5.3 Integration

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.3 Integration
Difficulty	Medium

Time allowed: 60

Score: /46

Percentage: /100

Question 1a

(a) Show that

$$(3-2x)^2 = 9 - 12x + 4x^2$$

[2 marks]

Question 1b

(b) Hence, or otherwise, find the indefinite integral for the following:

$$\int (3-2x)^2 \, \mathrm{d}x$$

[2 marks]

Question 2

Given

$$\int_k^5 (2x-1) \, \mathrm{d}x = 20$$

find the value of the positive constant k.

[4 marks]

Question 3a

A curve $y = f(x)$ passes through point $A(4, 2)$ and has a gradient of $f'(x) = 5x - 2$.

(a) Find the gradient of the curve at point A.

[2 marks]

Question 3b

(b) Find the equation of the tangent to the curve at point A.
Give your answer in the form $y = mx + c$.

[2 marks]

Question 3c

(c) Determine the equation of the curve $y = f(x)$.

[3 marks]

Question 4a

A point $P(3, 8)$ lies on the curve $y = f(x)$ that has a gradient of $f'(x) = -2x^2 + 11$.

(a) Find the gradient of the curve at point P.

[2 marks]

Question 4b

(b) Find the equation of the tangent to the curve at point P.
Give your answer in the form $y = mx + c$.

[2 marks]

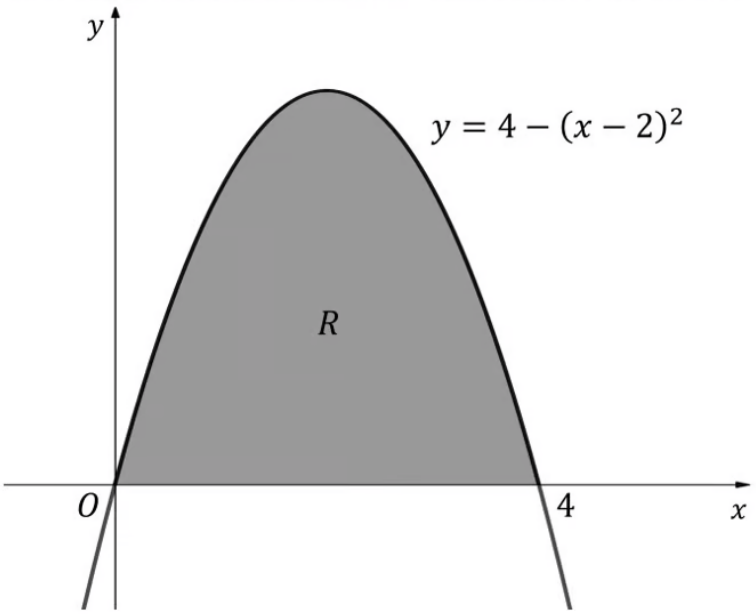
Question 4c

(c) Determine the equation of the curve $y = f(x)$.

[3 marks]

Question 5a

The diagram below shows part of the graph of $y = 4 - (x - 2)^2$.



(a) Write down the values of x where $y = 0$.

[1 mark]

Question 5b

(b) Show that

$$4 - (x - 2)^2 = 4x - x^2$$

[1 mark]

Question 5c

(c) Evaluate

$$\int_0^4 (4x - x^2)\, dx$$

[2 marks]

