

1.3 Sequences & Series

Question Paper

Course	DPIB Maths
Section	1. Number & Algebra
Topic	1.3 Sequences & Series
Difficulty	Medium

Time allowed: 130
Score: /105
Percentage: /100

Question 1a

The second term, u_2 , of a geometric sequence is 44 and the third term, u_3 , is 55.

(a) Find the common ratio, r , of the sequence.

[2 marks]

Question 1b

(b) Find the first term of the sequence, u_1 .

[2 marks]

Question 1c

(c) Find S_5 , the sum of the first 5 terms of the sequence.

[2 marks]

Question 2a

The sum of the first 16 terms of an arithmetic sequence is 920.

(a) Find the common difference, d , of the sequence if the first term is 27.5.

[3 marks]

Question 2b

(b) Find the first term of the sequence if the common difference, d , is 11.

[3 marks]

Question 3a

The sum of the first 5 terms of a geometric sequence is 461.12.

(a) Find the common ratio, r , of the sequence if the first term is 200, given that $r > 0$.

[3 marks]

Question 3b

- (b) Find the first term of the sequence if the common ratio, r , is -2 .
Give your answer correct to 2 decimal places.

[3 marks]

Question 4a

The table below shows information about the terms of four different sequences a_n , b_n , c_n and d_n .

	$n = 1$	$n = 2$	$n = 3$	$n = 4$
a_n		12	30	
b_n		12	30	
c_n	80			10
d_n	80			10

- (a) Calculate a_1 , a_4 and the common difference, d , given that a_n is an arithmetic sequence.

[2 marks]

Question 4b

- (b) Calculate b_1 , b_4 and the common ratio, r , given that b_n is a geometric sequence.

[2 marks]

Question 4c

(c) Calculate c_2, c_3 and the common difference, d , given that c_n is an arithmetic sequence.

[2 marks]

Question 4d

(d) Calculate d_2, d_3 and the common ratio, r , given that d_n is a geometric sequence.

[2 marks]

Question 5a

Students are arranged for a graduation photograph in rows which follows an arithmetic sequence. There are 20 students in the fourth row and 44 in the 10th row.

- (a) (i) Find the common difference, d , of the arithmetic sequence.
- (ii) Find the first term of the arithmetic sequence.

[3 marks]

Question 5b

(b) Given there are 20 rows of students in the photograph, calculate how many students there are altogether

[3 marks]

Question 6a

Marie is an athlete returning to running after an injury and wants to manage the number of kilometres she runs per week. She decides to run 4 km the first week and increase this by 1.5 km each week.

(a) Find the distance Marie ran in the 10th week.

[2 marks]

Question 6b

(b) Find the week in which Marie runs 26.5 km.

[3 marks]

Question 6c

Marie's coach says she can start preparing for her next race once she has run a total of 220 km.

(c) Find the week in which Marie will complete this.

[3 marks]

Question 7a

The eighth term, u_8 , of an arithmetic sequence is 18 and the common difference, d , is 2.

(a) (i) Find the first term of the arithmetic sequence.

(ii) Find the value of u_{17} .

[4 marks]

Question 7b

The first and 17th terms of the arithmetic sequence are the third and fifth terms respectively of a geometric sequence.

- (b) (i) Find the possible values for the common ratio, r , of the geometric sequence.
- (ii) Find the first term of the geometric sequence.

[4 marks]

Question 8a

In a geometric sequence, $u_3 = 160$ and the common ratio, r , is $\frac{1}{4}$.

- (a) (i) Find the first term, u_1 .
- (ii) Find u_6 .

[4 marks]

Question 8b

(b) Find the value of the infinite sum of the sequence.

[2 marks]

Question 8c

The first and third terms of the geometric sequence are the seventh and ninth terms respectively of an arithmetic sequence.

(c) (i) Find the common difference, d , of the arithmetic sequence.

(ii) Find the first term of the arithmetic sequence.

[4 marks]

Question 9a

A sequence can be defined by $a_n = 32 - 7n$, for $n \in \mathbb{Z}^+$.

- (a) Write an expression for $a_1 + a_2 + a_3 + \dots + a_{12}$ using sigma notation and find the value of the sum.

[3 marks]

Question 9b

- (b) Write an expression for $a_4 + a_5 + a_6 + \dots + a_{15}$ using sigma notation and find the value of the sum.

[3 marks]

Question 10a

A sequence can be defined by $g_n = 4 \times 3^{n-1}$, for $n \in \mathbb{Z}^+$.

- (a) Write an expression for $g_1 + g_2 + g_3 + \dots + g_{10}$ using sigma notation and find the value of the sum.

[3 marks]

Question 10b

- (b) Write an expression for $g_8 + g_9 + g_{10} + \dots + g_{18}$ using sigma notation and find the value of the sum.

[3 marks]

Question 11a

The kiwi is a flightless bird and is a national treasure in New Zealand. At the start of 2021 there were approximately 68 000 kiwi left, with the population decreasing by 2% every year.

- (a) Find the expected population size of kiwis in 2030 assuming the rate of decrease in kiwi population remains the same.

[3 marks]

Question 11b

- (b) Find the year in which the population of kiwis falls below 50 000 assuming the rate of decrease in kiwi population remains the same.

[3 marks]

Question 12a

Aaron is working on his cycling in preparation for a triathlon event in 10 months. He cycles a total of 240 km in the first month and plans to increase this by 12.5% each month.

(a) Find the distance Aaron cycles in the fifth month of preparation.

[3 marks]

Question 12b

(b) Calculate the total distance Aaron cycles until the triathlon.

[3 marks]

Question 13a

A geometric sequence has $u_1 = 0.5$ and $r = 3$.

(a) Find

(i) u_4

(ii) S_5 .

[2 marks]

Question 13b

An arithmetic sequence has the same u_4 and S_5 as the geometric sequence above.

(b) Find u_1 and d for the arithmetic sequence.

[4 marks]

Question 14a

Daniel and Jonah have each been given \$5000 to save for university.

Daniel invests his money in an account that pays a nominal annual interest rate of 2.24%, **compounded quarterly**.

- (a) Calculate the amount Daniel will have in his account after 8 years.
Give your answer to 2 decimal places.

[3 marks]

Question 14b

Jonah wants to invest his money in an account such that his investment will double in 10 years. Assume the account pays a nominal annual interest of $r\%$, **compounded half-yearly**.

- (b) Determine the value of r .

[3 marks]

Question 15a

On his 40th birthday, Robert invests \$15 000 into a savings account that pays a nominal annual interest rate of 4.78%, **compounded monthly**.

- (a) (i) Write an expression for the total value of the investment after n years.
Give your answer to 2 decimal places.
- (ii) Find the total amount in the savings account after 3 and 5 years.

[3 marks]

Question 15b

- (b) Find the age Robert will be when the amount of money in his account will be 1.5 times the initial amount.

[2 marks]

Question 16

The sum of the first two terms of a geometric sequence is 15.3 and the sum of the infinite geometric sequence is 30. Find the positive value of the common ratio, r .

[6 marks]

