4.3 Probability

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.3 Probability
Difficulty	Very Hard

Time allowed: 130

Score: /105

Percentage: /100

Question 1a

A game is played using a fair spinner with four sectors numbered 1 to 4, as well as a fair eight-sided dice with its sides numbered 1 to 8.

(a) Using an appropriate representation, describe the sample space of possible outcomes when the spinner is spun and the dice is rolled at the same time.

[3 marks]

Question 1b

When the game is played, the spinner is spun and the dice is rolled at the same time, and the player's score is determined as follows:

- if the number on the spinner is higher than the number on the dice, then the score is the sum of the two numbers;
- if the number on the spinner is lower than the number on the dice, then the score is the (positive) difference of the two numbers;
- if the numbers on the spinner and the dice are equal, then the score is the product of the two numbers.

(b) Find the probability of the score in the game being

(i) exactly 7

(ii) 10 or more

(iii) a triangular number (1, 3, 6, 10, 15, 21, ...).

[4 marks]

Question 1c

(c) The game is played 300 times. Find the expected number of times that a triangular number score will occur.

[2 marks]

Question 2a

Leofranc is the membership secretary of an ancient languages enthusiasts' society. He conducted a survey to discover what the main language was that society members had chosen to study while they were stuck at home during coronavirus lockdown. Some of the results of this survey are contained in the following table:

		Main language				
		Akkadian	**Hittite**	**Mycenaean Greek**	**Middle Persian**	**Old Church Slavonic**
Age	**13-17**	5	3	13	2	4
	18-30	9	11	10	15	13
	31-54	16	15	12		12
	55-70	10	10	11	5	5
	>70		5	9	4	3

Unfortunately Leofranc spilled gallic acid on the survey results, so the numbers that belong in the two empty boxes on the table can no longer be read. Leofranc remembers, however, that the number of people who had chosen Middle Persian as their main language was only half the number of those who had chosen Akkadian. Also, the events "had chosen Hittite as their main language" and "was between 18 and 30 years old" were independent.

(a) Use the above information to complete the table.

[7 marks]

Question 2b

(b) The society has a total of 1138 members (not all of whom responded to the survey). Assuming that the survey figures are representative of the society as a whole, estimate the number of members of the society who

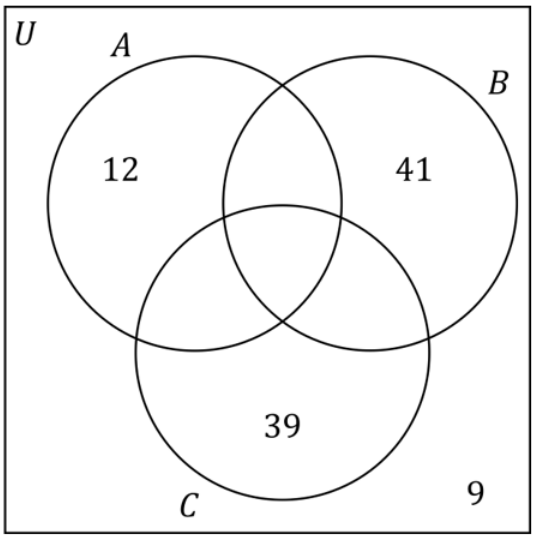
(i) had not chosen Akkadian as their main language during lockdown

(ii) were less than 55 years old and had chosen Hittite or Mycenaean Greek as their main language during lockdown.

[3 marks]

Question 3

120 students went on a school trip to the Thormton Manor theme park. A statistics student has begun filling in the following Venn diagram, showing the numbers of students who went on none, one or more of the park's three most terrifying rides: the Aquaplunge water slide (A), the Barnstormer rollercoaster (B), and the Really Scary Carousel (C).



A student is randomly chosen from the group that went to the theme park.

Given that 'went on Aquaplunge' and 'went on the Really Scary Carousel' were mutually exclusive events, while 'went on Aquaplunge' and 'went on the Barnstormer' were independent events, find the probability that the student:

(i) went on the Really Scary Carousel

(ii) did not go on exactly two of the rides

(iii) went on Aquaplunge, given that they went on the Barnstormer

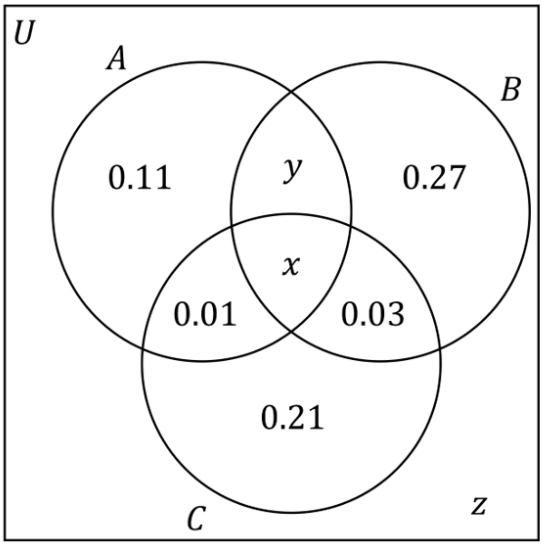
(iv) went on the Barnstormer, given that they went on less than two of the rides

(v) went on the Really Scary Carousel, given that they did not go on Aquaplunge.

[14 marks]

Question 4a

The Venn diagram below shows the probabilities of attendees at a charity pasta dinner having sampled one of the three pasta dishes on offer: alphabetty spaghetti (A), spaghetti Bolognese (B), and linguine carbonara (C).



