

3.5 Voronoi Diagrams

Question Paper

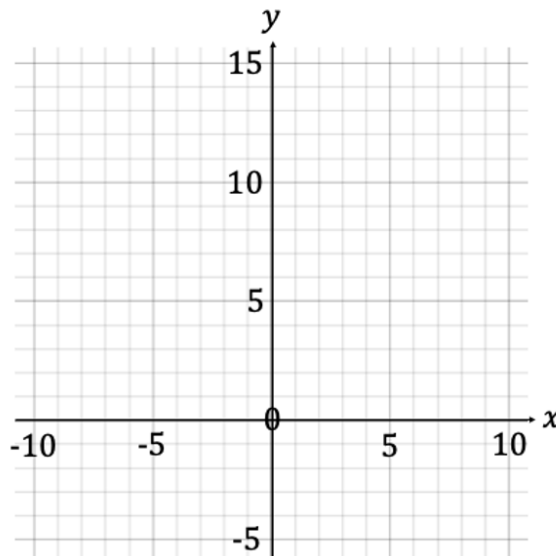
Course	DPIB Maths
Section	3. Geometry & Trigonometry
Topic	3.5 Voronoi Diagrams
Difficulty	Very Hard

Time allowed: 80
Score: /59
Percentage: /100

Question 1a

Three phone masts are erected in a triangular arrangement with the vertices of the triangle being $A(-1, 12)$, $B(7, 6)$ and $C(1, 2)$. A scientist is conducting some experiments in the area is using some sensitive electronic equipment and wishes to find a suitable location. The location that he chooses must be as far as possible from each of the three phone masts in order to minimise any potential interference. One unit on the graph represents 200 m.

(a) Draw a Voronoi diagram on the grid below.



[4 marks]

Question 1b

(b) Find the coordinates of the best position for the scientist to work.

[2 marks]

Question 1c

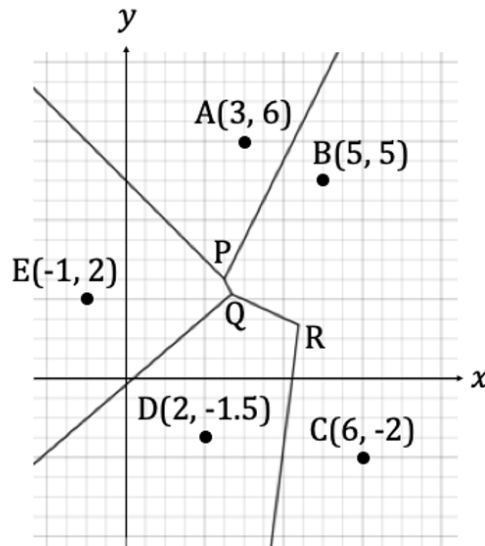
(c) Calculate the distance that the scientist will be from the nearest phone mast.

[2 marks]

Question 2a

The local government of an area wishes to install tsunami warning sirens in order to be able to alert residents in the case of natural disasters.

A Voronoi diagram is drawn to find the appropriate locations for the warning sirens to be installed, an incomplete copy can be seen below. Five towns lie at points $A(3, 6)$, $B(5, 5)$, $C(6, -2)$, $D(2, -1.5)$ and $E(-1, 2)$. The points of intersection of the perpendicular bisectors are P , Q and R . One unit on the grid represents 1 km.



- (a) Write down the equation of the line that will complete the cell containing town B. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

[4 marks]

Question 2b

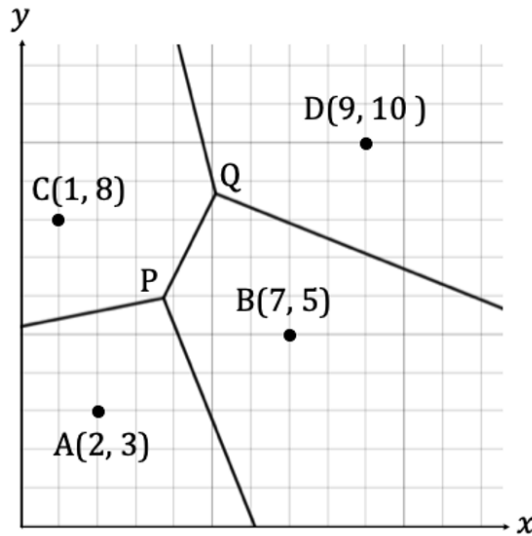
Each warning siren can be heard up to a distance of 4 km.

(b) State the minimum number of sirens required to reach all of the towns and write down their coordinates.

[6 marks]

Question 3a

A geography student wishes to observe the microclimate of their back garden to see if there are differences in how much rain falls in different areas. She places a rain gauge at four separate sites. The student decides to construct a Voronoi diagram, assuming that the rain collected at one site will be the same throughout the cell in which it is situated. The positions of the sites are indicated on the diagram below at point $A(2,3)$, $B(7,5)$, $C(1,8)$ and $D(9,10)$. One unit on the grid represents 10 m.



