2.5 Reciprocal & Rational Functions

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.5 Reciprocal & Rational Functions
Difficulty	Medium

Time allowed: 80

Score: /63

Percentage: /100

Question 1a

Let $f(x) = \dfrac{2x + 1}{x - 4}, x \neq 4.$

(a) For the graph of f, find the equation of:

    (i)   the vertical asymptote

    (ii)   the horizontal asymptote.

[3 marks]

Question 1b

(b) Find $f^{-1}(x)$.

[2 marks]

Question 1c

(c) Write down the equation of the vertical asymptote to the graph of $f^{-1}(x)$.

[1 mark]

Question 2

Consider $f(x) = \frac{ax+b}{3x+c}$, for $x \neq -\frac{c}{3}$, where $a, b, c \in \mathbb{Z}$.

$y = -1$ and $x = 2$ are the equations of the asymptotes of the graph of f. Point $\text{A}\left(-2, -\frac{2}{3}\right)$ lies on the graph.

Find the values of a, b and c.

[6 marks]

Question 3a

Consider the function f defined by $f(x) = \frac{3}{x+5} - 2$, $x \in \mathbb{R}$, $x \neq p$.

(a) Write down the value of p.

[1 mark]

Question 3b

(b) Write down the equation of the horizontal asymptote to the graph of $y = f(x)$.

[1 mark]

Question 3c

(c) Show that $\dfrac{3}{x+5} - 2 = \dfrac{ax+b}{x+5}$, where a and b are constants to be determined.

[2 marks]

Question 3d

(d) Sketch the graph of $y = f(x)$.

[3 marks]

Question 4a

Let $f(x) = \frac{4x-2}{2x+5}$, for $x \neq -\frac{5}{2}$.

(a) For the graph of f, find the coordinates of

  (i) the x-intercept

  (ii) the y-intercept.

[3 marks]

Question 4b

(b) For the graph of f, find the equation of

  (i) the vertical asymptote

  (ii) the horizontal asymptote.

[3 marks]

Question 5a

Consider the function f defined by $f(x) = \dfrac{2(3x-1)}{(x+3)(x-2)}$, $x \in \mathbb{R}$, $x \neq -3, 2$.

(a) Find the coordinates of the points where the graph of $y = f(x)$ intersects the coordinate axes.

[2 marks]

Question 5b

(b) Express $f(x)$ as partial fractions.

[3 marks]

Question 5c

(c) Hence find the equation of the horizontal asymptote to the graph of $y = f(x)$.

[2 marks]

Question 6a

Consider the function $f(x) = \dfrac{4x-12}{x^2-4x-5}$, $x \in \mathbb{R}$, $x \neq -1, 5$.

(a) Find the coordinates of the points where the graph of $y = f(x)$ intersects the

(i) x-axis

(ii) y-axis.

[2 marks]

Question 6b

(b) Write down the equations of

(i) the vertical asymptotes

(ii) the horizontal asymptote

to the graph of $y = f(x)$.

[3 marks]

Question 6c

(c) By considering the value of f for large positive and large negative values of x, sketch the graph of f. Be sure to indicate clearly the points of intersection with the coordinate axes, as well as any asymptotes.

[4 marks]

Question 7a

Consider the function $f(x) = \frac{x^2+5x+6}{x+1}$, $x \in \mathbb{R}$, $x \neq -1$.

(a) Find the coordinates of the points where the graph of $y = f(x)$ intersects the

    (i) x-axis,

    (ii) y-axis.

[3 marks]

Question 7b

(b) Write down the equation of the vertical asymptote to the graph of $y = f(x)$.

[1 mark]

Question 7c

(c) (i) Show that $\dfrac{x^2+5x+6}{x+1} = x + a + \dfrac{b}{x+1}$, where a and b are constants to be determined.

(ii) Hence write down the equation of the oblique asymptote to the graph of $y = f(x)$.

[4 marks]

Question 7d

(d) Sketch the graph of $y = f(x)$. Be sure to indicate clearly the points of intersection with the coordinate axes, as well as any asymptotes.

[3 marks]

Question 8a

Let f be a function defined by $f(x) = 3x - 4 + \dfrac{2}{x-1}$, $x \in \mathbb{R}$, $x \neq p$.

(a) Write down

    (i)   the value of p

    (ii) the equation of the vertical asymptote to the graph of $y = f(x)$

    (iii) the equation of the oblique asymptote to the graph of $y = f(x)$.

[3 marks]

Question 8b

(b) Show that $f(x)$ can be written in the form $\dfrac{ax^2 + bx + c}{x-1}$, where a, b and c are constants to be determined.

[2 marks]

Question 8c

(c) Use an algebraic method to show that the graph of $y = f(x)$ does not cross the x-axis.

[3 marks]

Question 8d

(d) Sketch the graph of $y = f(x)$ and hence write down the range of the function f.

[3 marks]