) Find an expression for $g(x)$.

[3 marks]

Question 1c

(c) Hence find the area of region R.

[3 marks]

Question 1d

The following diagram shows a cylindrical log with a wedge cut from it. The area of the major sector AOB is equal to the area of the shaded region S. The length l is equal to the distance between points E and F on the diagram above.

(d) Find the volume of the log.

[6 marks]

Question 2a

The average interest rates on accounts for all the different banks in a country can be modelled as a normal distribution with a mean μ% and a standard deviation σ%.

Given that 10% of banks have interest rates lower than 1.2% and 8% have interest rates higher than 2%,

(a) find two linear equations satisfied by μ and σ, and hence find the values of μ and σ to three significant figures.

[6 marks]

Question 2b

(b) Find the probability that a particular bank's average interest rate is over 1.8%.

[2 marks]

Question 2c

(c) Given that a bank's average interest rate is less than 1.8%, find the probability that its average interest rate is higher than 1.2%.

[4 marks]

Question 2d

(d) For a city in this country with 54 banks, find the expected number of banks that have average interest rates over 2.2%.

[3 marks]

Question 2e

For a child's second birthday, parents decide to put \$10 000 into a bank account paying a fixed rate of 2.2% annual interest, and also to put \$10 000 into an investment portfolio. The annual returns for the investment portfolio can be modelled as a normal distribution with mean 12% and standard deviation 5.5%.

(e) Given that the returns on the investment portfolio are independent of each other from year to year, find the expected combined value of the bank account and the investment portfolio when the child turns 18.

[5 marks]

Question 2f

(f) Find the probability that in a given year the combined annual return is over 12%.

[5 marks]

Question 3a

The first two terms of an infinite geometric sequence are $u_1 = 12$ and $u_2 = 9\cos^2\theta$, where $0 < \theta < 2\pi$ and $\theta \neq \frac{\pi}{2}$.

(a) Find the range of possible values for the common ratio r of the geometric sequence.

[5 marks]

Question 3b

(b) Show that the sum of the infinite sequence can be expressed as $\dfrac{96}{5-3\cos 2\theta}$.
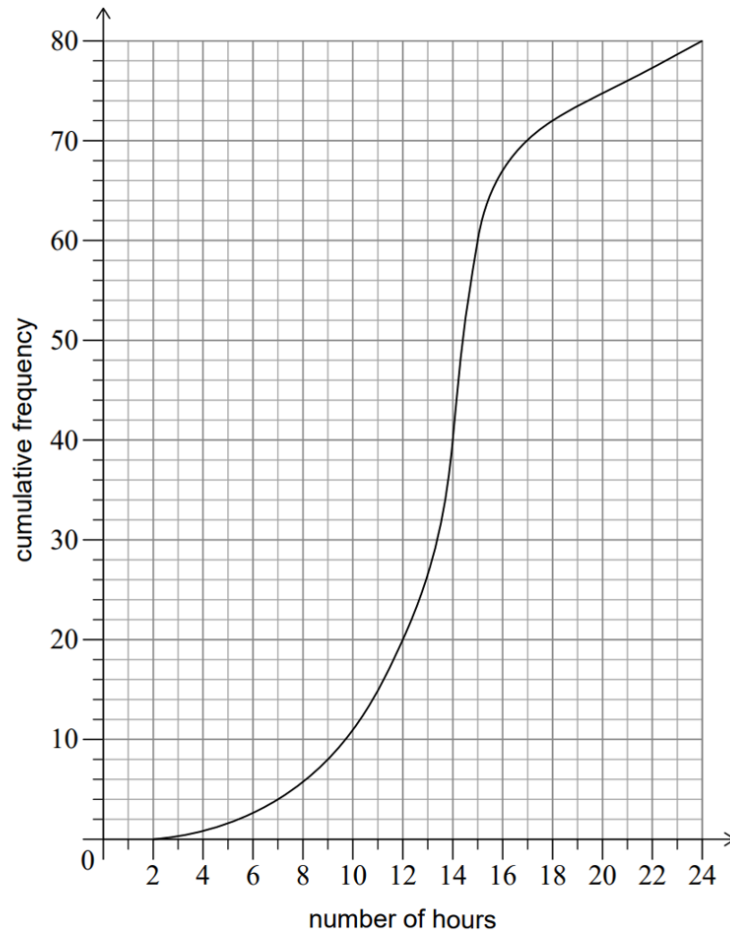
[5 marks]

Question 3c

(c) Find the value of θ that minimises the sum of the infinite sequence.

[5 marks]

Question 4a

As part of a study, a group of 80 people from a particular city were randomly selected and tested to find out the number of hours they spend on social media per week. The results are represented on the following cumulative frequency graph.

(a) Find the median number of hours these people spend on social media per week.

[2 marks]

Question 4b

(b) Given that 87.5% of these people spend less than k hours per week on social media, find the value of k.

[3 marks]

Question 4c

The same information is represented by the following table.

Number of hours, h	$2 < h \leq 7$	$7 < h \leq 9$	$9 < h \leq 15$	$15 < h \leq 21$	$21 < h \leq 24$
Frequency	4	a	b	16	c

(c) Find the values of a, b and c.

[3 marks]

Question 4d

There are 2 200 000 people living in the city.

(d) Use the results of the study to estimate the number of people that spend more than 15 hours per week on social media.

[3 marks]

Question 4e

(e) Explain why the sampling method used might not provide an accurate representation of the amount of time **all** people in the city spend on social media per week, and suggest a more appropriate sampling method.

[2 marks]

Question 5a

Consider a function f where line l_1 with equation $y = 2x - 8$ is the normal to the graph of f at the point where $x = 5$.

(a) Find the values of $f'(5)$ and $f(5)$.

[4 marks]

Question 5b

Let g be the function defined by $g(x) = f(2x^2 - 3)$, and let A be the point on the graph of g where $x = 2$. Line l_2 is the tangent to the graph of g at point A.

(b) Find the equation of l_2.

[4 marks]

Question 5c

(c) Find the coordinates of the point of intersection between lines l_1 and l_2. Give your answers for the coordinates as exact values.

[6 marks]

Question 6a

Let $f(x) = kx \ln(3x^4)$ for $x > 0$, where $k > 0$ is a constant.

(a) Given that $f(a) = 0$, find the value of a.

[3 marks]

Question 6b

(b) Find

    (i) $\ f'(x)$

    (ii) $\ f''(x)$.

[5 marks]

Question 6c

(c) Show that the graph of f has exactly one minimum point and determine its x-coordinate.

[5 marks]

Question 6d

(d) Given that the y-coordinate of the minimum point is $-4,$ find the value of k.

[3 marks]