IB Physics DP

YOUR NOTES

12. Quantum & Nuclear Physics (HL only)

CONTENTS

12.1 The Interaction of Matter with Radiation

12.1.1 Photons

YOUR NOTES

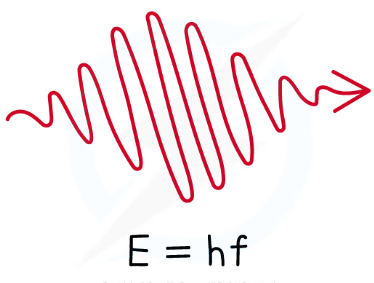
Photons

- In classical wave theory, electromagnetic (EM) radiation is assumed to behave as a **wave**
  - This is demonstrated by the fact EM radiation exhibits phenomena such as diffraction and interference
- However, experiments from the last century, such as discovering the photoelectric effect and atomic line spectra, can only be explained if the EM radiation is thought of as behaving as **particles**
- These experiments have formed the basis of **quantum theory**, which will be explored in detail in this section

Photons

- Photons are fundamental particles which make up all forms of electromagnetic radiation
- A photon is defined as

A massless "packet" or a "quantum" of electromagnetic energy



A wave packet (photon)

- It is also defined as:

A massless particle that possesses an energy equal to *E* = *hf*

- This means is that the energy of a photon is not transferred continuously, but as discrete packets of energy
- In other words, each photon carries a specific amount of energy, and transfers this energy all in one go, rather than supplying a consistent amount of energy
- The energy of a photon can be calculated using the formula:

$$E = hf$$

- Substituting in from the wave equation, energy can also be equal to:

$$E = \frac{hc}{\lambda}$$

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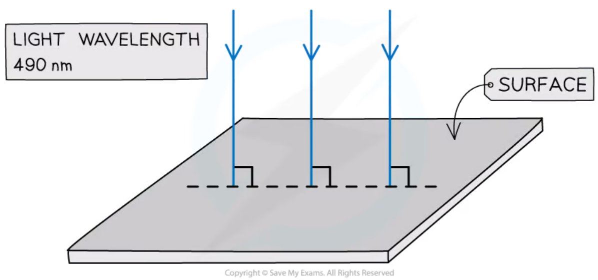
- Where:
  - E = energy of the photon (J)
  - h = Planck's constant (J s)
  - c = the speed of light (m s^{-1})
  - f = frequency in Hertz (Hz)
  - λ = wavelength (m)
- This equation tells us:
  - The higher the frequency of EM radiation, the higher the energy of the photon
  - The energy of a photon is inversely proportional to the wavelength
  - A long-wavelength photon of light has a lower energy than a shorter-wavelength photon

Worked Example

Light of wavelength 490 nm is incident normally on a surface, as shown in the diagram.



The power of the light is 3.6 mW. The light is completely absorbed by the surface.

Calculate the number of photons incident on the surface in 2.0 s.

Step 1: Write down the known quantities

- Wavelength, λ = 490 nm = 490 × 10^{-9} m
- Power, P = 3.6 mW = 3.6 × 10^{-3} W
- Time, t = 2.0 s

Step 2: Write the equations for wave speed and photon energy

$$\textit{wave speed}: \quad c = f\lambda \rightarrow f = \frac{c}{\lambda}$$

$$\textit{photon energy}: \quad E = hf \rightarrow E = \frac{hc}{\lambda}$$

Step 3: Calculate the energy of one photon

$$\mathrm{E} = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}) \times (3.00 \times 10^{8})}{490 \times 10^{-9}} = 4.06 \times 10^{-19}\ \mathrm{J}$$

Step 4: Calculate the number of photons hitting the surface every second

$$\frac{\text{power of light source}}{\text{energy of one photon}} = \frac{3.6 \times 10^{-3}}{4.06 \times 10^{-19}} = 8.9 \times 10^{15}\ s^{-1}$$

Step 5: Calculate the number of photons that hit the surface in 2 s

$$(8.9 \times 10^{15} \times 2) = 1.8 \times 10^{16}$$

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Photon Momentum

- Einstein showed that a photon travelling in a vacuum has momentum, despite it having no mass
- The momentum (p) of a photon is related to its energy (E) by the equation:

$$p = \frac{E}{c}$$

- Where
  - c = the speed of light
  - p = the momentum of the photon (kg m s^{-1})
  - E = the energy of the photon (J)

Worked Example

A 5.0 mW laser beam is incident normally on a fixed metal plate. The cross-sectional area of the beam is 8.0×10^{-6} m^2. The light from the laser has frequency 5.6×10^{14} Hz.

Assuming that all the photons are absorbed by the plate, calculate the momentum of the photon, and the pressure exerted by the laser beam on the metal plate.

Step 1: Write down the known quantities

- Power, $P = 5.0$ mW $= 5.0 \times 10^{-3}$ W
- Frequency, $f = 5.6 \times 10^{14}$ Hz
- Cross-sectional area, $A = 8.0 \times 10^{-6}$ m^2

Step 2: Write the equations for photon energy and momentum

$$\text{photon energy}: E = hf$$

$$\text{photon momentum}: p = \frac{E}{c} = \frac{hf}{c}$$

Step 3: Calculate the photon momentum

$$\text{p} = \frac{hf}{c} = \frac{(6.63 \times 10^{-34}) \times (5.6 \times 10^{14})}{3.00 \times 10^{8}} = 1.24 \times 10^{-27}\ N\,s$$

Step 4: Calculate the number of photons incident on the plate every second

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$$\frac{\text{power of light source}}{\text{energy of one photon}} = \frac{5.0 \times 10^{-3}}{hf}$$

$$= \frac{5.0 \times 10^{-3}}{(6.63 \times 10^{-34}) \times (5.6 \times 10^{14})} = 1.35 \times 10^{16}\, s^{-1}$$

Step 5: Calculate the force exerted on the plate in a 1.0 s time interval

$$force = rate\ of\ change\ of\ momentum$$

$$\rightarrow force = number\ of\ photons\ per\ second \times momentum\ of\ each\ photon$$

$$F = (1.35 \times 10^{16}) \times (1.24 \times 10^{-27}) = 1.67 \times 10^{-11}\, N$$

Step 6: Calculate the pressure

$$pressure = \frac{force}{area} = \frac{1.67 \times 10^{-11}}{8.0 \times 10^{-6}} = 2.1 \times 10^{-6}\, Pa$$

Exam Tip

Make sure you learn the definition for a photon: *discrete quantity / packet / quantum of electromagnetic energy* are all acceptable definitions

The values of Planck's constant and the speed of light will always be given to you in an exam, however, it helps to memorise them to speed up calculation questions!

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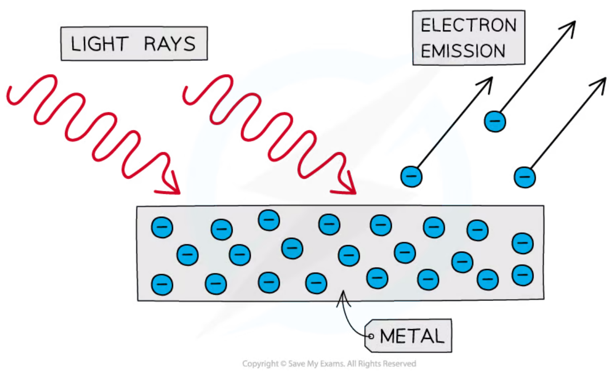
The Photoelectric Effect

- The **photoelectric effect** is the phenomena in which electrons are emitted from the surface of a metal **upon the absorption of electromagnetic radiation**
- Electrons removed from a metal in this manner are known as **photoelectrons**
- The photoelectric effect provides important evidence that light is quantised, or carried in discrete packets
  - This is shown by the fact each electron can absorb only a single photon
  - This means only the frequencies of light above a **threshold frequency** will emit a photoelectron



Photoelectrons are emitted from the surface of metal when light shines onto it

Threshold Frequency & Wavelength

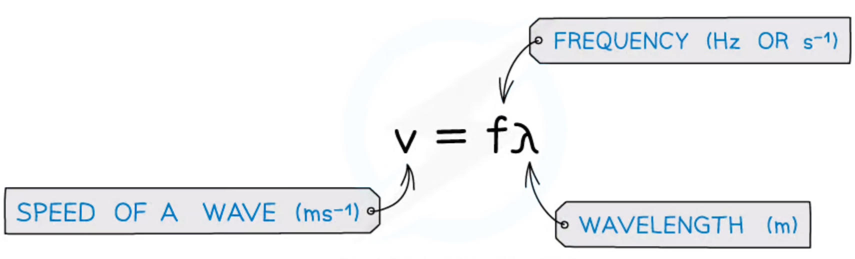
- The **threshold frequency** is defined as:

  The minimum frequency of incident electromagnetic radiation required to remove a photoelectron from the surface of a metal

- The **threshold wavelength**, related to threshold frequency by the wave equation, is defined as:

  The longest wavelength of incident electromagnetic radiation that would remove a photoelectron from the surface of a metal

- The frequency and wavelength are related by the equation



- Since photons are particles of light, $v = c$ (speed of light)

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- Threshold frequency and wavelength are properties of a material and vary from metal to metal

Threshold frequencies and wavelengths for different metals

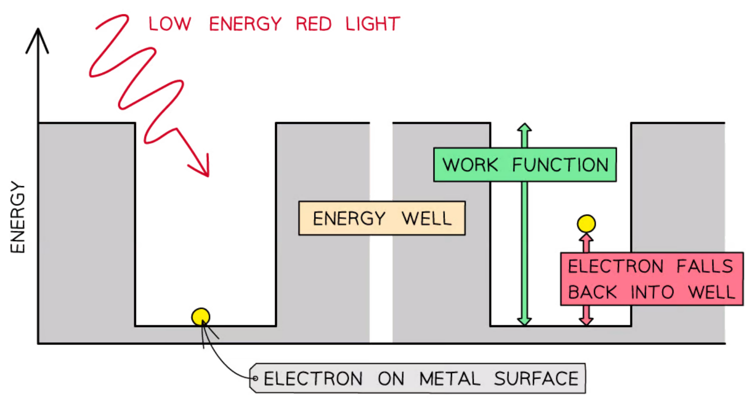
Metal	Threshold Frequency (f_0) / Hz	Threshold Wavelength (λ_0) / nm
Sodium	4.40×10^{14}	682
Potassium	5.56×10^{14}	540
Zinc	1.02×10^{15}	294
Iron	1.04×10^{15}	289
Copper	1.13×10^{15}	266
Gold	1.23×10^{15}	244
Silver	9.71×10^{15}	30.9

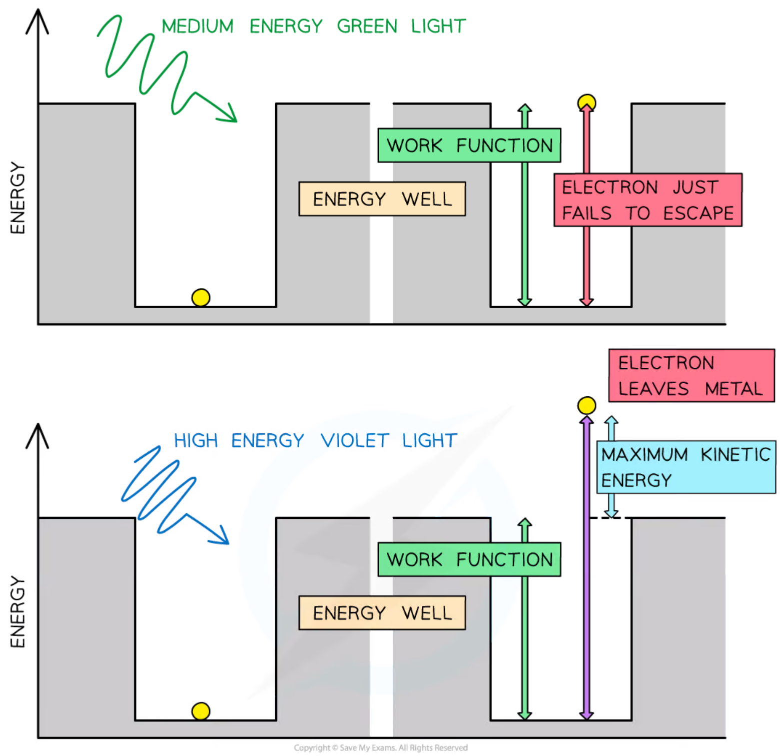
The Work Function

- The work function Φ, or threshold energy, of a material, is defined as:

The minimum energy required to release a photoelectron from the surface of a metal

- Consider the electrons in a metal as trapped inside an 'energy well' where the energy between the surface and the top of the well is equal to the work function Φ
- A single electron absorbs one photon
- Therefore, an electron can only escape from the surface of the metal if it absorbs a photon which has an energy equal to Φ or higher





In the photoelectric effect, a single photon may cause a surface electron to be released if it has sufficient energy

- Different metals have different threshold frequencies and hence different work functions
- Using the well analogy:
  - A more tightly bound electron requires **more** energy to reach the top of the well
  - A less tightly bound electron requires **less** energy to reach the top of the well
- Alkali metals, such as sodium and potassium, have threshold frequencies in the **visible light region**
  - This is because the attractive forces between the surface electrons and positive metal ions are relatively weak
- Transition metals, such as zinc and iron, have threshold frequencies in the **ultraviolet region**
  - This is because the attractive forces between the surface electrons and positive metal ions are much stronger

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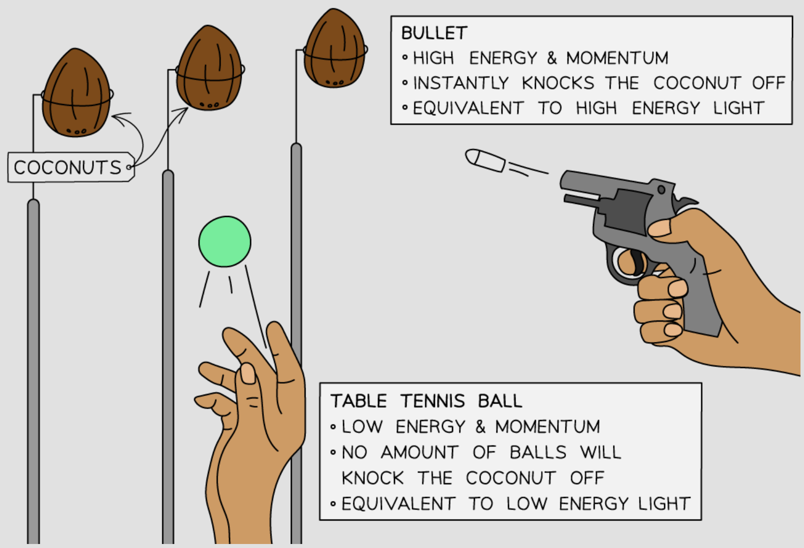
Exam Tip

A useful analogy for threshold frequency is a fairground coconut shy:

- One person is throwing table tennis balls at the coconuts, and another person has a pistol
- No matter how many of the table tennis balls are thrown at the coconut it will still stay firmly in place – this represents the **low frequency quanta**
- However, a single shot from the pistol will knock off the coconut immediately – this represents the **high frequency quanta**



