3.7 Inverse & Reciprocal Trig Functions

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.7 Inverse & Reciprocal Trig Functions
Difficulty	Medium

Time allowed: 80

Score: /64

Percentage: /100

Question 1a

(a)

State the value of $\arctan\left(\sqrt{3}\right)$.

[1 mark]

Question 1b

(b)

If $\arccos x = \frac{\pi}{6}$ find

(i)

the exact value of $\arcsin x$.

(ii)

the exact value of $\sec(\arccos x)$.

[6 marks]

Question 2

Find the exact values of the following expressions:

(i)

$\operatorname{cosec}\left(\frac{\pi}{3}\right)+\tan\left(\frac{\pi}{6}\right)$

(ii)

$3\sin\left(\frac{\pi}{4}\right)-\cot\left(\frac{\pi}{3}\right)$

[6 marks]

Question 3a

a)

Sketch the graph of $y=\cot x$ for $-\pi \leq x \leq \pi$.

[2 marks]

Question 3b

b)

Given that $\cot\theta = \frac{9}{7}$ and $\pi \leq \theta \leq \frac{3\pi}{2}$, find the values of $\cos\theta$, $\sin\theta$ and $\tan\theta$.

[5 marks]

Question 4

Solve $\tan^2 x = \sec x + 11$ for $0 \leq x \leq \pi$.

[5 marks]

Question 5a

a)
Show that the equation

$$\sec\theta - 5\cos\theta = 2\sqrt{2}$$

can be rewritten as

$$5\cos^2\theta + 2\sqrt{2}\cos\theta - 1 = 0$$

[3 marks]

Question 5b

b)

Hence, solve the equation $\sec\theta - 5\cos\theta = 2\sqrt{2}$ for all values of θ in the interval $-\pi \leq \theta \leq \frac{\pi}{2}$.

[3 marks]

Question 6a

A function f can be defined by $f(x) = 3x - 5x\arcsin(x)$, where $-1 \leq x \leq 1$.

a)
Sketch the graph of f indicating clearly any intercepts with the coordinate axes and the coordinates of any local maximum or minimum points.

[3 marks]

Question 6b

b)
State the domain and range of f.

[2 marks]

Question 6c

c)
Solve the inequality $3x - 5x \arcsin(x) > -2$.

[3 marks]

Question 7a

The function f is defined as $f(x) = \arccos x, -1 \leq x \leq 1$, and the function g is such that $g(x) = f(3x)$.

a)
Sketch the graph of $y = f(x)$ and state the range of f.

[3 marks]

Question 7b

b)
Sketch the graph of $y = g(x)$ and state the domain of g.

[3 marks]

Question 7c

c)
Find the inverse function $g^{-1}(x)$ and state its domain.

[2 marks]

Question 8a

a)
Show that $\sec\theta \cot\theta \equiv \operatorname{cosec}\theta$.

[2 marks]

Question 8b

b)
Hence solve in the range $0 \leq \theta \leq 2\pi$, the equation $\sec\theta \cot\theta = -2$

[3 marks]

Question 9a

a)
Show that the equation

$$\tan^2 x = 6 \sec x - 10$$

can be rewritten in the form

$$(\sec x - 3)^2 = 0$$

[3 marks]

Question 9b

b)
Hence, solve the equation $\tan^2 x = 6 \sec x - 10$ in the range $0 \leq x \leq 2\pi$.

[3 marks]

Question 10a

a)
Show that the equation

$$\cot^2 x = 9 - 3 \operatorname{cosec} x$$

can be rewritten in the form

$$(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 5) = 0.$$

[3 marks]

Question 10b

b)
Hence, solve the equation $\cot^2 x = 9 - 3 \operatorname{cosec} x$ in the interval $-180° \leq x \leq 180°$. Give your answers correct to 1 decimal place.

[3 marks]