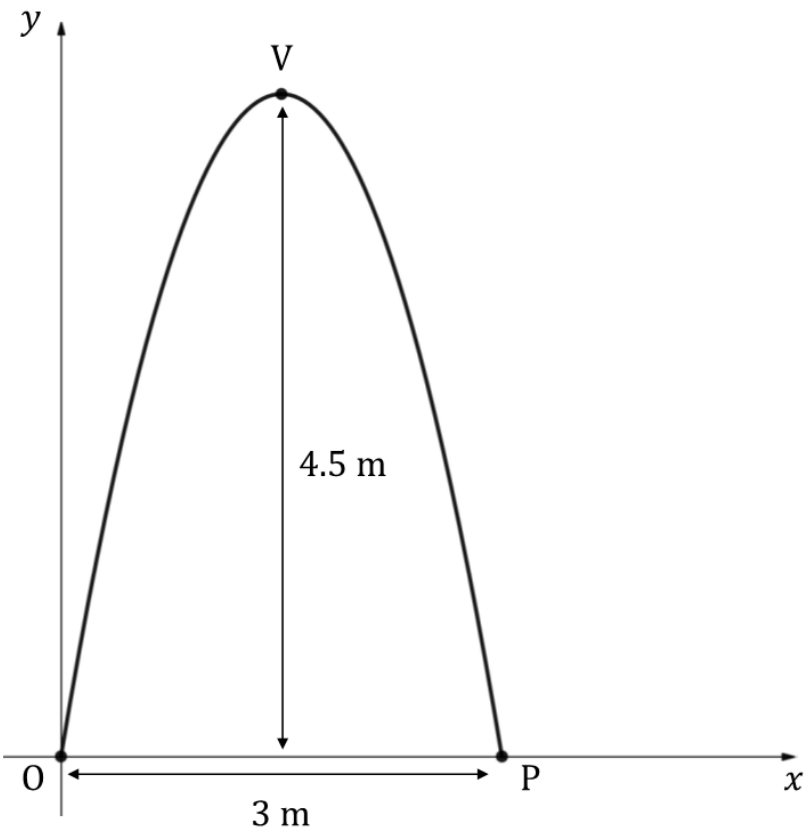
Question 5d

(d) Write down the area of the region labelled R.

[1 mark]

Question 6a

The following diagram shows an arch that is 4.5 m tall and 3 m wide. The arch crosses the x-axis at the origin, O, and at point P, and its vertex is at point V. The arch may be represented by a curve with an equation of the form $y = x(ax + 6)$, where all units are measured in metres.



(a) Find

(i) the coordinates of P

(ii) the coordinates of V

(iii) the value of a.

[4 marks]

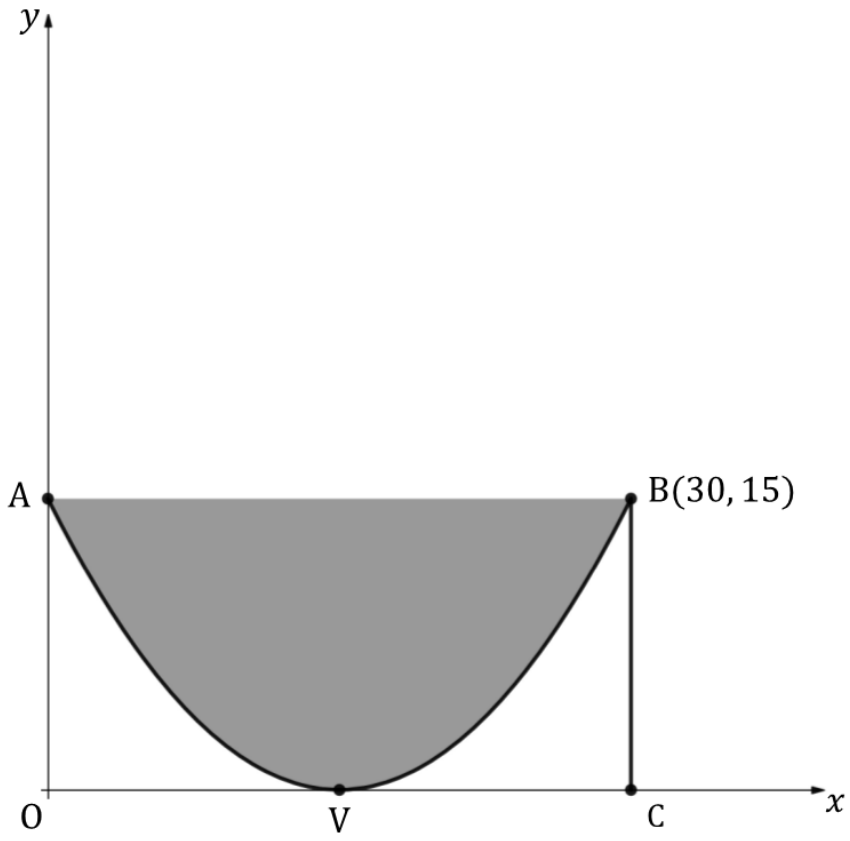
Question 6b

(b) Find the cross-sectional area under the arch.