12.1.3 Observing the Photoelectric Effect

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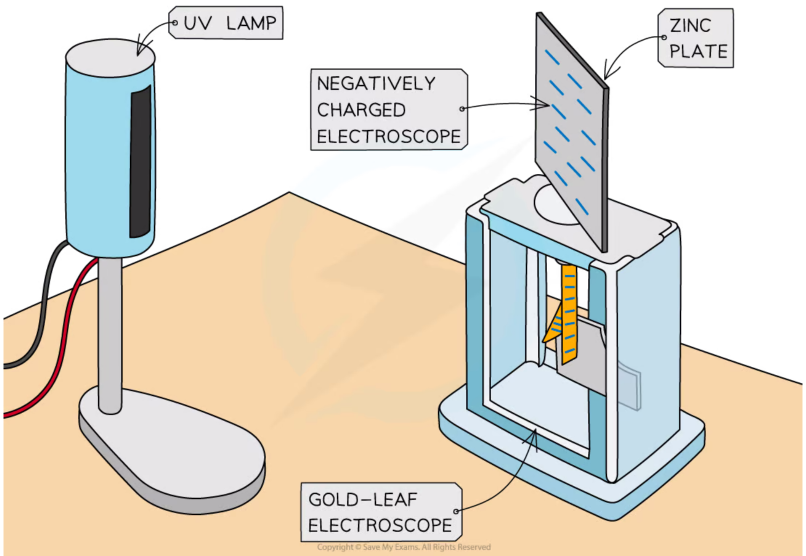
Observing the Photoelectric Effect

- The photoelectric effect can be observed on a **gold leaf electroscope**
- A plate of metal, usually **zinc**, is attached to a gold leaf, which initially has a negative charge, causing it to be repelled by a central negatively charged rod
  - This causes negative charge, or electrons, to build up on the zinc plate
- **UV light** is shone onto the metal plate, leading to the **emission** of photoelectrons
- This causes the extra electrons on the central rod and gold leaf to be removed, so, the gold leaf begins to fall back towards the central rod
  - This is because they become less negatively charged, and hence repel less
- Some notable observations:
  - Placing the UV light source closer to the metal plate causes the gold leaf to fall more quickly
  - Using a higher frequency light source does not change the how quickly the gold leaf falls
  - Using a filament light source causes no change in the gold leaf's position
  - Using a positively charged plate also causes no change in the gold leaf's position



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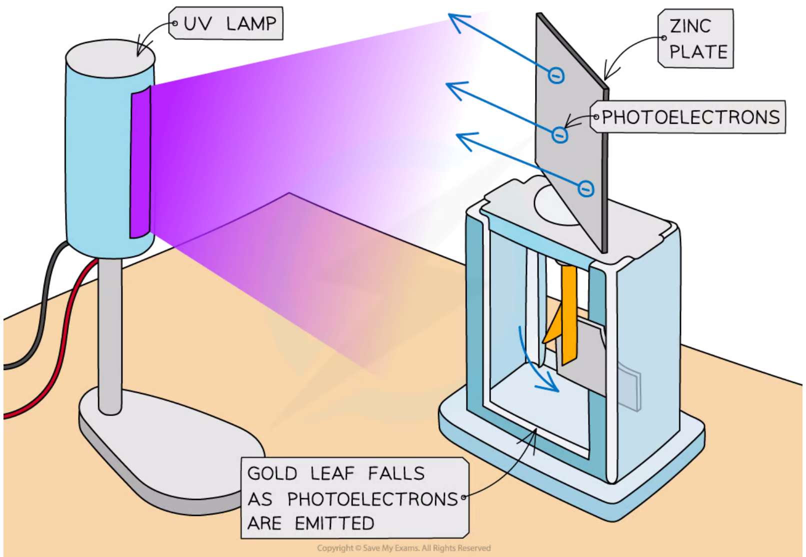

Typical set-up of the gold leaf electroscope experiment

Laws of Photoelectric Emission

- The photoelectric effect is evidence that light does not purely behave as a wave
- This is demonstrated by the following observations

- Observation 1:

  Placing the UV light source closer to the metal plate causes the gold leaf to fall more quickly

- Explanation 1:
  - Placing the UV source closer to the plate increases the intensity incident on the surface of the metal
  - Increasing the intensity, or brightness, of the incident radiation increases the number of photoelectrons emitted per second
  - Therefore, the gold leaf loses negative charge more rapidly

- Observation 2:

  Using a higher frequency light source does not change how quickly the gold leaf falls

- Explanation 2:
  - The maximum kinetic energy of the emitted electrons increases with the frequency of the incident radiation
  - In the case of the photoelectric effect, energy and frequency are **independent** of the intensity of the radiation
  - So, the intensity of the incident radiation affects how quickly the gold leaf falls, not the frequency

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- Observation 3:

  Using a filament light source causes no change in the gold leaf's position

- Explanation 3:
  - If the incident frequency is below a certain threshold frequency, no electrons are emitted, no matter the intensity of the radiation
  - A filament light source has a frequency below the threshold frequency of the metal, so, no photoelectrons are released

- Observation 4:

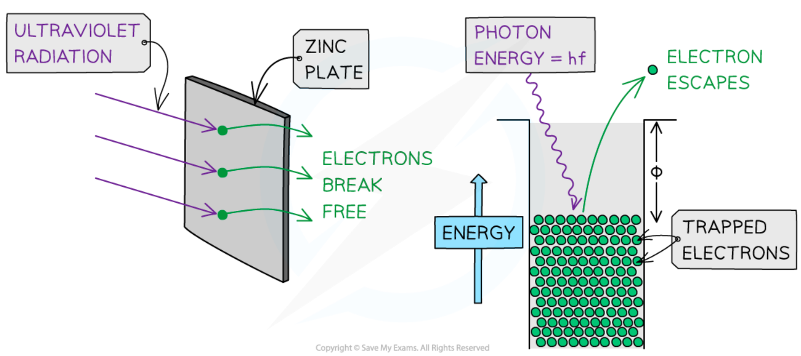
  Using a positively charged plate causes no change in the gold leaf's position

- Explanation 4:
  - If the plate is positively charged, that means there is an excess of positive charge on the surface of the metal plate
  - Electrons are negatively charged, so they will not be emitted unless they are on the surface of the metal
  - Any electrons emitted will be attracted back by positive charges on the surface of the metal

- Observation 5:

  Emission of photoelectrons happens as soon as the radiation is incident on the surface of the metal

- Explanation 5:
  - A single photon interacts with a single electron
  - If the energy of the photon is equal to the work function of the metal, photoelectrons will be released instantaneously



In the photoelectric effect, a single photon may cause a surface electron to be released if it has sufficient energy

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Worked Example

Describe how the photoelectric effect proves the particulate nature of light.

Step 1: Outline what wave theory predicts about the photoelectric effect

- Wave theory predicts:
  - If radiation strikes a metal surface and ejects an electron, the kinetic energy of the electron should depend on the intensity of the incident wave

Step 2: Outline an observation of the photoelectric effect experiment

- From observation:
  - There is no electron emitted if the frequency of the radiation is below a certain threshold frequency
  - The intensity of the radiation does not determine the energy of the emitted electron
  - Above the threshold frequency, the maximum kinetic energy of the electrons increases with the frequency of the radiation

Step 3: Suggest how this observation supports the particulate nature of light

- This suggests:
  - In its interaction with matter, an electromagnetic wave behaves like a stream of particles, or photons
  - These photons carry energy which is proportional to the frequency of the radiation

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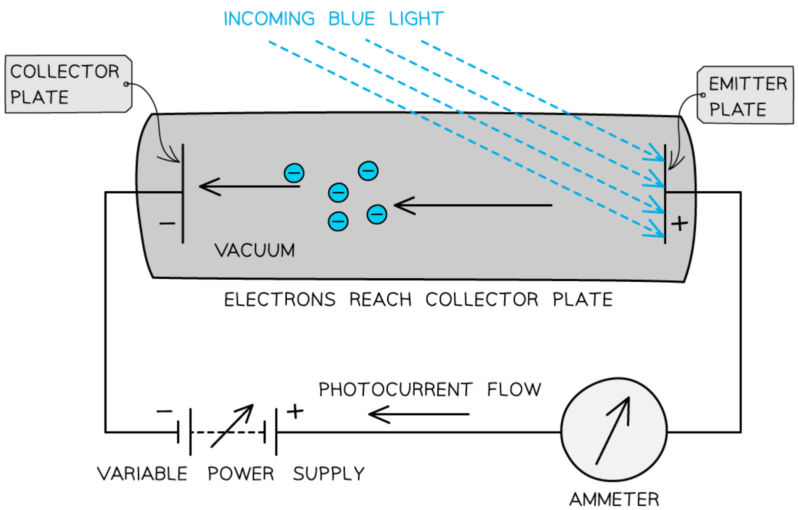
Stopping Voltage

- Stopping voltage, V_s, is defined as:

  The voltage required to stop photoelectron emission from occurring

- The photons arriving at the metal plate cause photoelectrons to be emitted
  - This is called the **emitter plate**
- The electrons that cross the gap are collected at the other metal plate
  - This is called the **collector plate**



This set-up can be used to determine the maximum kinetic energy of the emitted photoelectrons

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- The flow of electrons across the gap results in an e.m.f. between the plates that causes a current to flow around the rest of the circuit
  - Effectively, it becomes a photoelectric cell producing a **photoelectric current**
- If the e.m.f. of the variable power supply is initially zero, the circuit operates **only** on the photoelectric current
- As the supply is turned up, the emitter plate becomes more **positive** (because it is connected to the positive terminal of the supply)
- As a result, electrons leaving the emitter plate are attracted back towards it
  - This is because the p.d. across the tube **opposes** the motion of the electrons between the plates
- If any electrons escape with enough kinetic energy, they can overcome this attraction and cross to the collector plate
  - And if they don't have enough energy, they can't cross the gap
- By increasing the e.m.f. of the supply, eventually, a p.d. will be reached at which no electrons are able to cross the gap - this is the stopping voltage, V_s
- At this point, the energy needed to cross the gap is equal to the maximum kinetic energy KE_{max} of the electrons

$$KE_{max} = eV_S$$

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Exam Tip

The observations and explanations of the photoelectric effect are key findings in Physics, which led to a whole new branch of discovery. As such, they are favourites with Examiners. Make sure you have them at your fingertips!

It is important to note that the stopping voltage actually holds a **negative value**, but since we use it to determine the maximum kinetic energy of the emitted electrons, its sign is not important in calculations, it's acceptable to just quote its magnitude.

12.1.4 Solving Photoelectric Problems

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Solving Photoelectric Problems

- Since energy is always conserved, the energy of an incident photon is equal to:

The work function + the maximum kinetic energy of the photoelectron

- The energy within a photon is equal to ***hf***
- The fraction of the energy transferred to the electron to release it from a material is called the work function, and the remaining amount is given as kinetic energy to the emitted photoelectron

- This equation is known as the **photoelectric equation**:

$$E = hf = \varphi + \frac{1}{2}mv^2_{max}$$

- Where:
  - h = Planck's constant (J s)
  - f = the frequency of the incident radiation (Hz)
  - Φ = the work function of the material (J)
  - $½ mv^2_{max} = E_{k(max)}$ = the maximum kinetic energy of the photoelectrons (J)

- This equation demonstrates:
  - If the incident photons do not have a high enough frequency and energy to overcome the work function (Φ), then no electrons will be emitted
  - $hf_0 = \Phi$, where f_0 = threshold frequency, photoelectric emission only just occurs
  - $E_{k(max)}$ depends only on the frequency of the incident photon, and **not** the intensity of the radiation
  - The majority of photoelectrons will have kinetic energies **less than** $E_{k(max)}$

Graphical Representation of Work Function

- The photoelectric equation can be rearranged into the straight line equation:

$$y = mx + c$$

- Comparing this to the photoelectric equation:

$$E_{k_{(max)}} = hf - \Phi$$

- A graph of maximum kinetic energy $E_{k(max)}$ against frequency f can be obtained

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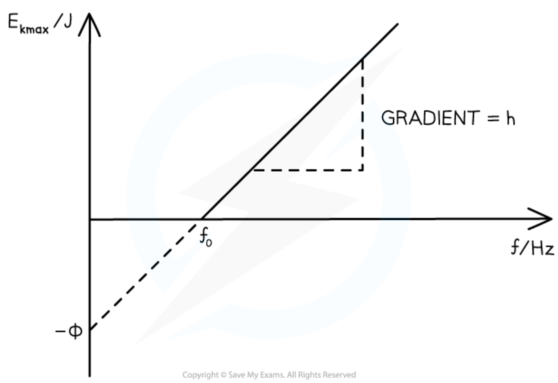

- The key elements of the graph:
  - The work function Φ is the **y-intercept**
  - The threshold frequency f_0 is the **x-intercept**
  - The **gradient** is equal to Planck's constant h
  - There are no electrons emitted below the threshold frequency f_0

Kinetic Energy & Intensity

- The **kinetic energy** of the photoelectrons is **independent of the intensity** of the incident radiation
- This is because **each electron can only absorb one photon**
- Kinetic energy is only dependent on the **frequency** of the incident radiation
- Intensity is a measure of the number of photons incident on the surface of the metal
- So, increasing the number of electrons striking the metal will not increase the kinetic energy of the electrons, it will increase the **number** of photoelectrons emitted

Why Kinetic Energy is a Maximum

- Each electron in the metal acquires the same amount of energy from the photons in the incident radiation
- However, the energy required to remove an electron from the metal varies because some electrons are on the surface whilst others are deeper in the metal
  - The photoelectrons with the maximum kinetic energy will be those on the **surface** of the metal since they do not require much energy to leave the metal
  - The photoelectrons with lower kinetic energy are those deeper within the metal since some of the energy absorbed from the photon is used to approach the metal surface (and overcome the work function)
  - There is less kinetic energy available for these photoelectrons once they have left the metal

Photoelectric Current

- The photoelectric current is the number of photoelectrons emitted per second
- **Photoelectric current** is proportional to the **intensity** of the radiation incident on the surface of the metal
- This is because the intensity is proportional to the number of photons striking the metal per second
- Since each photoelectron absorbs a single photon, the photoelectric current must be proportional to the intensity of the incident radiation

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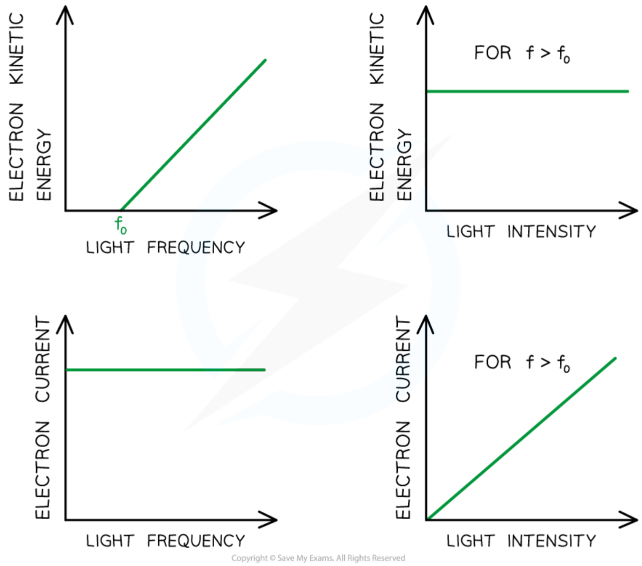
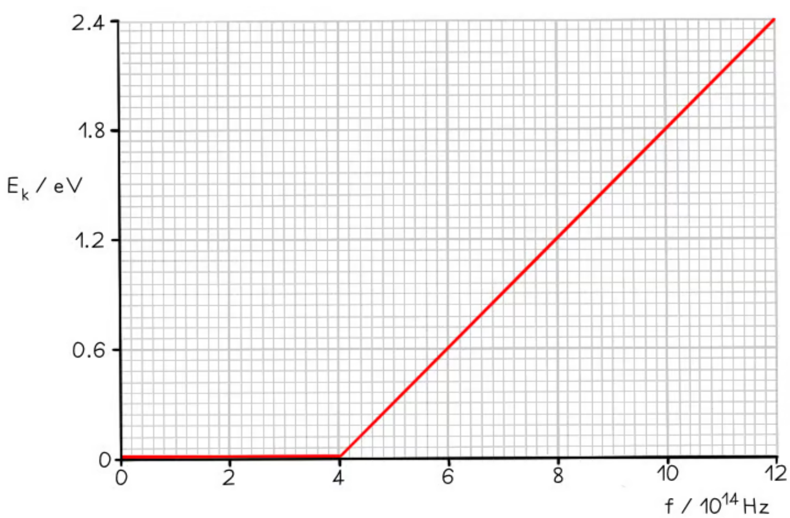


Graphs showing the variation of electron KE and photocurrent with the frequency of the incident light

Worked Example

The graph below shows how the maximum kinetic energy E_k of electrons emitted from the surface of sodium metal varies with the frequency f of the incident radiation.



