1.5 Complex Numbers

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.5 Complex Numbers
Difficulty	Hard

Time allowed: 110

Score: /86

Percentage: /100

Question 1a

Consider the quadratic equation $z^2 - 8z + 25 = 0,\ z \in \mathbb{C}$.

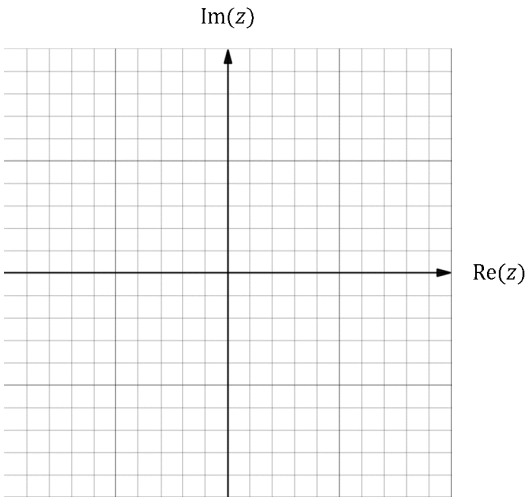
The roots of the equation are $z_1 = a + b\mathrm{i}$ and $z_2 = a - bi$ where $a, b \in \mathbb{Z}$.

(a)
Find the value of a and b.

[3 marks]

Question 1b

(b)
Sketch z_1, z_2, $z_1 + z_2$ and $z_1 - z_2$ on the Argand diagram below, be sure to include an appropriate scale.



[4 marks]

Question 2

Consider the complex numbers $z_1 = -3 + 2\mathrm{i}$ and $z_2 = 1 - 3\mathrm{i}$.

Find

(i)
$z_1 + z_2$
(ii)
$z_1 - z_2$
(iii)
$z_1 z_2$
(iv)
$\frac{z_1}{z_2}$

[7 marks]

Question 3

Consider the complex numbers $z_1 = 3 - \mathrm{i}$ and $z_2 = -2 - 3\mathrm{i}$.

Find the modulus and argument of $z_1 z_2^*$.

[7 marks]

Question 4a

Consider the complex numbers $z_1 = 1 - 2\mathrm{i}$ and $z_3 = -3 + 5\mathrm{i}$.

(a)

Work out the following:

(i)

$\mathrm{Re}(z_2 - z_1)$

(ii)

$\mathrm{Im}(z_1 z_2)$

(iii)

$\left(\frac{z_1}{z_2}\right)^*$

For part (iii) give your answer in the form $a + b\mathrm{i}$, where a and b are real numbers.

[6 marks]

Question 4b

(b)
Write down the complex conjugate of z_2 and describe the geometrical relationship between z_2 and z_2^*.

[2 marks]

Question 5

Find all possible real values for a and b such that

(i)
$(a + b\mathrm{i})(2 - 3\mathrm{i}) = 8 + \mathrm{i}$
(ii)
$a(2 + b\mathrm{i}) = b(-6 + \mathrm{i})$
(iii)
$(2a + 3\mathrm{i})(3 + b\mathrm{i}) = 12 + 21\mathrm{i}$

[7 marks]

Question 6a

(a)

For a general complex number $z = x + \mathrm{i}y$, where $x, y \in \mathbb{R}$, show that

(i)

$$\mathrm{Re}(z) = \frac{z + z^*}{2\mathrm{i}}$$

(ii)

$$\mathrm{Im}(z) = \frac{z - z^*}{2\mathrm{i}}$$

[3 marks]

Question 6b

(b)

For the complex numbers $z_1 = a_1 + b_1\mathrm{i}$ and $z_2 = a_2 + b_2\mathrm{i}$, where $a_1, a_2, b_1, b_2 \in \mathbb{R}$, show that

$$|z_1 z_2| = |z_1||z_2|$$

[6 marks]

Question 7

Consider the complex numbers $w = 2\mathrm{i}z$ and $w - z = 5 - 5\mathrm{i}$.

Find

(i)

$|z|$

(ii)

$\arg w$

(iii)

$\mathrm{Re}(z + w)$

(iv)

$\mathrm{Im}(z - w)$

[8 marks]

Question 8a

Consider the complex numbers $z_1 = a - 6\mathrm{i}$, $z_2 = 1 + b\mathrm{i}$ and $z_1 z_2 = -17 - 9\mathrm{i}$ where $a, b \in \mathbb{R}$.

(a)
Find the possible values of a and b.

[4 marks]

Question 8b

(b)

Using the answers gained in part (a), write down values for c and d that will satisfy the equation

$$-(3+\mathrm{i})(c+d\mathrm{i}) = 17-9\mathrm{i}$$

[2 marks]

Question 9a

Consider the complex numbers $z = 3+5\mathrm{i}$ and $w = -2+3\mathrm{i}$.

(a)

Represent the complex numbers z and w on an Argand diagram.

[2 marks]

Question 9b

The points $z+w$ and $z-w$ are represented by the points A and B on the Argand diagram respectively.

(b) Find the angle $\mathrm{A\widehat{O}B}$.

[5 marks]

Question 10a

Consider the complex numbers $z = -4 - 3\mathrm{i}$, $w = ai$ and $\frac{z}{w} = b + 2a\mathrm{i}$, where $a, b \in \mathbb{R}$.

(a)
Find the possible values of a and b.

[4 marks]

Question 10b

(b)

Find the modulus of $\frac{w}{z}$.

[3 marks]

Question 11a

Let $\omega_1 = 3 - \mathrm{i}$ and $\omega_2 = 1 + 2\mathrm{i}$.

(a)

Given that $\frac{1}{\omega_1} + \frac{1}{\omega_2} = \frac{1}{z}$, express z in the form $a + b\mathrm{i}$, where $a, b \in \mathbb{R}$.

[4 marks]

Question 11b

(b)

Find $\omega_1 \omega_2 z^*$, giving your answer in the form $a + b\mathrm{i}$, where $a, b \in \mathbb{R}$.

[2 marks]

Question 12a

Consider the complex number $z = \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}\mathrm{i}$.

(a)

Use technology to find the values of z^2 and z^3. Give your answers in the form $a + b\mathrm{i}$, where $a, b \in \mathbb{R}$.

[2 marks]

Question 12b

b)
Draw z, z^2 and z^3 on an Argand diagram.

[3 marks]

Question 12c

c)
Find the smallest integer $k > 3$ such that z^k is a real number.

[2 marks]