(a) Given that half the attendees sampled the linguine carbonara, and that 38% of the attendees sampled at least two of the three dishes, determine the values of x, y and z.

[4 marks]

Question 4b

An attendee from the dinner is chosen at random.

(b) Determine the probability that the attendee

(i) had sampled exactly two of the three dishes

(ii) had sampled at least one of the three dishes but not all three of them.

(iii) had not sampled any of the pasta dishes, given that they had sampled less than two of them.

[4 marks]

Question 5

A and B are events such that $\mathrm{P}(A) = 0.58,\ \mathrm{P}(B) = 0.71,$ and $\mathrm{P}((A \cup B)') = 0.27.$

Find:

(i) $\mathrm{P}(A' \cup B)$

(ii) $\mathrm{P}(A' \cap B')$

(iii) $\mathrm{P}(A' \cup B')$

[6 marks]

Question 6

A, B and C are three events such that $\mathrm{P}(A) = 0.2$, $\mathrm{P}(B) < 0.5$, and events B and C are independent. Additionally, $\mathrm{P}(A \cap B) = 0.01$ and $\mathrm{P}(A \cap C) = 0.14$.

Given that $\mathrm{P}(B \cap C') = 0.03$, $\mathrm{P}(B' \cap C) = 0.38$ and $\mathrm{P}(A \cap B \cap C) = \mathrm{P}(A' \cap B \cap C)$, find:

(i) $\mathrm{P}(B \cap C)$

(ii) $\mathrm{P}(C')$

(iii) $\mathrm{P}(A'|C)$

(iv) $\mathrm{P}(A \cup B \cup C|B')$

[10 marks]

Question 7a

A bag contains 10 black tokens and 6 white tokens. A token is drawn from the bag and its colour recorded, and then a fair coin is flipped. If the coin lands on heads then a second token is drawn from the bag without replacing the first token. If the coin lands on tails then the first token is replaced in the bag before a second token is drawn.

(a) Draw a tree diagram to represent this experiment.

[4 marks]

Question 7b

(b) Find the probability that the second token drawn is white.

[3 marks]

Question 7c

(c) Explain why the events "both tokens drawn were the same colour" and "the coin landed on tails" are not independent events.

[3 marks]

Question 8a

The game Undead Redemption is played using three fair dice – a four-sided dice with the sides numbered 1 to 4, a six-sided dice with the sides numbered 1 to 6, and an eight-sided dice with the sides numbered 1 to 8.

In the game your character is battling a zombie. The battle can last between one and three rounds, and it is resolved as follows:

- In the first round, you and the zombie each roll the four-sided dice. If your roll is greater than or equal to the zombie's roll then the zombie is destroyed and the battle is over. Otherwise your character is wounded and the battle goes on to the second round.
- In the second round, you roll the four-sided dice and the zombie rolls the six-sided dice. If your roll is greater than or equal to the zombie's roll then the zombie is destroyed and the battle is over. Otherwise your character is wounded again and the battle goes on to the third round.
- In the third round, you roll the four-sided dice and the zombie rolls the eight-sided dice. If your roll is greater than or equal to the zombie's roll then the zombie is destroyed and the battle is over. Otherwise your character is wounded for the third time and dies.

(a) Draw a tree diagram to represent this information.

[6 marks]

Question 8b

(b) Find the probability that

(i) the zombie is destroyed

(ii) your character dies

(iii) the zombie is destroyed, given that your character is wounded one or more times

[4 marks]

Question 8c

(c) In the context of the question, give an example of two mutually exclusive events. Be sure to justify that they are mutually exclusive.

[2 marks]

Question 9

For two events, A and B, the conditional probability of A occurring given that B has occurred is given by

$$\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$$

If in addition events A and B are independent, then it will also be true that

$$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$$

Given that events C and D are independent, use the results above to show that

(i) $\mathrm{P}(C|D') = \mathrm{P}(C)$

(ii) $\mathrm{P}(D|C') = \mathrm{P}(D)$

[6 marks]

Question 10

Events A and B are independent. Given that $\mathrm{P}(A|B)$ is twice as large as $\mathrm{P}(B|A')$, and also that $\mathrm{P}((A \cup B)') = 0.28$, find $\mathrm{P}(A)$ and $\mathrm{P}(B)$.

[7 marks]

Question 11a

A pharmaceutical company is testing out its “regular” antiseptic cream against a new version it has developed named “Lots-on”.
In trials, 40% of volunteers were given the regular cream whilst the rest were given “Lots-on”.
Of those testing the regular cream on a cut on their finger, 15% reported the cut healed within 2 days; 35% reported the cut took 2 - 4 days to heal and the rest reported it took longer than 4 days.
Of those testing “Lots-on”, 30% reported the cut healed within 2 days; 45% reported the cut took 2 - 4 days to heal and the rest reported it took longer than 4 days.

(a) Draw a tree diagram to represent this information.

[3 marks]

Question 11b

(b) (i) Find the probability that a cut taking between 2 and 4 days to heal had “Lots-on” cream applied to it.

(ii) Find the probability that “Lots-on” cream was used for a cut taking longer than 4 days to heal.

[4 marks]

Question 12

Three independent events, B_1, B_2 and B_3 are such that $\mathrm{P}(B_1) = 0.2$ and $\mathrm{P}(B_2) = a$.
It is also known that for event A, $\mathrm{P}(A'|B_1) = 0.58$, $\mathrm{P}(A|B_2) = 2a$ and $\mathrm{P}(A|B_3) = 0.5a$.
Find the value of a, given that $\mathrm{P}(B_2|A) = \frac{256}{457}$. Fully justify your solution.

[6 marks]