Calculate the work function of sodium in eV.

Step 1: Write out the photoelectric equation and rearrange it to fit the equation of a straight line

$$E = hf = \Phi + \tfrac{1}{2} mv^2_{max} \quad \rightarrow \quad E_{k(max)} = hf - \Phi$$

$$y = mx + c$$

Step 2: Identify the threshold frequency from the x-axis of the graph

- When $E_k = 0$, $f = f_0$
- Therefore, $f_0 = 4 \times 10^{14}$ Hz

Step 3: Calculate the work function

$$\text{From the graph at } f_0,\ \frac{1}{2} m v^2_{max} = 0$$

$$\Phi = hf_0 = (6.63 \times 10^{-34}) \times (4 \times 10^{14}) = 2.652 \times 10^{-19}\ J$$

Step 4: Convert the work function into eV

$$1\ eV = 1.6 \times 10^{-19}\ J$$

$$\text{for } J \rightarrow eV\text{: divide by } 1.6 \times 10^{-19}$$

$$E = \frac{2.652 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.66\ eV$$

Exam Tip

When using the photoelectric effect equation, hf, Φ and $E_{k(max)}$ must all have the **same** units; Joules.

All values given in eV need to be converted into Joules by multiplying by 1.6×10^{-19}. Do this right away, in the same way as you would convert into SI units before calculating.

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12.1.5 Matter Waves

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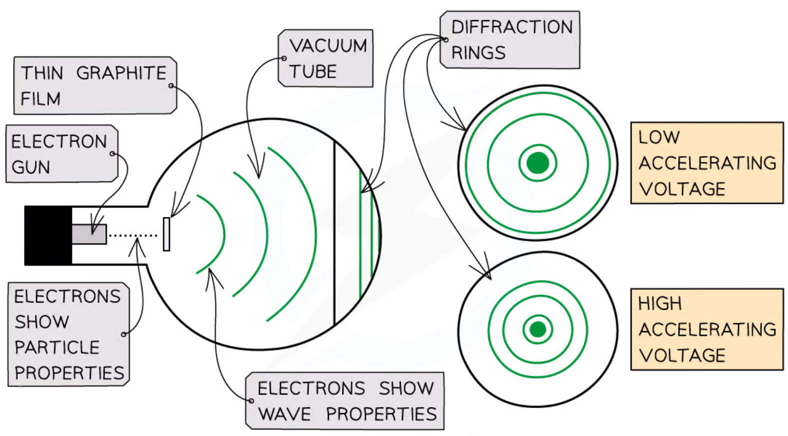
Matter Waves

- De Broglie proposed that electrons travel through space as a wave
  - This would explain why they can exhibit behaviour such as diffraction
- He therefore suggested that electrons must also hold wave properties, such as wavelength
  - This came to be known as the de Broglie wavelength
- However, he realised **all particles** can show wave-like properties, not just electrons
  - He hypothesised that all moving particles have a **matter wave** associated with them
- This is known as the de Broglie wavelength, and can be defined as:

  The wavelength associated with a moving particle

- The majority of the time, and for everyday objects travelling at normal speeds, the de Broglie wavelength is far too small for any quantum effects to be observed
  - A typical electron in a metal has a de Broglie wavelength of about 10 nm
- Therefore, quantum mechanical effects will only be observable when the width of the sample is around that value



- The electron diffraction tube can be used to investigate how the wavelength of electrons depends on their speed
  - The smaller the radius of the rings, the smaller the de Broglie wavelength of the electrons
- As the voltage is increased:
  - The energy of the electrons increases
  - The radius of the diffraction pattern decreases
- This shows as the **speed of the electrons increases**, the **de Broglie wavelength of the electrons decreases**
- Using ideas based upon the quantum theory and Einstein's theory of relativity, de Broglie suggested that the momentum (p) of a particle and its associated wavelength (λ) are

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related by the equation:

$$\lambda = \frac{h}{p}$$

- Since momentum p = mv, the de Broglie wavelength can be related to the speed of a moving particle (v) by the equation:

$$\lambda = \frac{h}{mv}$$

Kinetic Energy

- Since kinetic energy $E = ½ mv^2$
- Momentum and kinetic energy can be related by:

$$E = \frac{p^2}{2m} \rightarrow p = \sqrt{2mE}$$

- Combining this with the de Broglie equation gives a form which relates the de Broglie wavelength of a particle to its kinetic energy:

$$\lambda = \frac{h}{\sqrt{2mE}}$$

- Where:
  - λ = the de Broglie wavelength (m)
  - h = Planck's constant (J s)
  - p = momentum of the particle ($kg\ m\ s^{-1}$)
  - E = kinetic energy of the particle (J)
  - m = mass of the particle (kg)
  - v = speed of the particle ($m\ s^{-1}$)

Worked Example

A proton and an electron are each accelerated from rest through the same potential difference.

Determine the ratio: $\frac{\text{de Broglie wavelength of the proton}}{\text{de Broglie wavelength of the electron}}$

- Mass of a proton = 1.67×10^{-27} kg
- Mass of an electron = 9.11×10^{-31} kg

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Step 1: Consider how the proton and electron can be related via their masses

The proton and electron are accelerated through the same p.d., therefore, they both have the same kinetic energy

Step 2: Write the equation relating the de Broglie wavelength of a particle to its kinetic energy

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

$$\lambda \propto \frac{1}{\sqrt{m}}$$

Step 3: Calculate the ratio

$$\frac{\text{de Broglie wavelength of the proton}}{\text{de Broglie wavelength of the electron}} = \frac{1}{\sqrt{m_p}} \div \frac{1}{\sqrt{m_e}}$$

$$\sqrt{\frac{m_e}{m_p}} = \sqrt{\frac{9.11 \times 10^{-31}}{1.67 \times 10^{-27}}} = 2.3 \times 10^{-2}$$

This means the de Broglie wavelength of the proton is 0.023 times smaller than that of the electron **OR** the de Broglie wavelength of the electron is about 40 times larger than that of the proton

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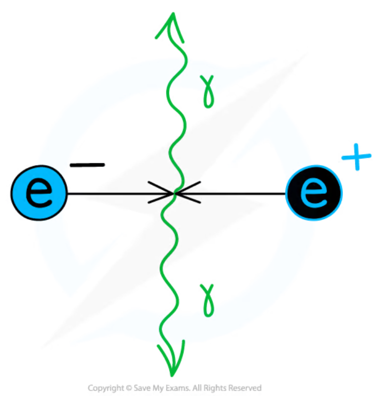
Pair Production & Annihilation

Annihilation

- When a particle meets its antiparticle pair, the two will **annihilate**
- Annihilation is:

When a particle meets its equivalent anti-particle they both are destroyed and their mass is converted into energy in the form of two gamma ray photons



When an electron and positron collide, their mass is converted into energy in the form of two photons emitted in opposite directions

- The minimum energy of **one** photon after annihilation is the total rest mass energy of **one** of the particles is:

$$E_{min} = hf_{min} = E$$

- Where:
  - E_{min} = minimum energy of one of the photons produced (J)
  - h = Planck's Constant (J s)
  - f_{min} = minimum frequency of one of the photons produced (Hz)
  - E = rest mass energy of one of the particles (J)

- To conserve momentum, the two photons will move apart in **opposite** directions
- To conserve energy, the two photons will have the same energy, which means they also have the same frequency because

$$\mathbf{E = hf}$$

Pair Production

- Pair production is the opposite of annihilation
- Pair production is:

When a photon interacts with a nucleus or atom and the energy of the photon is used to create a particle–antiparticle pair

YOUR NOTES
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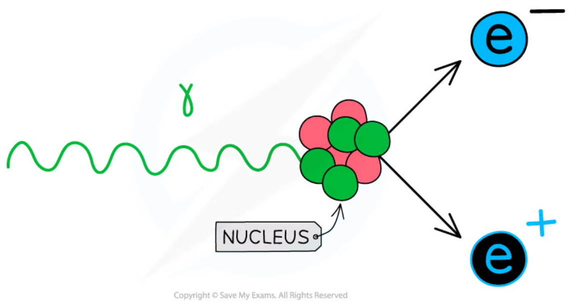
- The presence of a nearby neutron is essential in pair production so that the process conserves both energy and momentum
- A single photon alone cannot produce a particle-anti-particle pair or the conservation laws would be broken
- Pair creation is a case of energy being converted into matter



When a photon with enough energy interacts with a nucleus it can produce an electron-positron pair

- This means the energy of the photon must be above a certain value to provide the total rest mass energy of the particle-antiparticle pair
- The minimum energy for a photon to undergo pair production is the total rest mass energy of the particles produced:

$$E_{min} = hf_{min} = 2E$$

- Where:
  - E_{min} = minimum energy of the incident photon (J)
  - h = Planck's Constant (J s)
  - f_{min} = minimum frequency of the photon (Hz)
  - E = rest mass energy of one of the particles (J)

- To conserve momentum, the particle and anti-particle pair move apart in **opposite** directions
- Remember for both calculations, the frequency f is also defined by the wave equation, $\mathbf{v = f\lambda}$

- Using this equation, the wavelength λ can also be calculated
  - Remember that v is c (the speed of light) for gamma ray photons

? Worked Example

Calculate the maximum wavelength of one of the photons produced when a proton and antiproton annihilate each other.

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Step 1: Write down the known quantities

Rest mass energy of a proton (and antiproton) = 938.257 MeV

$$1 \text{ MeV} = 1.60 \times 10^{-13} \text{ J}$$

Step 2: Write the equation for the minimum photon energy

$$E_{min} = hf_{min} = E$$

Step 3: Write energy in terms of wavelength

$$f_{min} = \frac{c}{\lambda_{max}}$$

$$E_{min} = \frac{hc}{\lambda_{max}} = E$$

Step 4: Rearrange for wavelength

$$\lambda_{max} = \frac{hc}{E}$$

Step 5: Substitute in values

$$\lambda_{max} = \frac{\left(6.63 \times 10^{-34}\right) \times (3.0 \times 10^{8})}{938.257 \times \left(1.60 \times 10^{-13}\right)} = 1.32 \times 10^{-15} \text{ m}$$

Exam Tip

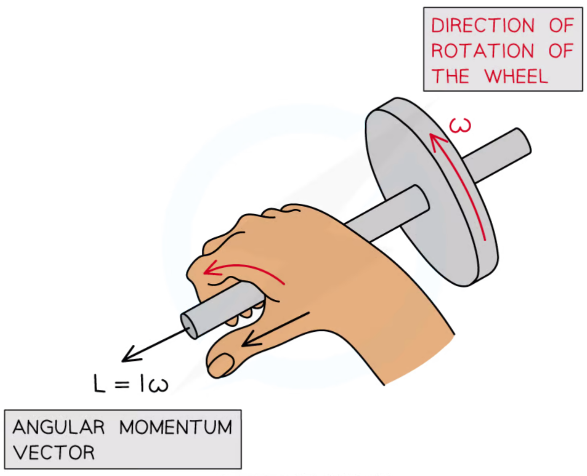
Since the Planck constant is in Joules (J) remember to always convert the rest mass-energy from MeV to J.

YOUR NOTES

Quantization of Angular Momentum

- Angular momentum is a property of any spinning or rotating body, very similar to linear momentum
  - In linear motion, momentum is the product of mass and velocity
- In rotational motion the momentum is the product of moment of inertia and angular speed

- Angular momentum is a vector, this means:
  - The **magnitude** is equal to the momentum of the particle times its radial distance from the centre of its circular orbit
  - The **direction** of the angular momentum vector is normal to the plane of its orbit with the direction being given by the corkscrew rule



Angular momentum acts at right angles to the direction of rotation

- Niels Bohr proposed that the angular momentum, *L*, of an electron in an energy level is quantised in integer multiples of $\frac{h}{2\pi}$
- Where:
  - n = an integer (n = 1, 2, 3...)
  - *h* = Planck's constant
- Sometimes ***h*** may be written as the reduced Planck's constant, **ħ**, which is equal to

$$\hbar = \frac{h}{2\pi}$$

- Hence the angular momentum for an electron in a circular orbit is **constant**

- De Broglie proposed that an electron with momentum *p* = *mv* has a wavelength λ given by

$$\lambda = \frac{h}{p}$$

- For an electron moving in a straight line, the matter wave takes a familiar wave shape consisting of peaks and troughs

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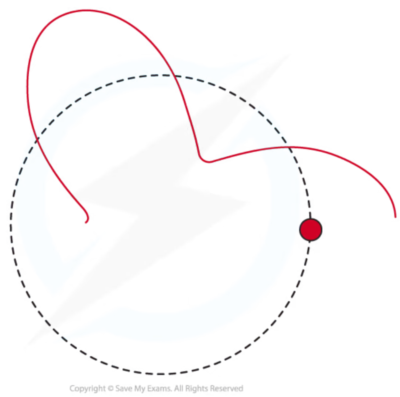
- Although the electron itself isn't oscillating up and down, only the matter wave

de Broglie matter wave for an electron moving in a straight line at constant speed

- For the same electron moving in a circle, the matter wave still has a sinusoidal shape but is wrapped into a circle

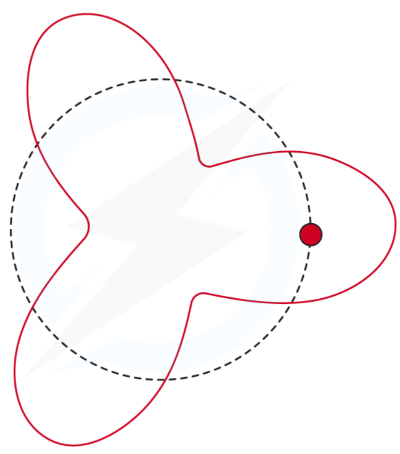


de Broglie matter wave for an electron moving in a circular orbit at constant speed

