5.2 Further Differentiation

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.2 Further Differentiation
Difficulty	Medium

Time allowed: 130

Score: /104

Percentage: /100

Question 1

Differentiate $\dfrac{5x^7}{\sin 2x}$ with respect to x.

[4 marks]

Question 2a

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ for each of the following:

(a) $y = \cos(x^2 - 3x + 7) + \sin(e^x)$

[4 marks]

Question 2b

(b) $y = \ln(2x^3)$

[3 marks]

Question 2c

(c) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

[3 marks]

Question 3a

Differentiate with respect to x, simplifying your answers as far as possible:

(a) $(4\cos x - 3\sin x)e^{3x-5}$

[3 marks]

Question 3b

(b) $(x^3 - 4x^2 + 7)\ln x$

[3 marks]

Question 3c

(c) $\sin\left(x^{\frac{1}{3}} + x^{-\frac{4}{5}} + \pi\right)$

[3 marks]

Question 4a

A curve has the equation $y = \mathrm{e}^{-3x} + \ln x,\ x > 0$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

[2 marks]

Question 4b

(b) Hence find the gradient of the normal to the curve at the point $(1,\ e^{-3})$, giving your answer correct to 3 decimal places.

[2 marks]

Question 5a

Consider the curve with equation $y = e^{3x^2 + 5x - 2}$.

(a) Find $\frac{dy}{dx}$.

[1 mark]

Question 5b

(b) Hence find the equation of the tangent to the curve at the point $(-2,\ 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[3 marks]

Question 6

Let $f(x) = \frac{g(x)}{h(x)}$, where $g(2) = 4$, $h(2) = -1$, $g'(2) = 0$ and $h'(2) = 2$.

Find the equation of the tangent of f at $x = 2$.

[6 marks]

Question 7a

A curve has the equation $y = x^3 - 12x + 7$.

(a) Find expressions for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

[3 marks]

Question 7b

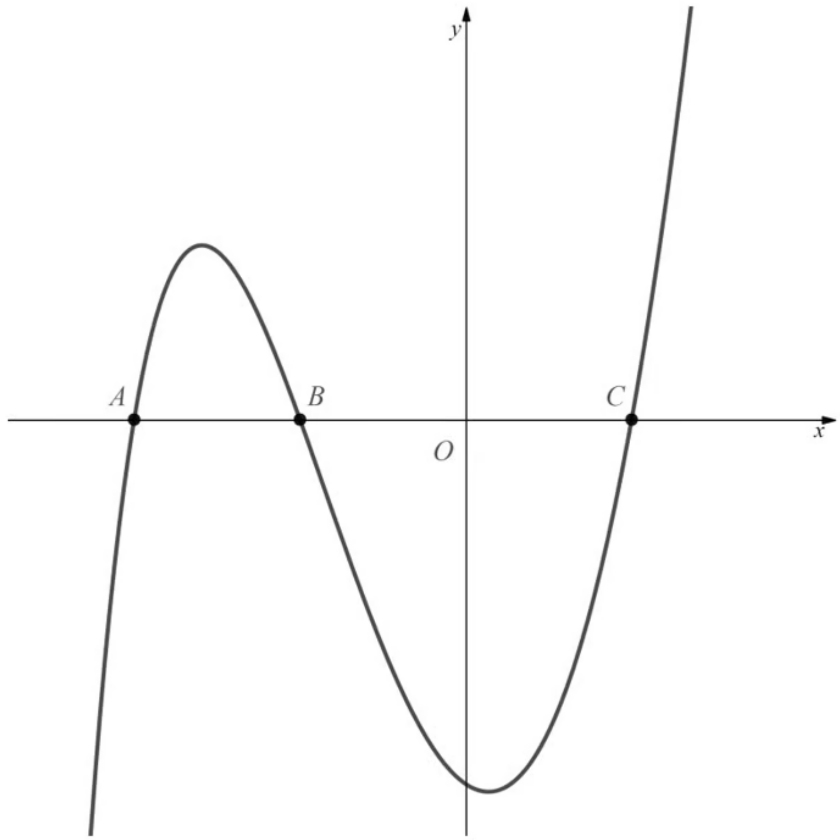
(b) Determine the coordinates of the local minimum of the curve.

