1.7 Permutations & Combinations

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.7 Permutations & Combinations
Difficulty	Very Hard

Time allowed: 90

Score: /73

Percentage: /100

Question 1a

Consider a circular clock with the numbers 1 through 12 spaced evenly around the outside of the clock face.

Lines are drawn connecting each of the 12 numbers with each of the other numbers.

(a)
Find the number of lines that
(i)
are drawn from the number 12.
(ii)
are drawn in total.

[3 marks]

Question 1b

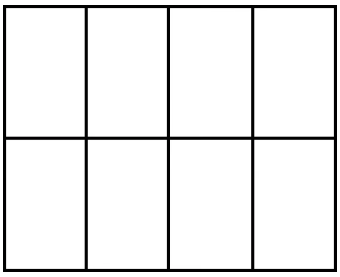
Consider the triangles formed on the face of the clock by groups of three of the of the above lines, where the vertices of each triangle lie at three distinct numbers on the clock face. For example, the lines from 12 to 4, and 4 to 7, and 7 to 12 will form one of these triangles.

(b)
Find the number of triangles that
(i)
have one vertex at the number 4
(ii)
are made by all of the lines.

[4 marks]

Question 2a

A horse-riding club uses 8 small fields arranged in two rows of four to allow its horses to graze.



The riding club has 8 horses in total, but three of the horses – named Angel, Blue and Chance – get agitated if they are placed too close to each other for too long.

For each of the following sets of conditions, find the number of possible ways of placing the 8 horses in the 8 fields.

(a)
Each field may contain up to 6 horses. Angel, Blue and Chance, however, must each be in a separate field.

[4 marks]

Question 2b

(b)
Each field may contain at most one horse. Additionally Angel must be in one of the corner fields, and Angel, Blue and Chance must not be placed in fields which share any boundaries (it is okay if the fields meet at a corner, but they may not share a side).

[4 marks]

Question 3a

Consider the ten letters in the word QUARANTINE.

(a)
Find the number of ways that four letters can be chosen at random from the word QUARANTINE if the selection may contain
(i)
no As and no Ns
(ii)
exactly one A and no Ns
(iii)
exactly one A.

[4 marks]

Question 3b

(b)
Find the number of distinct ways four out of the ten letters of the word QUARANTINE can be arranged if in each case all four letters are different from each other.

[3 marks]

Question 4a

Consider the eight letters in the word CAMPFIRE.

a)
Find the number of ways in which the eight letters may be arranged if
(i)
there are exactly two letters in between the letter A and the letter R
(ii)
reading left to right, the letter C is before the letter A and the letter A is before the letter M
(iii)
the word FIRE is written uninterrupted somewhere within the arrangement.

[5 marks]

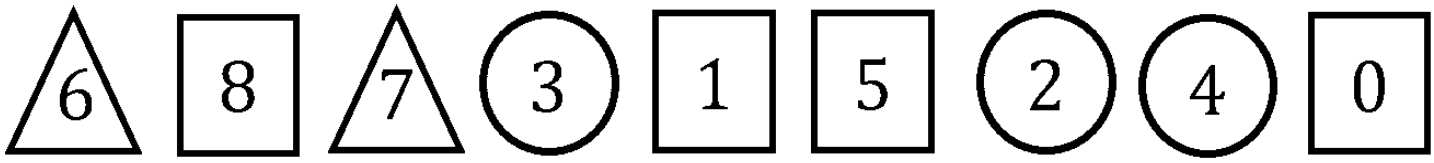
Question 4b

(b)
Find the number of ways in which the eight letters in the word CAMPFIRE may be arranged, given that at least two of the vowels must be next to each other

[3 marks]

Question 5a

Ahmed is playing a game with the following nine cards:



(a)
Ahmed arranges the cards to form a 9–digit number. Find how many different 9–digit numbers can be made such that the number is a multiple of five and the circular cards are not all together.

[4 marks]

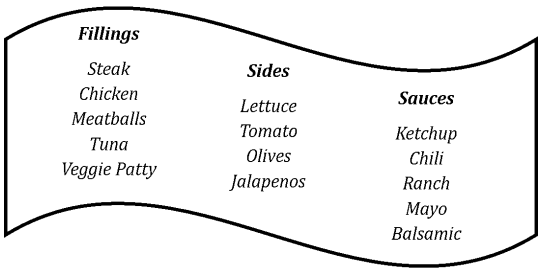
Question 5b

(b)
Ahmed chooses one triangular card, one circular card and one rectangular card at random and arranges them to make a 3–digit code. Find the number of different 3–digit codes Ahmed could make.