- As the electron continues to orbit in a circle two possibilities may occur:

1. On completing one oscillation, the waves overlap in phase

- The waves will continue in phase over many orbits giving rise to constructive interference and a standing wave



de Broglie matter wave where 3λ is less than the orbits circumference

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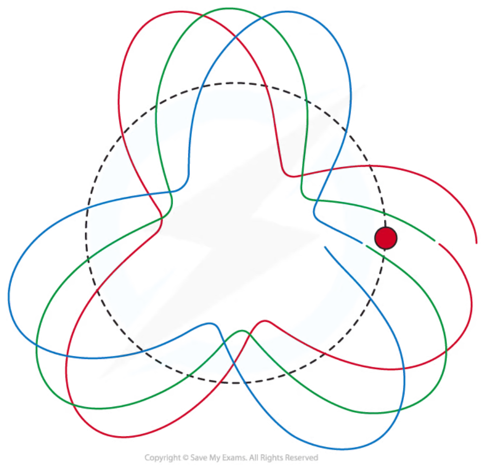
2. On completing one oscillation, the waves overlap but they are not in phase

- In other words, peak overlaps with peak, trough with trough
- This means that where the waves overlap, destructive interference occurs and as a result, no such electron orbit is allowed



de Broglie matter wave where n = 3. Here the circumference of the circular orbit is 3λ

- Hence, the circumference of the orbit $(2\pi r)$ must equal an integer number of wavelengths $(n\lambda)$ for a standing wave to form:

$$n\lambda = 2\pi r$$

- Using the de Broglie relation, $\lambda = \frac{h}{p}$:

$$n\frac{h}{p} = 2\pi r$$

- Since momentum is equal to $p = mv$:

$$n\frac{h}{mv} = 2\pi r$$

- Rearranging for angular momentum, mvr:

$$n\frac{h}{2\pi} = mvr$$

Bohr Condition

- The **Bohr Condition** is given by the relation:

$$n\frac{h}{2\pi} = mvr$$

- Where:
  - n = energy level
  - h = Planck's constant (J s)
  - m = mass (kg)
  - v = velocity (m s^{-1})

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- r = radius (m)
- The condition essentially states that an electron can only move in fixed orbits
- Neils Bohr and Ernest Rutherford found that atomic orbits are only allowed when:
  - The angular momentum of the electron is an integer multiple of $\frac{h}{2\pi}$
- The Bohr condition also leads to the restriction of the **electron radius** to certain values
- Another implication is that the **discrete** or **quantised energy** of the electron follows this rule:

$$E = -\frac{13.6}{n^2}\text{eV}$$

Worked Example

Determine the velocity of the electron in the first Bohr orbit of the hydrogen atom (n = 1).

You may use the following values:

- Mass of an electron = 9.1×10^{-31} kg
- Radius of the orbit = 0.529×10^{-10} m
- Planck's constant = 6.63×10^{-34} kg m^2 s^{-1}

Step 1: List the known quantities

- Mass of an electron, $m = 9.1 \times 10^{-31}$ kg
- Radius of the orbit, $r = 0.529 \times 10^{-10}$ m
- Planck's constant, $h = 6.63 \times 10^{-34}$ kg m^2 s^{-1}

Step 2: Write the Bohr Condition equation and rearrange for velocity, *v*

$$n\frac{h}{2\pi} = mvr$$

$$v = \frac{nh}{2\pi \times mr}$$

Step 3: Substitute in the values and calculate, *v*

$$v = \frac{1 \times (6.63 \times 10^{-34})}{2\pi \times (9.1 \times 10^{-31}) \times (0.529 \times 10^{-10})} = 2.2 \times 10^6 \ m\ s^{-1}$$

Step 4: Write the final answer

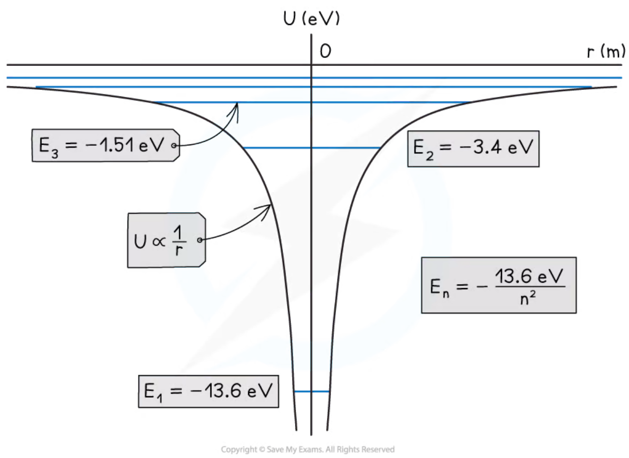
The velocity of the electron = **2.2×10^6 m s^{-1}**

12.1.8 The Wave Function

YOUR NOTES

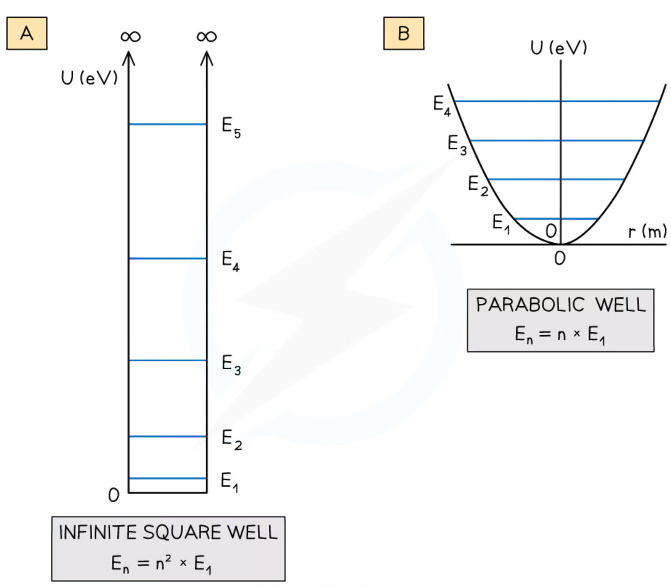
The Wave Function

- Bohr showed that the angular momentum of an electron orbiting a hydrogen atom was quantised
  - He also showed that the allowed electron energies were also quantised
  - Unfortunately, this original model only worked for hydrogen
- **Schrodinger** then developed an equation that could be used to calculate the quantised energy levels for other atoms
  - The only information the equation needs is the shape of the particle's potential energy function



The potential energy curve for hydrogen and the quantised energy levels for n = 1, 2, 3, etc

- On the potential energy curve for hydrogen:
  - The **higher** the energy levels, the **closer** together they become in energy



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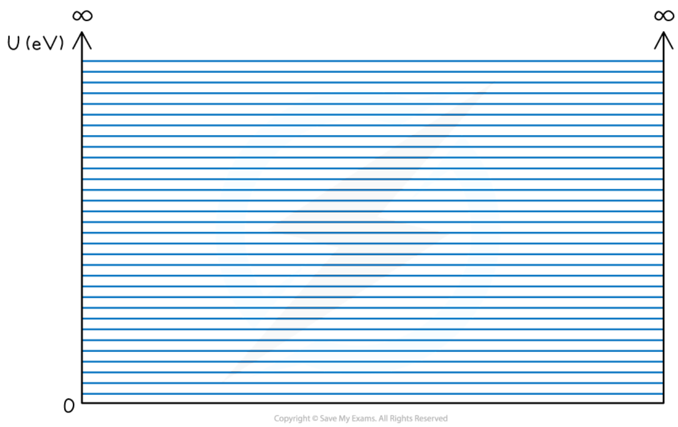
The potential energy curves for (a) an infinite square well and (b) a parabolic well and their associated quantised energy levels

- The shapes of these graphs also produce quantised energy levels but with a different energy distribution
  - For the **parabolic well**, the energy levels are equally spaced
  - For the **infinite square well**, the energy levels at larger energies are further apart – this is different to the case for hydrogen

- Finally, as the infinite square well gets **wider**, the quantised energy levels get **closer** together



The "swimming pool" square well where the width is very large. For a wide well, the quantised levels get so close together that the allowed levels are effectively "continuous"

- The Schrodinger equation predicts not only the distribution of energy levels for a particular potential energy curve
  - Each energy level also has a wavefunction, usually given the symbol Ψ (Psi)

- Wavefunctions of the infinite square well are shown below
  - They look similar to standing waves on a stretched string

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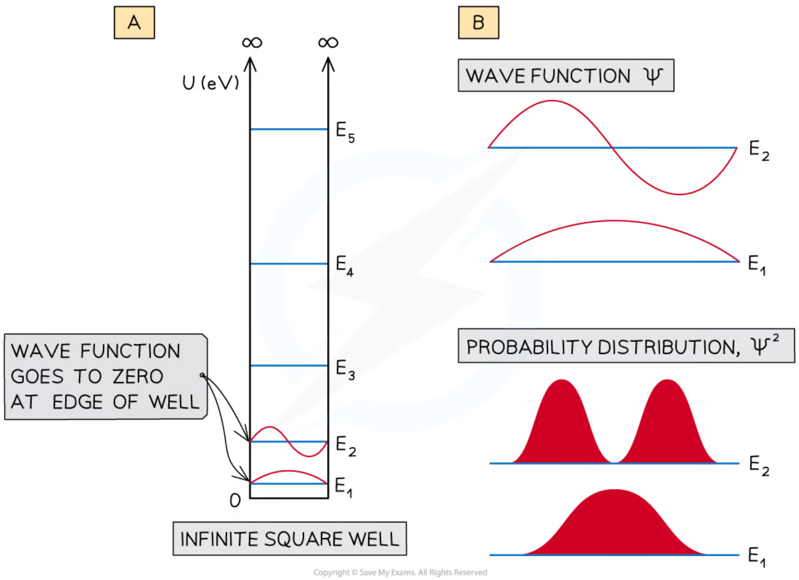

(a) Wave functions for E1 and E2 of an infinite square well. (b) an expanded view of the wavefunctions Ψ, and the probability

- The physical significance of the wavefunction Ψ is that:
  - The square of the wavefunction, Ψ^2 dr gives the probability of finding the particle in the region of width *dr*
- The wavefunction is said to be **normalised**
  - This means the probability of finding a particle somewhere in the well at a particular energy level is equal to **one**
- The **infinite square well** shows the wave function going to zero at the edges of the well
  - This means that the probability of finding the particle close to the walls of the well is **small**
- For the energy level E_1 the probability of finding it at the centre of the well in large
  - However, for E_2 the probability of finding it at the centre of the well is small

- A different result occurs for a square well that is not infinitely deep
- For two energy levels in a square well, the wave functions penetrate the barrier regions as decaying exponential curves

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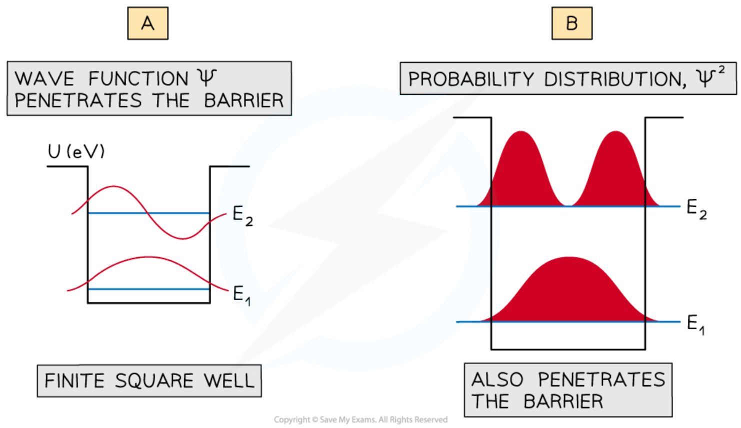

(a) Wave functions for E1 and E2 for a finite square well. (b) the wavefunctions Ψ, and the probability distribution Ψ^2 are seen to penetrate the barrier

- This means that the probability of finding the particle in the classically forbidden barrier region is **not zero**

12.1.9 The Uncertainty Principle

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The Uncertainty Principle

- The quantisation of energy levels and their associated wave functions leads to some surprising outcomes
  - One of these outcomes is known as the **uncertainty principle**
- The uncertainty principle states that:

  There are certain pairs of physical quantities that are cannot be known precisely at the same time

- This leads to two consequences related to:
  - Position and momentum
  - Energy and time

Position & Momentum

- One example is that it is **not possible to know the position and the momentum** of a quantum particle precisely
- This can be represented in equation form as:

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

- Where:
  - Δx = change in displacement
  - Δp_x = change in momentum
  - h = Planck's constant
- This equation shows that:
  - The better the position of the particle is known, the less precise is the knowledge of its momentum
- The value of Planck's constant is small so this limitation only becomes important for small particles within the quantum regime - it does not apply in the Newtonian mechanics that you have used to calculate momentum for larger objects

Energy & Time

- There is a similar relationship between **energy and time** where both cannot be known precisely

$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$$

- Where:
  - ΔE = change in energy
  - Δt = change in time
  - h = Planck's constant

Wavefunctions

- This principle can be demonstrated by looking at the wavefunctions associated with quantum particles that are nearly 'free'

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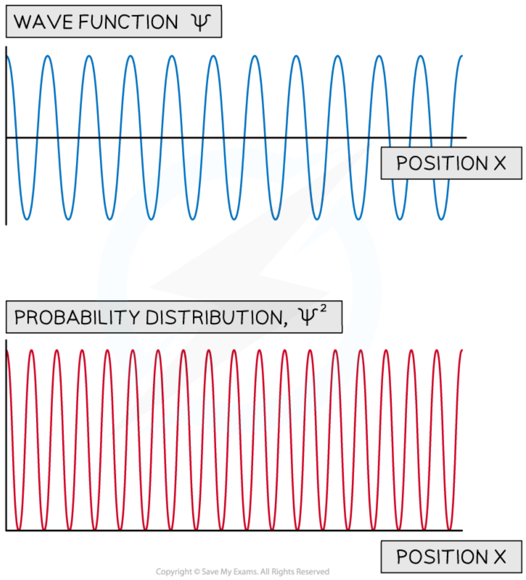
- For example, those associated with the energy levels in a 'swimming pool' potential energy well

- The graph of wavefunction vs position shows that while the **momentum** is known precisely the **position** of the particle is unknown



The wave function of a free particle with a fixed energy. The wave function shows it has a single value of momentum while the probability distributions shows that the particle can be anywhere in space.