[2 marks]

Question 7a

A trough has a cross-sectional area shown by the shaded region of the diagram below, where the x and y values are in centimetres. The curved bottom of the trough has an equation in the form $y = r(x - 15)^2$. Point O is the origin, and points O, A, B and C are the vertices of a rectangle. Point V, the deepest point of the trough, is situated on the x-axis.



(a) Determine the value of r.

[2 marks]

Question 7b

(b) Find the cross-sectional area of the trough.

[4 marks]

Question 7c

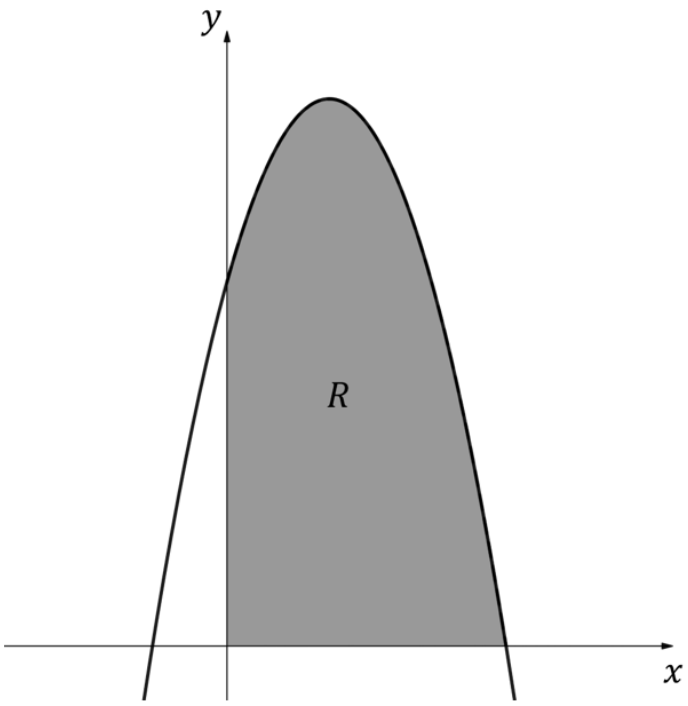
The length of the trough is 1.2 m.

(c) Find the volume of the trough. Give your answer in cm^3.

[2 marks]

Question 8a

The following diagram shows part of the graph of $f(x) = (5 - 2x)(2 + 3x),\ x \in \mathbb{R}$. The shaded region R is bounded by the x-axis, the y-axis and the graph of f.



(a) Write down an integral for the area of region R.

[2 marks]

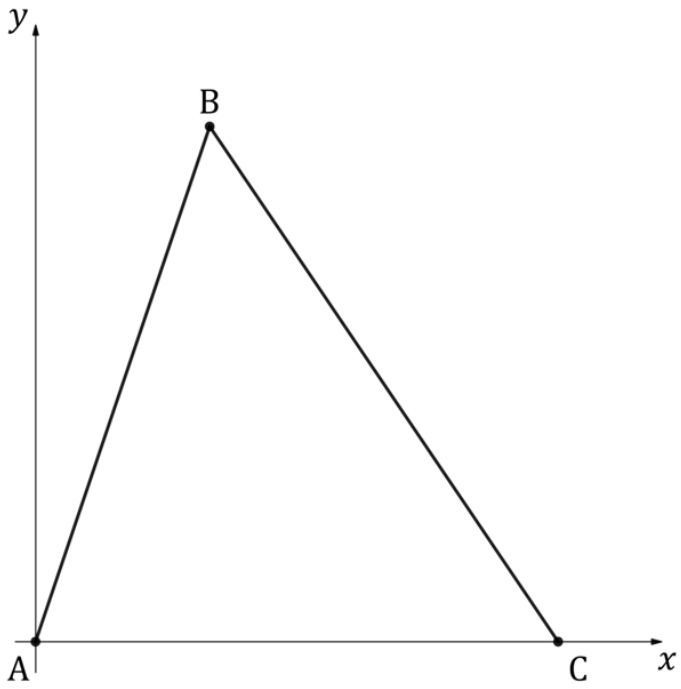
Question 8b

(b) Find the area of region R.

[1 mark]

Question 8c

The three points $A(0, 0)$, $B(4, h)$ and $C(9, 0)$ define the vertices of a triangle.



(c) Find the value of h, the y-coordinate of B, given that the area of the triangle is equal to the area of region R.

[2 marks]