[3 marks]

Question 6a

Jonni always struggles to decide which combination of fillings, sides and sauces he wants to put in his sandwich at his favourite sandwich shop. The available options are listed below:



To avoid having to make up his mind, each day Jonni instead sets himself a rule for how many of each option he will put in his sandwich, and then uses an app he has designed to choose one sandwich at random from among those that his rule allows. Note that choosing the same filling, side or sauce more than once is never allowed.

(a)
If Jonni's rule on a given day is that he will have one filling, three sides and one type of sauce in his sandwich, find the probability that Jonni has either steak or chicken in his sandwich.

[1 mark]

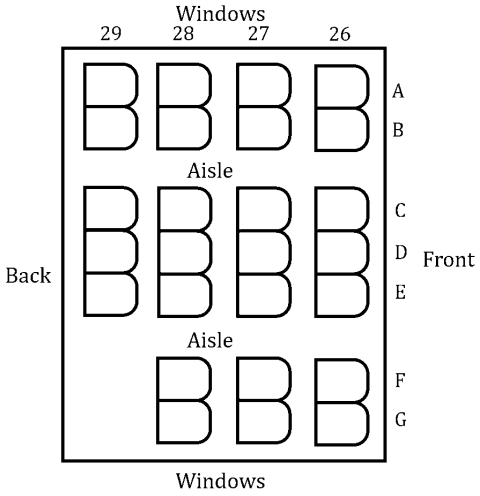
Question 6b

(b)
If Jonni's rule on a given day is that he will have two fillings, one side and three types of sauce in his sandwich, find the probability that he gets either tuna with tomato, or jalapenos with ketchup and mayo, as a part of his sandwich choice.

[4 marks]

Question 7a

Twenty-six people are travelling on an airplane. Seven of them are businesspeople on their way to a facilities management conference. Three of them are spies on their way to take up positions as 'consular officials' in an unnamed embassy. The other sixteen are cryptozoologists on their way to follow up some clues regarding a recent unicorn sighting. These passengers have been randomly allocated the seats from 26A to 29E inclusive, as depicted in the diagram below.



(a)
Find the number of ways that all twenty-six people could be seated on the plane if the three spies are all sat in the back row and the seven businesspeople are all in seats by a window.

[4 marks]

Question 7b

(b)
Given that each of the seven window seats has a cryptozoologist in it, find the probability that the three spies are not all sat together in one of the groups of three seats in between the two aisles.

[5 marks]

Question 8a

The following four family groups, consisting of 6 adults and 12 children in total, are all going to the cinema together:

- Mr and Mrs Mitchell and their two children
- Mrs and Mrs Lee and their four children
- Mr Kim and his three children
- Ms Miller and her three children

(a)
Find the number of different ways that all 18 people can sit in a row of 18 seats if
(i)
no adults are sat next to each other
(ii)
each family group sits together.

[4 marks]

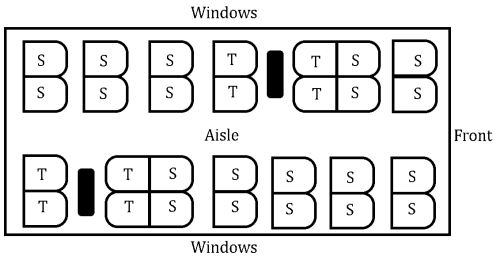
Question 8b

(b)
If instead the 18 people are sat randomly in three rows of six, what is the probability that all of the adults will be sat in the back row?

[3 marks]

Question 9a

The diagram below shows part of a train carriage. There are twenty-eight seats consisting of twenty normal seats, S, and eight table seats, T. In this part of the train carriage there is a group of twelve schoolchildren and two teachers, along with a married couple, four businessmen, and three backpackers.



(a)
Find the number of ways that the passengers could be arranged in this part of the train carriage if
(i)
the passengers are arranged completely at random
(ii)
all four businessmen want to sit together facing each other around one of the tables.

[4 marks]

Question 9b

(b)
Given that everybody is arranged completely at random within this part of the train carriage, find the probability that
(i)
the three backpackers are sat in the front and the two teachers are sat together on one side of the aisle
(ii)
the schoolchildren are all on one side of the aisle and the married couple are sat together on the other side of the aisle.

[5 marks]

Question 10

A total of *n* tasks are to be shared among *m* people, where each of the people is uniquely identified by a number between 1 and m. Person 1 is to receive k_1 tasks, Person 2 is to receive k_2 tasks, and so on, with the integers $k_1, k_2, ..., k_m$ being such that $k_1, k_2 ..., k_m \geq 0$ and $k_1 + k_2 + ... + k_m = n$.

Show that the number of different ways in which the tasks can be assigned according to the above rules is

$$\frac{n!}{k_1!k_2!...k_m!}$$

[6 marks]