- For a particle in a 'swimming pool' potential well, its energy levels are close together
  - Hence, the wave function can be a mixture of the wavefunctions associated with a number of closely spaced energy levels

- For a range of five neighbouring energy levels, the wavefunction can be obtained by adding the individual wavefunctions
  - This is a result of the principle of superposition

- This gives rise to a wave packet, meaning the particle is more likely to be in a particular region of space
- This can be shown by a graph of the probability distribution vs position
  - This helps to visualise how the **approximate uncertainty** in the position, Δx and momentum Δp_x can be determined

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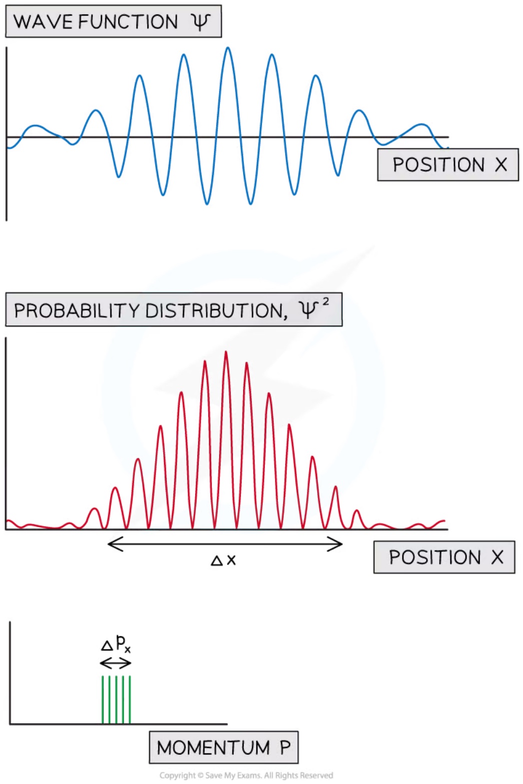

A wave packet made from the wavefunctions associated with five closely spaced energy levels. The probability distribution is shown along with the uncertainty in position and momentum.

- Consider the wavefunctions associated with five and ten energy levels with the probability distribution and the estimates of Δx and $Δp_x$
- Comparing the two sets of graphs, it can be seen that:
  - As the momentum becomes **less well known** the position becomes **more well known**

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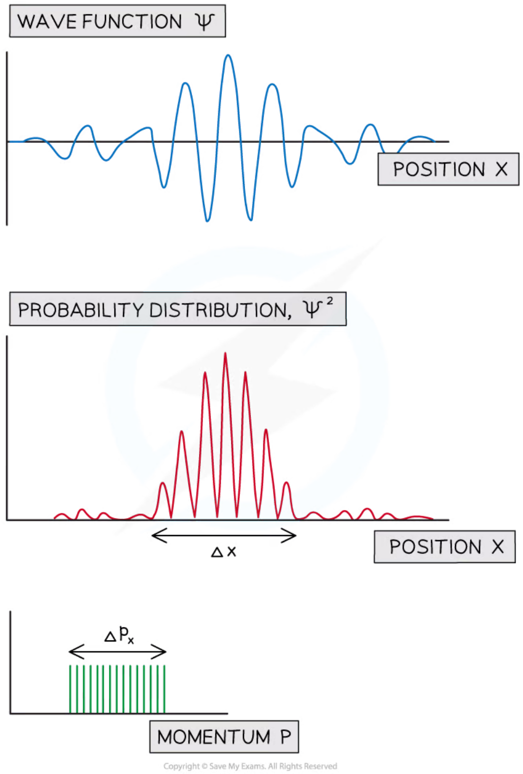

A wave packet made from the wavefunctions associated with five closely spaced energy levels. The probability distribution is shown along with the uncertainty in position and momentum.

Worked Example

A narrow beam of electrons with velocity 10^6 m s^{-1} are directed towards a slit which is 10^{-9} m wide.

The electrons are observed on a screen placed 2.0 m from the slit.

Estimate the length of the area of the screen where electrons will be seen in appreciable numbers.

Step 1: Write down the uncertainty equation for position and momentum

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

Step 2: Identify the known uncertainty from the question

- The slit is 10^{-9} m wide, so the uncertainty in the vertical component of the position of the electrons at the slit, $\Delta x = 10^{-9}$

Step 3: Rearrange the equation to find Δp and calculate

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$$\Delta p = \frac{h}{4\pi\Delta x} = \frac{(6.63 \times 10^{-34})}{4\pi \times (10^{-9})} = 5.28 \times 10^{-26}\ N\ s$$

Step 4: Calculate the momentum, p

- Mass of electron, m = 9.1 × 10^{-31} kg
- Momentum, p = mv = 9.1 × 10^{-31} × 10^{6} = 9.1 × 10^{-25} N s

Step 5: Substitute change in momentum (step 3) and momentum (step 4) into the equation for change in direction

$$d\theta = \frac{\Delta p}{p} = \frac{5.28 \times 10^{-26}}{9.1 \times 10^{-25}} = 0.06\ rad$$

Step 6: Apply the deviation of angle to find the length on the screen

- Distance from slits to screen, D = 2.0 m
- Angle of deviation from line of beam, θ = 0.06 rad
- Therefore:

$$\text{length on screen} = 2\theta \times D = (2 \times 0.06) \times 2.0 = 0.24\ m$$

Step 7: Write the final answer out in full

- The length of the screen where appreciable numbers of electrons are expected to be observed = 24 cm

12.2 Nuclear Physics

12.2.1 Rutherford Scattering & Nuclear Radius

YOUR NOTES
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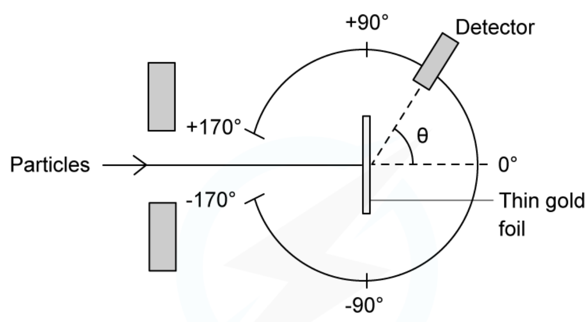
Rutherford Scattering & Nuclear Radius

- In the Rutherford scattering experiment, alpha particles are fired at a thin gold foil
- Initially, before interacting with the foil, the particles have kinetic energy,

$$E_k = \frac{1}{2}mv^2$$

- Some of the alpha particles are found to come straight back from the gold foil
- This indicates that there is **electrostatic repulsion** between the alpha particles and the gold nucleus



Experimental set up of the Rutherford alpha scattering experiment

- At the point of closest approach, *d*, the repulsive force reduces the speed of the alpha particles to zero momentarily, before any change in direction
  - At this point, the initial kinetic energy of an alpha particle, E_k, **is equal to electric potential energy,** E_p
- The radius of the closest approach can be found be equating the initial kinetic energy to the electric potential energy

$$E_p = k\frac{Qq}{d}$$

- Where:
  - Charge of an alpha particle, Q = 2e
  - Charge of a target nucleus, q = Ze
  - Z = proton number
  - e = charge on an electron (or proton)
- Substituting into the equation:

$$E_p = k\frac{(2e)(Ze)}{d}$$

- This gives an expression for the **potential energy** at the point of **repulsion:**

$$E_p = k\frac{2Ze^2}{d}$$

YOUR NOTES
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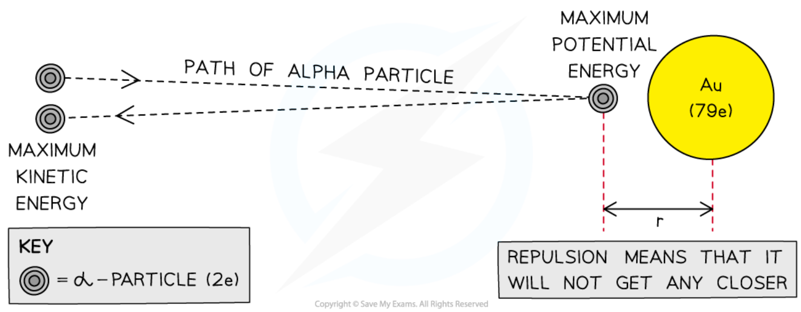
- Due to conservation of energy:
  - This expression also gives the initial kinetic energy possessed by the alpha particle
- Rearranging and calculating for the distance, *d*, gives a value for the radius of the nucleus when the alpha particle is fired with high energy

$$d = k\frac{2Ze^2}{E_p}$$



The closest approach method of determining the size of a gold nucleus

Nuclear Radius

- The radius of nuclei depends on the nucleon number, *A* of the atom
- This makes sense because as more nucleons are added to a nucleus, more space is occupied by the nucleus, hence giving it a larger radius
- The exact relationship between the radius and nucleon number can be determined from experimental data
- By doing this, physicists were able to deduce the following relationship:

$$R = R_0 A^{\frac{1}{3}}$$

- Where:
  - R = nuclear radius (m)
  - A = nucleon / mass number
  - R_0 = constant of proportionality = 1.20 fm

Nuclear Density

- Assuming that the nucleus is spherical, its volume is equal to:

$$V = \frac{4}{3}\pi R^3$$

- Where R is the nuclear radius, which is related to mass number, A, by the equation:

$$R = R_0 A^{\frac{1}{3}}$$

- Where R_0 is a constant of proportionality
- Combining these equations gives:

$$V = \frac{4}{3}\pi \left(R_0 A^{\frac{1}{3}}\right)^3 = \frac{4}{3}\pi R_0^3 A$$

- Therefore, the nuclear volume, V, is proportional to the mass of the nucleus, A

YOUR NOTES
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- Mass (m), volume (V), and density (ρ) are related by the equation:

$$\rho = \frac{m}{V}$$

- The mass, *m*, of a nucleus is equal to:

$$m = Au$$

- Where:
  - *A* = the mass number
  - *u* = atomic mass unit
- Using the equations for mass and volume, nuclear density is equal to:

$$\rho = \frac{Au}{\frac{4}{3}\pi R_0^3 A} = \frac{3u}{4\pi R_0^3}$$

- Since the mass number *A* cancels out, the remaining quantities in the equation are all constant
- Therefore, this shows the density of the nucleus is:
  - Constant
  - Independent of the radius
- The fact that nuclear density is constant shows that nucleons are evenly separated throughout the nucleus regardless of their size
- The accuracy of nuclear density depends on the accuracy of the constant R_0, as a guide nuclear density should always be of the order 10^{17} kg m^{-3}
- Nuclear density is significantly larger than atomic density, this suggests:
  - The majority of the atom's mass is contained in the nucleus
  - The nucleus is very small compared to the atom
  - Atoms must be predominantly empty space

Worked Example

Determine the value of nuclear density.

You may take the constant of proportionality, R_0, to be 1.20×10^{-15} m.

Step 1: Derive an expression for nuclear density
- Using the equation derived above, the density of the nucleus is:

$$\rho = \frac{3u}{4\pi R_0^{\,3}}$$

Step 2: List the known quantities
- Atomic mass unit, $u = 1.661 \times 10^{-27}$ kg
- Constant of proportionality, $R_0 = 1.20 \times 10^{-15}$ m

Step 3: Substitute the values to determine the nuclear density

YOUR NOTES

$$\rho = \frac{3 \times (1.661 \times 10^{-27})}{4\pi (1.20 \times 10^{-15})^3} = \mathbf{2.3 \times 10^{17}\ kg\ m^{-1}}$$

Exam Tip

Make sure you're comfortable with the calculations involved with the alpha particle closest approach method, as this is a common exam question.

You will be expected to remember that the charge of an α is the charge of 2 protons (2 × the charge of an electron)

12.2.2 Nuclear Scattering

YOUR NOTES

Nuclear Scattering

- Electrons accelerated to close to the speed of light have wave-like properties such as the ability to diffract and have a de Broglie wavelength equal to:

$$\lambda = \frac{h}{mv}$$

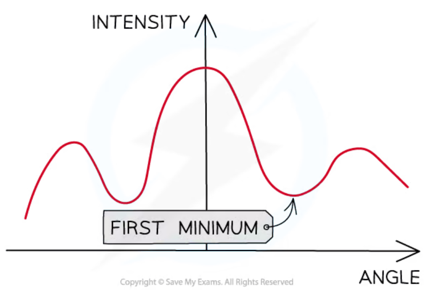
- Where:
  - h = Planck's constant
  - m = mass of an electron (kg)
  - v = speed of the electrons (m s^{-1})

- When beams of neutrons or electrons are directed at a nucleus they will diffract around it
- The pattern formed by this diffraction has a predictable minimum which forms at an angle θ to the original direction according to the equation

$$\sin\theta = \frac{\lambda}{b}$$

- The diffraction pattern forms a central bright spot with dimmer concentric circles around it
- From this pattern, a graph of intensity against diffraction angle can be used to find the diffraction angle of the first minimum

- The graph of intensity against angle obtained through electron diffraction is as follows:



The first minimum of the intensity-angle graph can be used to determine nuclear radius

- Using this, the size of the atomic nucleus, R, can be determined using:

$$\sin\theta = \frac{\lambda}{2R}$$

- Where:
  - θ = angle of the first minimum (degrees)
  - λ = de Broglie wavelength (m)
  - R = radius of the nucleus (m)

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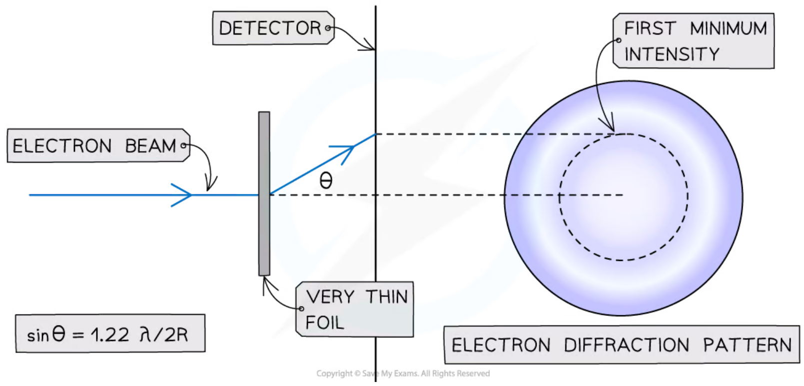
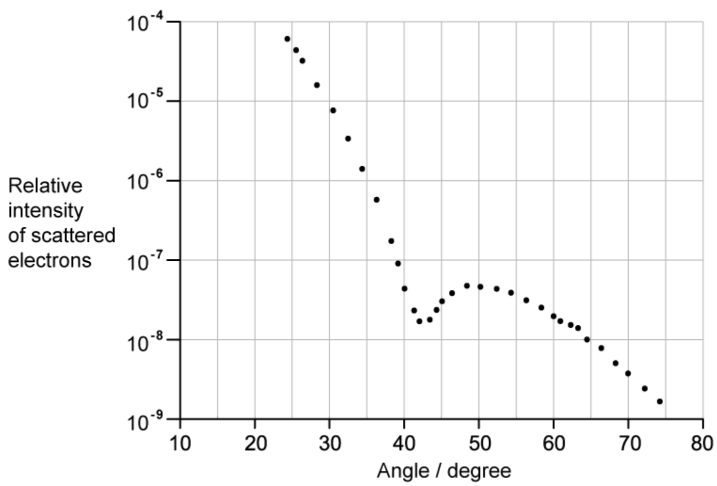


Geometry of electron diffraction

Worked Example

The graph shows how the relative intensity of the scattered electrons varies with angle due to diffraction by the oxygen-16 nuclei. The angle is measured from the original direction of the beam.