(a) Find the coordinates of the points P and Q where the cell boundaries intersect.

[5 marks]

Question 3b

The rainfall at the different sites over a 24-hour period is recorded in the table below.

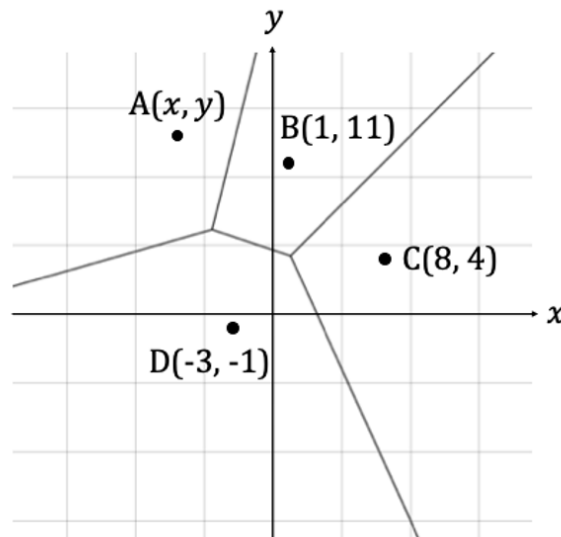
Site	Site A	Site B	Site C	Site D
Rainfall (mm)	22	24	3	26

(b) Give a reason that could explain why the rainfall at site C is different to the rainfall measured at the other sites.

[1 mark]

Question 4a

The Voronoi diagram below shows the location of four whirlpools at points $A(x, y)$, $B(1, 11)$, $C(8, 4)$ and $D(-3, -1)$. One unit on the diagram represents 500 m.



The perpendicular bisector of $[AB]$ has the equation $4x - y + 24 = 0$.

(a) Find the coordinates, (x, y) , at point A.

[6 marks]

Question 4b

A viewing station is to be constructed between the four whirlpools. The perpendicular bisector of $[BC]$ has the equation $y = x + 3$ and the perpendicular bisector of $[BD]$ has the equation $x + 3y - 14 = 0$.

- (b) (i) Find the coordinates of the point which is furthest from the whirlpools and therefore the safest location in which to construct the station.
- (ii) Write down the distance from the viewing station to the nearest whirlpool.

[4 marks]

Question 5a

A group of four pirates claim a circular island of radius 8 km, with the centre of the island located at point $P(0, 0)$ on their map. One unit on the map represents 1 km.

The pirates want to set up their bases with boundaries that are formed by finding the perpendicular bisectors between the pirates.

- (a) Write down the coordinates of the locations of each of the pirates' bases if they are all to have the same area of land in their cell.

[4 marks]

Question 5b

- (b) Calculate the area of land that is in each pirate's cell.

[2 marks]

Question 5c

A fifth pirate, the Captain who had been lost at sea, arrives at the island and establishes his base at point P. None of the other pirates move their bases.

(c) Calculate the percentage of their land that each of the original four pirates has to hand over to the Captain.

[4 marks]

Question 6

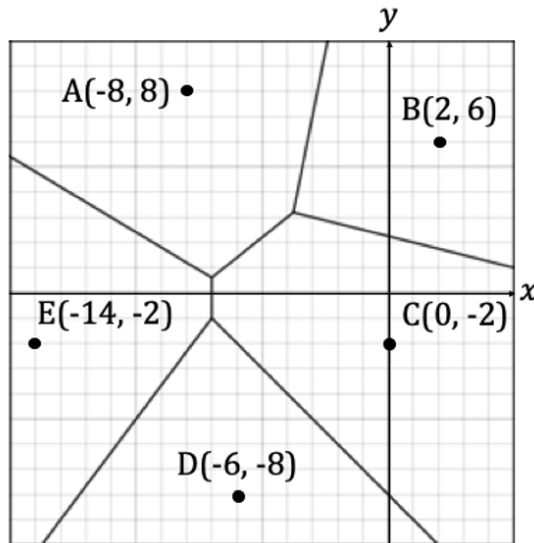
An energy company supplies five towns with gas in an area of 20 km by 20 km. The Voronoi diagram below shows the location of the towns at $A(-8, 8)$, $B(2, 6)$, $C(0, -2)$, $D(-6, -8)$ and $E(-14, -2)$. The boundaries of the cells are the main pipelines running between the towns, to which the towns will be connected.

The perpendicular bisector of $[AE]$ has the equation $3x + 5y + 18 = 0$

The perpendicular bisector of $[AC]$ has the equation $4x - 5y + 31 = 0$

The perpendicular bisector of $[DE]$ has the equation $4x - 3y + 25 = 0$

Each unit on the map represents 1 km.



Calculate the total length of the main pipelines in the area.

[15 marks]