[3 marks]

Question 8a

The diagram below shows part of the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is the function defined by

$$\mathrm{f}(x) = (x^2 - 1)\ln(x + 3), \quad x > -3$$



Points A, B and C are the three places where the graph intercepts the x-axis.

(a) Find $\mathrm{f}'(x)$.

[4 marks]

Question 8b

(b) Show that the coordinates of point A are $(-2, 0)$.

[2 marks]

Question 8c

(c) Find the equation of the tangent to the curve at point A.

[3 marks]

Question 9a

Let $f(x) = x^2 e^x$.

(a) Find $f'(x)$.

[3 marks]

Question 9b

(b) Find $f''(x)$.

[3 marks]

Question 9c

(c) Determine the ranges of x-values for which the graph of f is

(i) concave-up

(ii) concave-down

giving all boundary values for the ranges as exact values.

[4 marks]

Question 9d

(d) Hence find the exact x coordinates of the points of inflection for the graph of f. Be sure to show that any points identified are indeed points of inflection.

[2 marks]

Question 10a

Let $f(x) = 2e^{2cosx}$, where $-\pi \leq x \leq \pi$.

(a) Find the number of points containing a horizontal tangent.

[1 mark]

Question 10b

(b) Show algebraically that the gradient of the tangent at $x = \frac{\pi}{2}$ is -4.

[4 marks]

Question 10c

(c) State the gradient of the tangent at $x = \frac{3\pi}{2}$.

[1 mark]

Question 10d

It can be found that as the function, f, undergoes a transformation $f(kx)$, the number of stationary points found between $-\pi \leq x \leq \pi$ increases.

(d) Find the number of stationary points on f after a transformation of $f(2x)$ and hence, state the general rule representing the number of stationary points in terms of k where $k \in Z^+$.

[3 marks]

Question 11

Let $f(x) = \sin x$ and $g(x) = sin^2 x$, for $0 \leq x \leq 2\pi$.

Solve $f'(x) = g'(x)$.

[5 marks]

Question 12a

(a) Use the quotient rule to show that the derivative of $\tan x$ is $\frac{1}{\cos^2 x}$.

[3 marks]

Question 12b

Consider the function f defined by $f(x) = x \tan x,\ -\frac{3\pi}{2} \le x \le \frac{3\pi}{2}$.

(b) Find $f'(x)$.

[2 marks]

Question 12c

(c) Show that

$$f''(x) = \frac{2}{\cos^2 x}(1 + x\tan x)$$

[5 marks]

Question 12d

(d) Using your answers to parts (b) and (c), determine the x-coordinates of any

(i) local minima or maxima

(ii) points of inflection

on the curve $y = f(x)$.

[5 marks]

Question 13a

An international mission has landed a rover on the planet Mars. After landing, the rover deploys a small drone on the surface of the planet, then rolls away to a distance of 6 metres in order to observe the drone as it lifts off into the air. Once the rover has finished moving away, the drone ascends vertically into the air at a constant speed of 2 metres per second.

Let D be the distance, in metres, between the rover and the drone at time t seconds.

Let h be the height, in metres, of the drone above the ground at time t seconds. The entire area where the rover and drone are situated may be assumed to be perfectly horizontal.

a)
Show that

$$D = \sqrt{h^2 + 36}$$

[2 marks]

Question 13b

b)
(i)
Explain why $\frac{\mathrm{d}h}{\mathrm{d}t} = 2$.

(ii)
Hence use implicit differentiation to show that

$$\frac{\mathrm{d}D}{\mathrm{d}t} = \frac{2h}{\sqrt{h^2 + 36}}$$

[5 marks]

Question 13c

c)
Find

(i)
the rate at which the distance between the rover and the drone is increasing at the moment when the drone is 8 metres above the ground.

(ii)
the height of the drone above the ground at the moment when the distance between the rover and the drone is increasing at a rate of $1\ \text{ms}^{-1}$.

[4 marks]