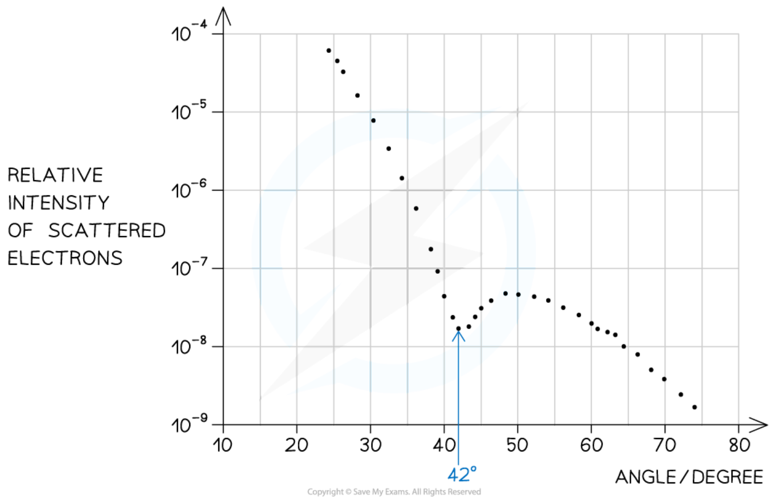


The de Broglie wavelength λ of each electron in the beam is 4.22×10^{-15} m.

Calculate the radius of an oxygen-16 nucleus using information from the graph.

Step 1: Identify the first minimum from the graph

YOUR NOTES



- Angle of first minimum, θ = 42°

Step 2: Write out the equation relating the angle, wavelength, and nuclear radius

$$\sin\theta = \frac{\lambda}{2R}$$

Step 3: Calculate the nuclear radius, *R*

$$R = \frac{\lambda}{2\sin\theta} = \frac{4.22 \times 10^{-15}}{2 \times \sin(42)} = 3.15 \times 10^{-15}\ \text{m}$$

Exam Tip

In the data booklet, you may notice the relation between angle and diameter appears a couple of times - in the Resolution topic, you are given the full equation for calculating the Rayleigh criterion

$$\theta = 1.22\frac{\lambda}{b}$$

However, in this topic, you are given the approximated version

$$\sin\theta \approx \frac{\lambda}{D}$$

This means you only need to use that version in exam questions about nuclear scattering, omitting the factor of 1.22 is acceptable and likely expected unless asked!

12.2.3 Deviations from Rutherford Scattering

YOUR NOTES

Deviations from Rutherford Scattering

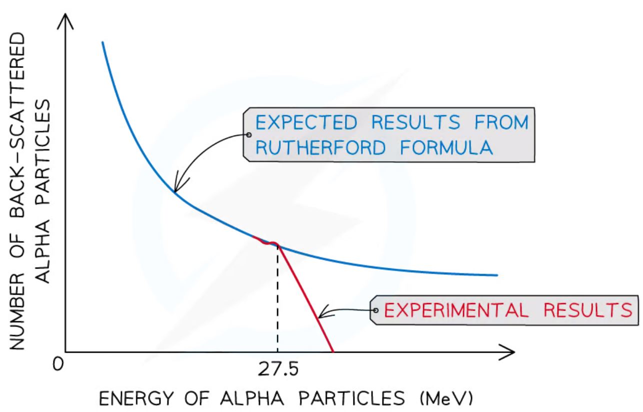
- Rutherford's alpha scattering experiment originally assumed that the alpha particles are **only** interacting through **electrostatic repulsion**
  - However, if the energy of the alpha particles exceeds **27.5 – 28 MeV**, then they will be close enough to **interact** with the nucleus via the **strong nuclear force**
- The Rutherford formula describes this as it states that as the angle of scattering angle increases, the number of alpha particles scattered at that angle sharply decreases

$$N \alpha \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

- Where:
  - N = number of alpha particles
  - θ = angle of scattering
- Rearranging this equation shows that it should be the case that:

$$N\sin^4\left(\frac{\theta}{2}\right) = k$$

- Where:
  - k = a constant
- Experimental data bears this out, suggesting that the Rutherford formula is correct
- A number of assumptions are made in deriving this formula, with the main one being that the only force we need to consider is the electric force
- Deviations from Rutherford scattering are evidence of the strong nuclear force



The observed back-scattering from alpha particles strongly deviates from the predicted relationship based only on electromagnetic repulsion at 27.5 MeV

YOUR NOTES

Worked Example

Alpha particles undergo scattering after being fired at a thin gold $^{197}_{79}\mathrm{Au}$ foil. The gold is then replaced to make a comparison.

Describe the predicted difference in the scattering pattern when the foil is replaced with aluminium $^{30}_{13}\mathrm{Al}$ foil of the same thickness.

Step 1: Compare the relative charges of the nuclei

- Gold has 79 protons, aluminium has 13 protons.
- The electric force between the nuclei is proportional to the charge, since,

$$E = k\frac{qQ}{r^2}$$

- Therefore the alpha particle will approach the aluminium nucleus at a much closer distance

Step 2: Consider what causes deviation from Rutherford scattering

- Deviations from Rutherford scattering occur when forces apart from the electric force between the nuclei are at play
- At much smaller distances the effect of the strong nuclear force will be felt

Step 3: Deduce the solution

- The alpha particles get closer to the aluminium nucleus, at a distance the strong nuclear force begins to act
- More deviation will be seen with aluminium foil than with gold foil

Exam Tip

The greatest deviations from Rutherford scattering occur when the energy of the alpha particles is high and the radius of the target nuclei is small (meaning it has a small nucleon number)

12.2.4 Nuclear Energy Levels

YOUR NOTES
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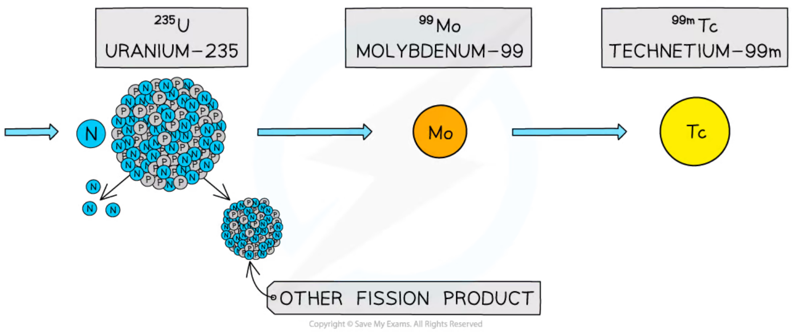
Nuclear Energy Levels

- In the same way that electrons can exist in excited states, nuclei can also exist in **excited states**
- After an unstable nucleus emits an alpha particle, beta particle or undergoes electron capture, it may emit any remaining energy in the form of a **gamma photon** (γ)
  - Emission of a γ photon does not change the number of protons or neutrons in the nucleus, it only allows the nucleus to **lose energy**
- This happens when a daughter nucleus is in an excited state after a decay
- This excited state is usually very **short-lived**, and the nucleus quickly moves to its **ground state**, either directly or via one or more lower-energy excited states



- One common application of this is the use of technetium-99m as a γ source in medical diagnosis
  - The 'm' stands for **metastable** which means the nucleus exists in a particularly stable excited state
- Technetium-99m is the decay product of molybdenum-99, which can be found as a product in nuclear reactors
- The decay of molybdenum-99 is shown below:

$$^{99}_{42}Mo \rightarrow {}^{99m}_{43}Tc + {}^{0}_{-1}\beta + \overline{\nu_e}$$

$$^{99m}_{43}Tc \rightarrow {}^{99}_{43}Tc + \gamma$$

- The half-life of molybdenum-99 is **66 hours**
  - This is long enough for the sample to be transported to hospitals
  - Subsequently, the technetium-99m can be separated at the hospital
- Technetium-99m has a short half-life of **6 hours**
  - This is an adequate timeframe for examining a patient
  - Plus, it is short enough to minimise damage to the patient

Nuclear Energy Level Diagrams

YOUR NOTES
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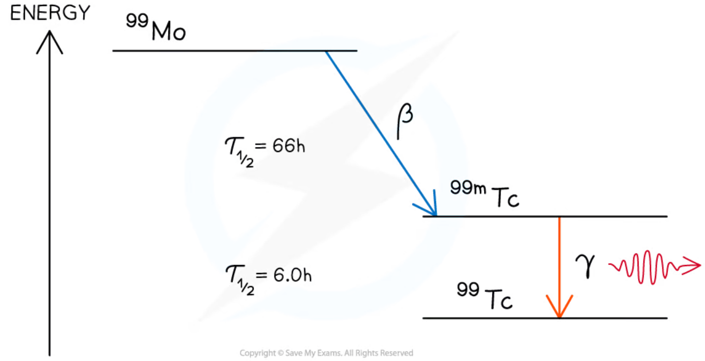
- Nuclear energy levels are similar to electron energy levels
- The nuclear energy level diagram of molybdenum-99 can be represented as follows:



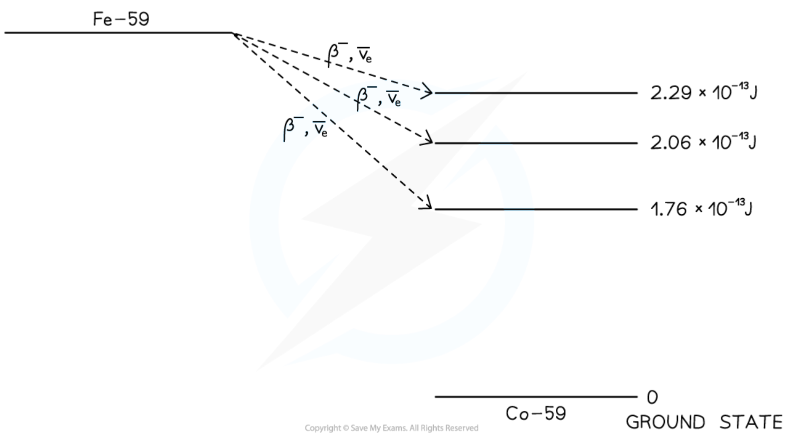
- The decay mode (usually alpha or beta) is represented by a diagonal line
- The excited state, or states, are generally stacked in descending energy order to the right of the decay

YOUR NOTES

Worked Example

A nucleus of iron Fe-59 decays into a stable nucleus of cobalt Co-59. It decays by β^- emission followed by the emission of γ-radiation as the Co-59 nucleus de-excites into its ground state.

The total energy released when the Fe-59 nucleus decays is 2.52×10^{-13} J. The Fe-59 nucleus can decay to one of three excited states of the cobalt-59 nucleus as shown below. The energies of the excited states are shown relative to the ground state.



Following the production of excited states of Co-59, γ-radiation of discrete wavelengths is emitted.

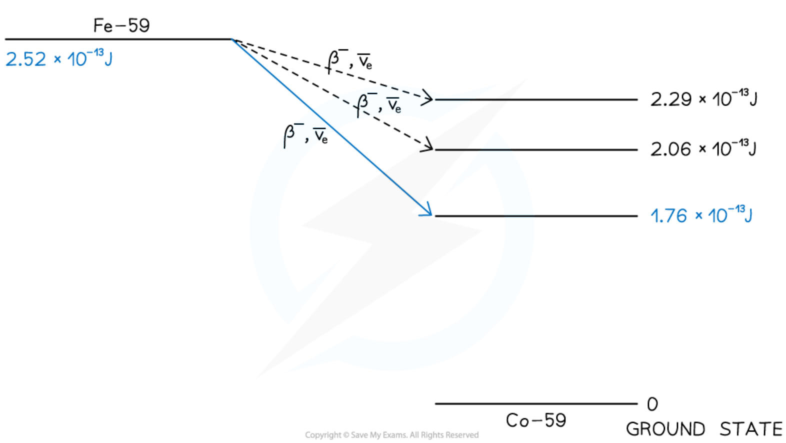
(a) Calculate the maximum possible kinetic energy of the $\beta-$ particle emitted in MeV

(b) State the maximum number of discrete wavelengths that could be emitted

(c) Calculate the longest wavelength of the emitted γ-radiation

Part (a)

Step 1: Identify the beta emission with the largest energy gap



YOUR NOTES

Step 2: Calculate the energy difference

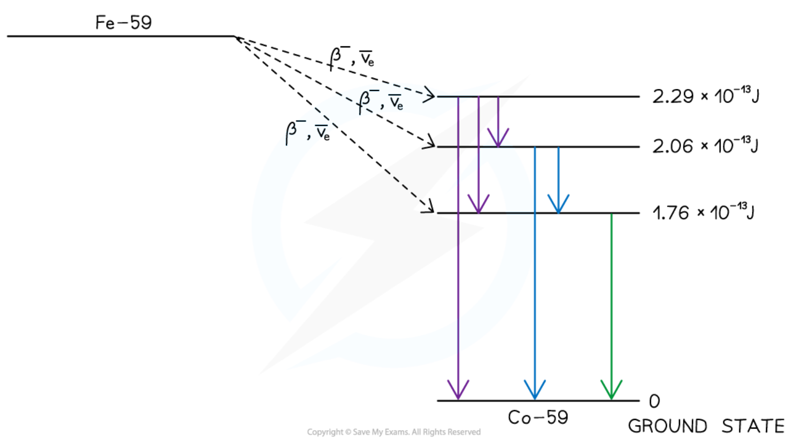
$$\Delta E = (2.52 - 1.76) \times 10^{-13} = 7.6 \times 10^{-14} \text{ J}$$

Step 3: Convert from J to MeV

$$\Delta E = \frac{7.6 \times 10^{-14}}{1.6 \times 10^{-13}} = 0.475 = 0.48 \text{ MeV}$$

- $1 \text{ MeV} = 1.6 \times 10^{-13}$ J

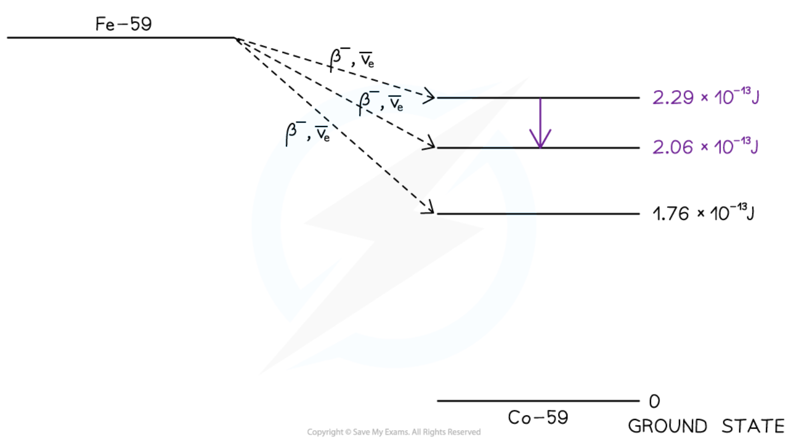
Part (b)



- There are 6 possible transitions, hence 6 discrete wavelengths could be emitted

Part (c)

Step 1: Identify the emission with the longest wavelength / smallest energy gap



- Longest wavelength = lowest frequency = smallest energy

Step 2: Calculate the energy difference

$$\Delta E = (2.29 - 2.06) \times 10^{-13} = 2.3 \times 10^{-14} \text{ J}$$

Step 3: Write down de Broglie's wavelength equation

YOUR NOTES
⬇

$$E = \frac{hc}{\lambda}$$

Where:

- h = Planck's constant
- c = speed of light

Step 4: Calculate the wavelength associated with the energy change

$$\lambda = \frac{hc}{E} = \frac{\left(6.63 \times 10^{-34}\right) \times \left(3.0 \times 10^{8}\right)}{2.3 \times 10^{-14}} = 8.6 \times 10^{-12} \text{ m}$$

YOUR NOTES
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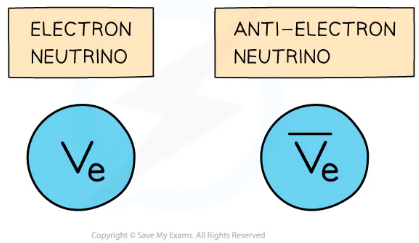
12.2.5 The Neutrino

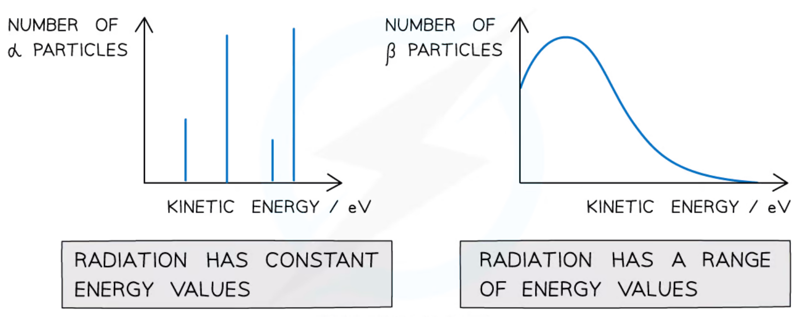
The Neutrino

- An electron neutrino is a type of subatomic particle with no charge and negligible mass which is also emitted from the nucleus
- The anti-neutrino is the antiparticle of a neutrino
  - Electron anti-neutrinos are produced during β– decay
  - Electron neutrinos are produced during β+ decay



- Although the neutrino has no charge and negligible mass, its existence was hypothesised to account for the conservation of **energy** in beta decay

- When the number of α particles is plotted against kinetic energy, there are clear spikes that appear on the graph
- This demonstrates that **α-particles have discrete energies** (only certain values)



Alpha particles have discrete energy levels whilst beta particles have a continuous range of energies

- When the number of β particles is plotted against kinetic energy, the graph shows a curve
- This demonstrates that **beta particles (electrons or positrons) have a continuous range of energies**
- This is because the energy released in beta decay is shared between the **beta particles** (electrons or positrons) and **neutrinos** (or anti-neutrinos)
- This was one of the first clues of the neutrino's existence
- The principle of conservation of momentum and energy applies in both alpha and beta emission

Exam Tip

One way to remember which particle decays into which depends on the type of beta emission, think of beta '**p**lus' as the '**p**roton' that turns into the neutron (plus an electron neutrino)

YOUR NOTES
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12.2.6 The Law of Radioactive Decay

YOUR NOTES
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The Law of Radioactive Decay

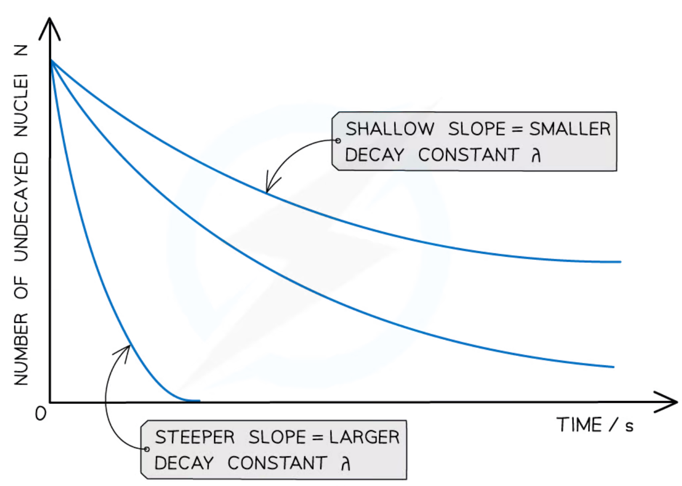
- Since radioactive decay is spontaneous and random, it is useful to consider the average number of nuclei which are expected to decay per unit of time
  - This is known as the **average decay rate**
- As a result, each radioactive element can be assigned a **decay constant**
- The decay constant λ is defined as:

The probability that an individual nucleus will decay per unit of time

- When a sample is highly radioactive, this means the number of decays per unit time is very high
  - This suggests it has a high level of **activity**
- Activity, or the number of decays per unit time can be calculated using:

$$A = -\frac{\Delta N}{\Delta t} = \lambda N$$

- Where:
  - A = activity of the sample (Bq)
  - ΔN = number of decayed nuclei
  - Δt = time interval (s)
  - λ = decay constant (s^{-1})
  - N = number of nuclei remaining in a sample
- In radioactive decay, the number of undecayed nuclei falls very rapidly, without ever reaching zero
  - Such a model is known as **exponential decay**
- The graph of number of undecayed nuclei against time has a very distinctive shape:



Radioactive decay follows an exponential pattern. The graph shows three different isotopes each with a different rate of decay

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- **The key features of this graph are:**
  - The steeper the slope, the larger the decay constant λ (and vice versa)
  - The decay curves always start on the y-axis at the initial number of undecayed nuclei (N_0)
- The law of radioactive decay states:

  The rate of decay of a nuclide is proportional to the amount of radioactive material remaining

- The number of undecayed nuclei N can be represented in exponential form by the equation:

$$N = N_0 e^{-\lambda t}$$

- Where:
  - N_0 = the initial number of undecayed nuclei (when t = 0)
  - N = number of undecayed nuclei at a certain time t
  - λ = decay constant (s^{-1})
  - t = time interval (s)
- The number of nuclei can be substituted for other quantities
- For example, the activity A is directly proportional to N, so it can also be represented in exponential form by the equation:

$$A = A_0 e^{-\lambda t}$$

- Where:
  - A = activity at a certain time t (Bq)
  - A_0 = initial activity (Bq)
- The received count rate C is related to the activity of the sample, hence it can also be represented in exponential form by the equation:

$$C = C_0 e^{-\lambda t}$$

- Where:
  - C = count rate at a certain time t (counts per minute or cpm)
  - C_0 = initial count rate (counts per minute or cpm)

Exam Tip

The symbol e represents the exponential constant - it is approximately equal to e = 2.718

On a calculator, it is shown by the button e^x

The inverse function of e^x is ln(y), known as the natural logarithmic function - this is because, if e^x = y, then x = ln(y)

Make sure you are confident using the exponential and natural logarithmic functions, they are a major component of the mathematics in this topic!

Problems Involving the Radioactive Decay Law

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⬇

Worked Example

Strontium-90 decays with the emission of a β-particle to form Yttrium-90. The decay constant of Strontium-90 is 0.025 year^{-1}.

Determine the activity A of the sample after 5.0 years, expressing the answer as a fraction of the initial activity A_0.

Step 1: Write out the known quantities

- Decay constant, $\lambda = 0.025$ year^{-1}
- Time interval, $t = 5.0$ years
- Both quantities have the same unit, so there is no need for conversion

Step 2: Write the equation for activity in exponential form

$$A = A_0 e^{-\lambda t}$$

Step 3: Rearrange the equation for the ratio between *A* and A_0

$$\frac{A}{A_0} = e^{-\lambda t}$$

Step 4: Calculate the ratio A/A_0

$$\frac{A}{A_0} = e^{-(0.025 \times 5)} = 0.88$$

- Therefore, the activity of Strontium-90 decreases by a factor of 0.88, or 12%, after 5 years

Worked Example

Americium-241 is an artificially produced radioactive element that emits α-particles.

In a smoke detector, a sample of americium-241 of mass 5.1 μg is found to have an activity of 5.9×10^5 Bq. The supplier's website says the americium-241 in their smoke detectors initially has an activity level of 6.1×10^5 Bq.

(a)
Determine the number of nuclei in the sample of americium-241.
(b)
Determine the decay constant of americium-241.
(c)
Determine the age of the smoke detector in years.

Part (a)

Step 1: Write down the known quantities

- Mass = 5.1 μg = 5.1×10^{-6} g

YOUR NOTES
⬇

- Molecular mass of americium = 241
- N_A = the Avogadro constant

Step 2: Write down the equation relating to the number of nuclei, mass and molecular mass

$$\text{Number of nuclei} = \frac{\text{mass} \times N_A}{\text{molecular mass}}$$

Step 3: Calculate the number of nuclei

$$\text{Number of nuclei} = \frac{(5.1 \times 10^{-6}) \times (6.02 \times 10^{23})}{241} = 1.27 \times 10^{16}$$

Part (b)

Step 1: Write down the known quantities

- Activity, $A = 5.9 \times 10^5$ Bq
- Number of nuclei, $N = 1.27 \times 10^{16}$

Step 2: Write the equation for activity

Activity, $A = \lambda N$

Step 3: Rearrange for decay constant λ and calculate the answer

$$\lambda = \frac{A}{N} = \frac{5.9 \times 10^5}{1.27 \times 10^{16}} = 4.65 \times 10^{-11}\ \text{s}^{-1}$$

Part (c)

Step 1: Write down the known quantities

- Activity, $A = 5.9 \times 10^5$ Bq
- Initial activity, $A_0 = 6.1 \times 10^5$ Bq
- Decay constant, $\lambda = 4.65 \times 10^{-11}\ \text{s}^{-1}$

Step 2: Write the equation for activity in exponential form

$$A = A_0 e^{-\lambda t}$$

Step 3: Rearrange for time t

$$\frac{A}{A_0} = e^{-\lambda t}$$

$$\ln\left(\frac{A}{A_0}\right) = -\lambda t$$

$$t = -\frac{1}{\lambda}\ln\left(\frac{A}{A_0}\right)$$

YOUR NOTES
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Step 4: Calculate the age of the smoke detector and convert to years

$$t = -\frac{1}{4.65 \times 10^{-11}} \ln\left(\frac{5.9 \times 10^5}{6.1 \times 10^5}\right) = 7.169 \times 10^8 \text{ s}$$

$$t = \frac{7.169 \times 10^8}{24 \times 60 \times 60 \times 365} = 22.7 \text{ years}$$

- Therefore, the smoke detector is 22.7 years old

12.2.7 Measuring Half-Life

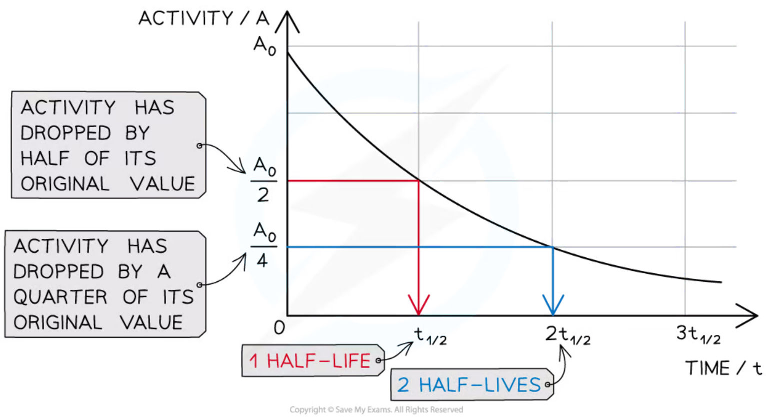
YOUR NOTES

Measuring Half-Life

- Half-life is defined as:

The time taken for the initial number of nuclei to halve for a particular isotope

- This means when a time equal to the half-life has passed, the **activity** of the sample will also half
- This is because the activity is proportional to the number of undecayed nuclei, $A \propto N$



When a time equal to the half-life passes, the activity falls by half, when two half-lives pass, the activity falls by another half (which is a quarter of the initial value)

- To find an expression for half-life, start with the equation for exponential decay:

$$N = N_0 e^{-\lambda t}$$

- Where:
  - N = number of nuclei remaining in a sample
  - N_0 = the initial number of undecayed nuclei (when t = 0)
  - λ = decay constant (s^{-1})
  - t = time interval (s)

- When time t is equal to the half-life $t_{½}$, the activity N of the sample will be half of its original value, so $N = ½ N_0$

$$\frac{1}{2} N_0 = N_0 e^{-\lambda t_{½}}$$

- The formula can then be derived as follows:

Divide both sides by N_0: $\frac{1}{2} = e^{-\lambda t_{½}}$

Take the natural log of both sides: $\ln\left(\frac{1}{2}\right) = -\lambda t_{½}$

Apply properties of logarithms: $\lambda t_{½} = \ln(2)$

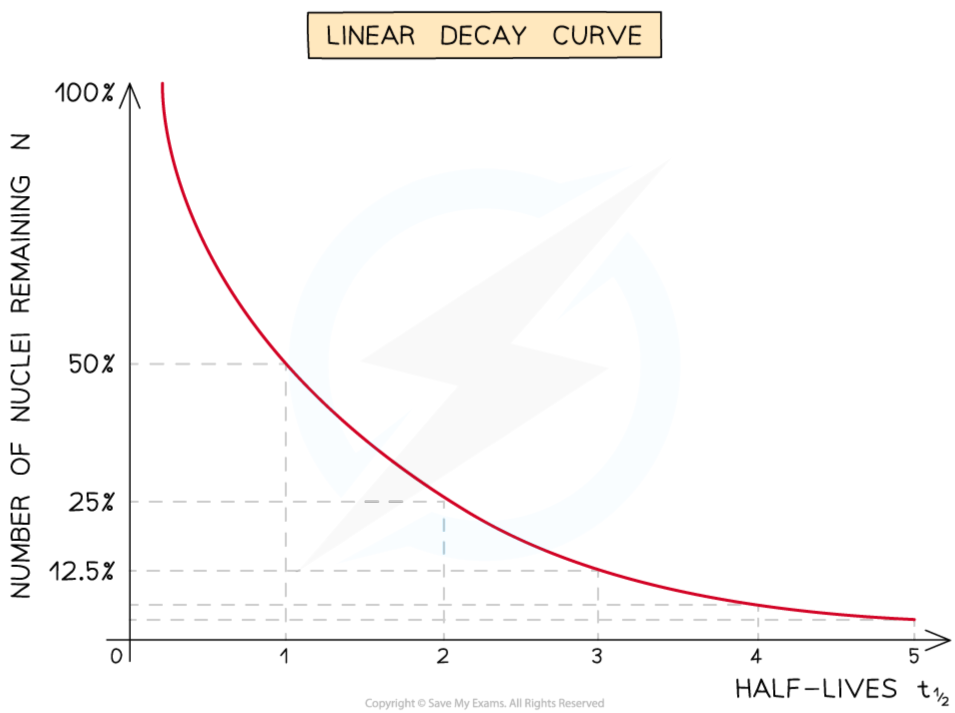
- Therefore, half-life $t_{½}$ can be calculated using the equation:

$$t_{½} = \frac{ln\,2}{\lambda} \simeq \frac{0.693}{\lambda}$$

YOUR NOTES

- This equation shows that half-life $t_{½}$ and the radioactive decay rate constant λ are inversely proportional
  - Therefore, the **shorter** the half-life, the **larger** the decay constant and the **faster** the decay

- The half-life of a radioactive substance can be determined from decay curves and log graphs
- Since half-life is the **time taken for the initial number of nuclei, or activity, to reduce by half**, it can be found by
  - Drawing a line to the curve at the point where the activity has dropped to half of its original value
  - Drawing a line from the curve to the time axis, this is the half-life



A linear decay curve. This represents the relationship: $\frac{\Delta N}{\Delta t} = -\lambda N$

Measuring Long Half-Lives

- For nuclides with long half-lives, on the scale of years, this can be measured by:
  - Measuring the mass of the nuclide in a pure sample
  - Determining the number of atoms N in the sample using $N = nN_A$
  - Measuring the total activity A of the sample using the counts collected by a detector
  - Determining the decay constant using $\lambda = \frac{A}{N}$
  - Calculating half-life using $t_{1/2} = \frac{\ln 2}{\lambda}$

YOUR NOTES
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- **Note:** The sample must be sufficiently large enough in order for a significant number of decays to occur per unit time so that an accurate measure of activity can be made

Measuring Short Half-Lives

- For nuclides with short half-lives, on the scale of seconds, hours or days, this can be measured by:
  - Measuring the background count rate in the laboratory (to subtract from each reading)
  - Taking readings of the count rate against time until the value equals that of the background count rate (i.e. until all of the sample has decayed)
  - Plotting a graph of activity, A, against time, t (as corrected count rate ∝ activity, A)
  - Making at least 3 estimates of half-life from the graph and taking a mean

  OR

  - Plotting a graph of ln N against time, t (as corrected count rate ∝ number of nuclei in the sample, N)
  - Finding the gradient of this graph, which gives $-\lambda$
  - Calculating half-life using $t_{1/2} = \frac{\ln 2}{\lambda}$

- Straight-line graphs tend to be more useful than curves for interpreting data
  - Due to the exponential nature of radioactive decay logarithms can be used to achieve a straight line graph

- Take the exponential decay equation for the number of nuclei

$$N = N_0 e^{-\lambda t}$$

- Taking the natural logs of both sides

$$\ln N = \ln (N_0) - \lambda t$$

- In this form, this equation can be compared to the equation of a straight line

$$y = mx + c$$

- Where:
  - $\ln (N)$ is plotted on the y-axis
  - t is plotted on the x-axis
  - gradient = $-\lambda$
  - y-intercept = $\ln (N_0)$

- Half-lives can be found in a similar way to the decay curve but the intervals will be regular as shown below:

YOUR NOTES
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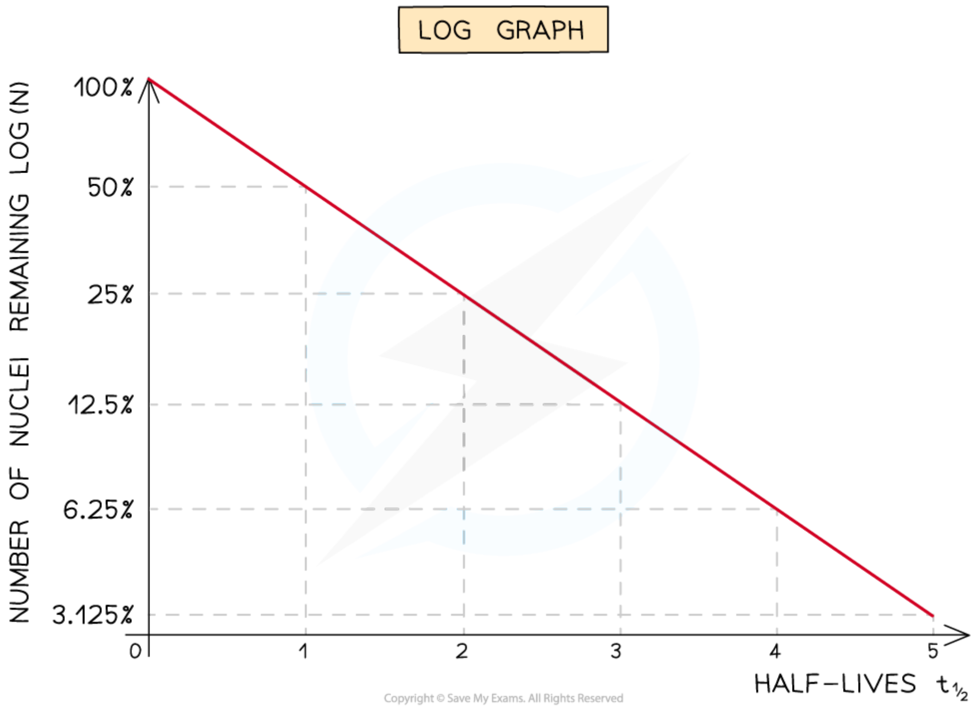

A logarithmic graph. This represents the relationship: $\ln N = -\lambda t + \ln N_0$

- Note: experimentally, the measurement generally taken is the count rate of the source
  - Since count rate ∝ activity ∝ number of nuclei, the graphs will all take the same shapes when plotted against time (or number of half-lives) linearly or logarithmically

Worked Example

Strontium-90 is a radioactive isotope with a half-life of 28.0 years. A sample of Strontium-90 has an activity of 6.4×10^9 Bq.

(a)
Calculate the decay constant λ, in year^{-1}, of Strontium-90.
(b)
Determine the fraction of the sample remaining after 50 years.

Part (a)

Step 1: List the known quantities

- Half-life, $t_{1/2}$ = 28 years

Step 2: Write the equation for half-life

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Step 3: Rearrange for λ and calculate

YOUR NOTES

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{28} = 0.025 \text{ year}^{-1}$$

Part (b)

Step 1: List the known quantities

- Decay constant, $\lambda = 0.025$ year^{-1}
- Time passed, $t = 50$ years

Step 2: Write the equation for exponential decay

$$N = N_0 \, e^{-\lambda t}$$

Step 3: Rearrange for $\frac{N}{N_0}$ and calculate

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-(0.025) \times 50} = 0.287$$

- Therefore, **28.7%** of the sample will remain after 50 years

YOUR NOTES
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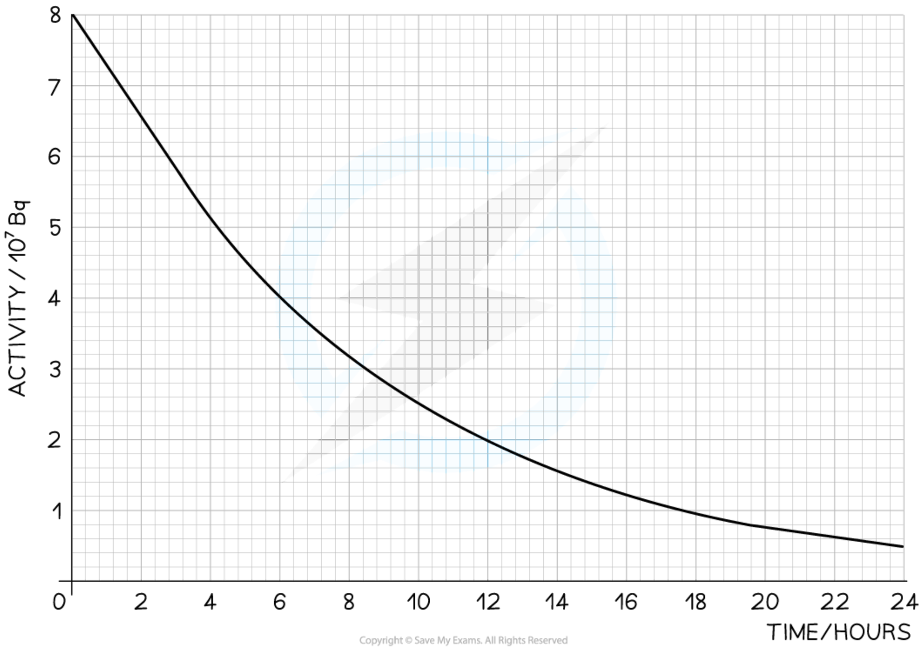
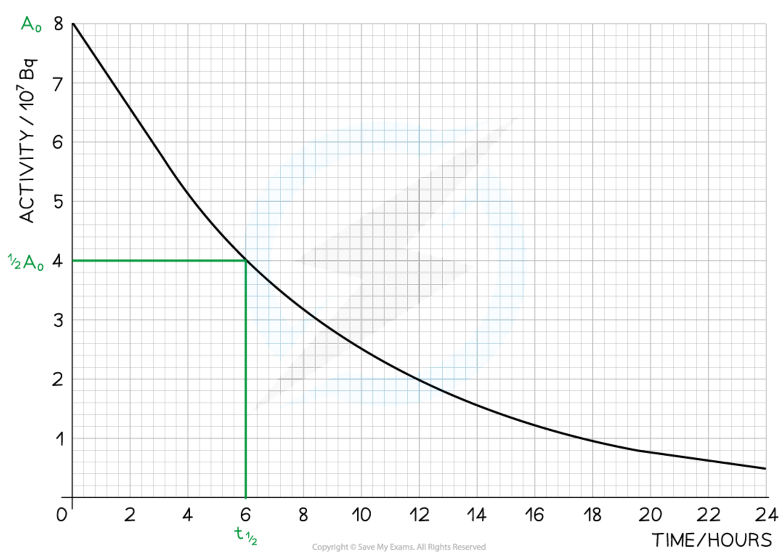
Worked Example

The radioisotope technetium is used extensively in medicine. The graph below shows how the activity of a sample varies with time.



Determine the number of technetium atoms remaining in the sample after 24 hours.

Step 1: Draw lines on the graph to determine the time it takes for technetium to drop to half of its original activity



Step 2: Read the half-life from the graph and convert to seconds

- $t_{1/2}$ = 6 hours = 6 × 60 × 60 = 21 600 s

YOUR NOTES
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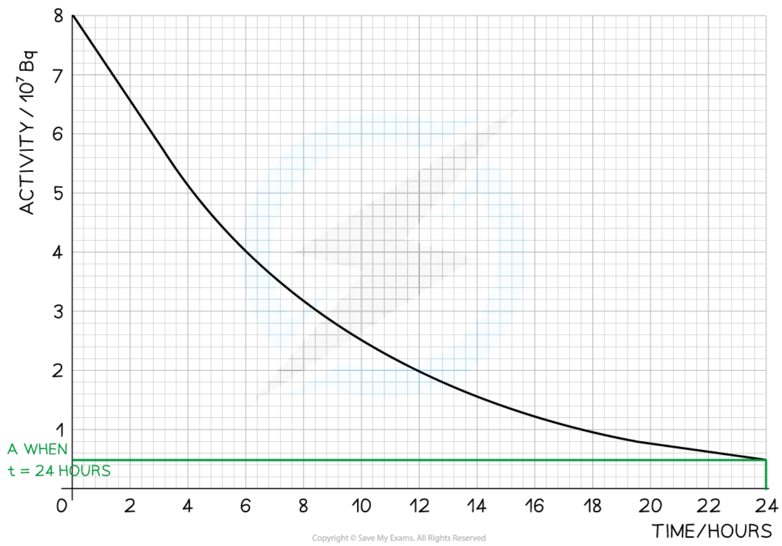
Step 3: Write out the half life equation

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Step 4: Calculate the decay constant

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{21\ 600} = 3.2 \times 10^{-5}\ s^{-1}$$

Step 5: Draw lines on the graph to determine the activity after 24 hours



- At t = 24 hours, $A = 0.5 \times 10^7$ Bq

Step 6: Write out the activity equation

$$A = \lambda N$$

Step 7: Calculate the number of atoms remaining in the sample

$$N = \frac{A}{\lambda} = \frac{0.5 \times 10^7}{3.2 \times 10^{-5}} = 1.56 \times 10^{11}$$

YOUR NOTES
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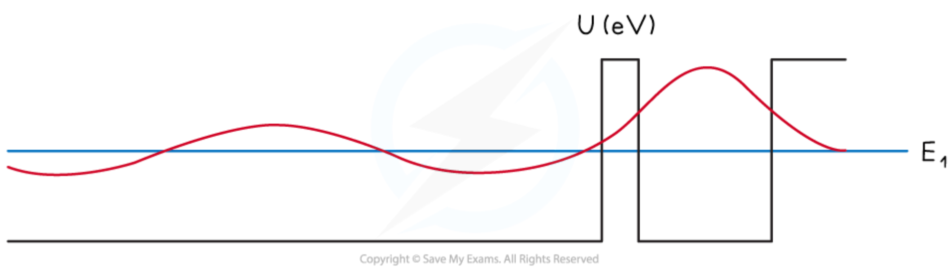
12.1 The Interaction of Matter with Radiation

12.1.10 Tunnelling

Tunnelling

- **Single potential wells** lead to quantised energy levels and their associated wavefunctions
  - The wavefunction extends throughout space
- However, for **infinitely deep square wells** the wavefunctions are localised within the well region
  - The probability of finding the quantum particle at the barrier is zero
- For a **finite barrier**, the wavefunction can penetrate the barrier
  - So, the particle has some probability of being in a "**classically forbidden region**"
- If there are two well-like regions, the solution of Schrodinger's equation gives an energy level and wavefunction that extends over the whole region of the potential well
- When the red wave function is squared it gives the probability of finding the particle in a particular region of space
- Since the wave function extends through the barrier this means there is a finite probability of finding the particle in **either** of the two well regions



A thin barrier or classically forbidden region can result in tunnelling

- Consequently, if a quantum particle were placed in the narrow well on the right, it is possible at some time later to find it in the region on the left
  - The particle is said to have **tunnelled** through the **narrow barrier**

Tunnelling & Alpha Decay

- The **strong nuclear force** within the nucleus is represented by the **square well**
  - While the $\frac{1}{r}$-dependence of the Coulombic repulsion dominates **outside the well**
- Nucleons in the nucleus have **quantised energy levels** and **wave functions**
  - An alpha particle can gain energy and occupy an excited energy level where the **barrier width** is **smaller**
- As a result, the alpha particle can tunnel through the classically forbidden region
  - This greatly **increases** the **probability** of the alpha particle being emitted

YOUR NOTES
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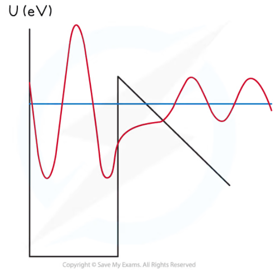


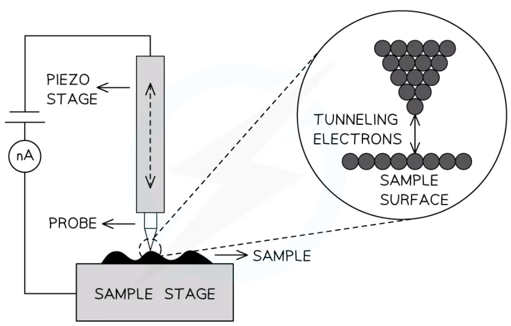
Alpha decay through quantum mechanical tunnelling

Uses of Quantum Tunnelling

- Quantum tunnelling is utilised in several systems, for example in:
  - Semiconductor devices
  - Fusion reactions in the Sun
  - A scanning tunnelling microscope

- In one mode of operation of a scanning tunnelling microscope, a sharp point, one atom thick, is maintained close to a surface
  - This is so that a small tunnelling **current** between the tip and the surface remains **constant**
  - In this case, the gap between the tip and the sample surface acts as the barrier that the **electrons** must tunnel through

- The tip is moved up and down and across the surface by **piezoelectric transducers** allowing the sample surface to be mapped out



Simplified schematic of a scanning tunnelling microscope