# 2.2 Quadratic Functions & Graphs

## Question Paper

| | |
|---|---|
| Course | DP IB Maths |
| Section | 2. Functions |
| Topic | 2.2 Quadratic Functions & Graphs |
| Difficulty | Very Hard |

**Time allowed:** 100

**Score:** /81

**Percentage:** /100

## Question 1a

Consider $f(x) = x^2 + bx + c$. The graph of $y = f(x)$ has no real roots.

(a) Show that $b^2 < 4c$ and explain why $c$ must be a positive value.

**[2 marks]**

## Question 1b

The minimum point on the graph of $y = f(x)$ is $(3, 2)$.

(b) Find the value of $b$ and the value of $c$.

**[2 marks]**

## Question 1c

(c) Sketch the graph of $y = f(x)$, clearly labelling the minimum point and any points where the graph intersects coordinate axes.

**[3 marks]**

## Question 2

Consider the function $f(x) = \log_b(6x - x^2)$, for $0 < x < 6$, where $b > 0$.

The equation $f(x) = 2$ has exactly one solution.

Find the value of $b$.

**[6 marks]**

## Question 3

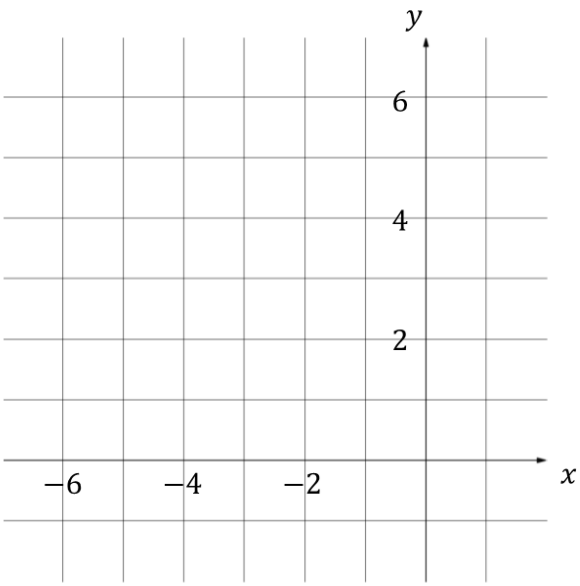
Let $f(x) = 2k + \frac{9}{x}$, for $x \neq 0$, where $k$ is a constant. The line $y = k - x$ does not intersect the graph of $y = f(x)$.

Find the possible values of $k$.

**[6 marks]**

## Question 4a

The graph of a quadratic function has equation $y = \frac{1}{4}x^2 + bx + c$, where $b, c \in \mathbb{Z}$, and the axis of symmetry is $x = -4$.




(a) Draw the axis of symmetry on the grid above.

**[1 mark]**

## Question 4b

The graph of the quadratic function intersects the $x$-axis at points $A(-6, 0)$ and B.

(b) (i) Write down the coordinates of B.

(ii) Find the values of $b$ and $c$.

**[3 marks]**

## Question 4c

(c) (i) Mark and label A and B on the grid above.

(ii) Write down the coordinates of the Vertex, V, and label it on the grid above.

(iii) Write down the coordinates of the $y$-intercept, C, and label it on the grid above.

(iv) Draw the graph of the quadratic function on the grid above.

**[4 marks]**

## Question 5a

Let $f(x) = 2kx^2 - (k+10)x + \frac{5}{2} + \frac{5}{8}k$, for $x \in \mathbb{R}$, where $k \in \mathbb{Z}$.

(a) Show that the discriminant of $f$ is $100 - 4k^2$.

**[3 marks]**

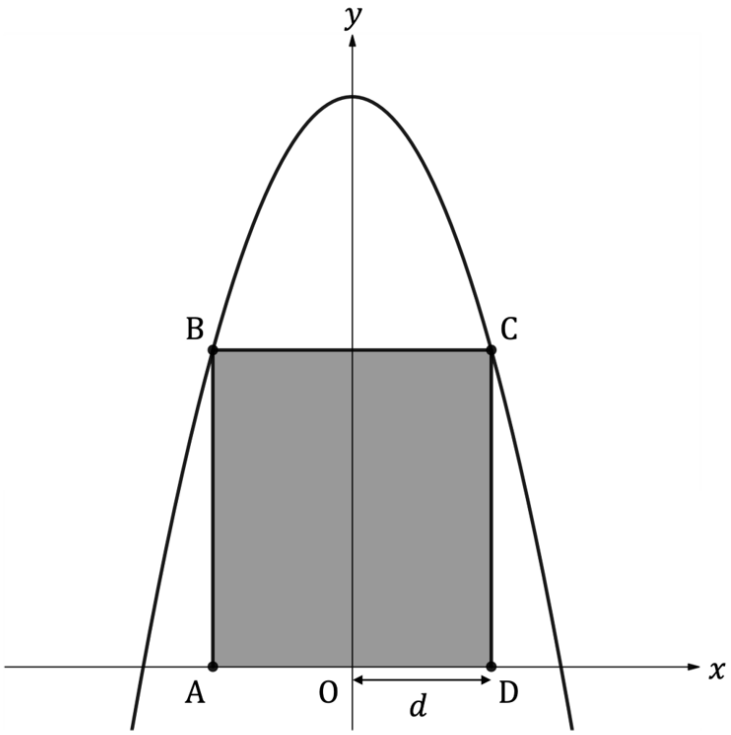
## Question 5b

(b) Find the values of $k$ such that the graph of $f$ has two **equal** roots.

**[3 marks]**

## Question 6a

Let $f(x) = 9 - x^2$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of $f$. Let $\text{OD} = d$.




(a) Find an expression in terms of $d$ for the area of the rectangle ABCD.

**[3 marks]**

## Question 6b

The coordinates of A are $(-2, 0)$.

(b) Find the area of ABCD.

**[2 marks]**

## Question 6c

Let $g(x) = (x-3)^2 + k$, for $x \in \mathbb{R}$, where $k$ is a constant.

(c) Given that the graphs of $f$ and $g$ intersect exactly once, find the value of $k$.

**[5 marks]**

## Question 7a

A tunnel is being constructed and its opening can be modelled by the quadratic function

$$h(x) = ax(b-x), \qquad x \geq 0,$$

where $h$ is the height of the tunnel, in metres, and $x$ is the width of the tunnel, in metres.

It is given that $h(10) = 10$ and $h(20) = 15$.

(a) Find the values of $a$ and $b$.

**[3 marks]**

## Question 7b

The height required for a lane of traffic is 5 m and each lane requires a width of 2.8 m.

(b) Find the number of lanes of traffic the tunnel can fit.

**[4 marks]**

## Question 8a

A company sells 55 cars per month for a sale price of $2000, whilst incurring costs for supplies, production and delivery of $890 per car. Reliable market research shows that for each increase (or decrease) of the sale price by $50 the company will sell 5 cars less (or more) and vice versa.

(a) Find an expression for the total profit, P, in terms of the sale price, $x$.

**[3 marks]**

## Question 8b

(b) Find the values of $x$ when $P(x) = 0$ and explain their significance in the context of the question.

**[2 marks]**

## Question 8c

(c) Calculate

(i) the maximum monthly profit, giving your answer to the nearest dollar.

(ii) the sale price needed to generate the maximum monthly profit.

(iii) the number of cars sold to generate the maximum monthly profit.

**[3 marks]**

## Question 9a

A company sells $L$ litres of water per month and their total monthly profit, $P$, can be modelled by the function

$$P(x) = (x - 0.45) \times N(x),$$

where $x$ is the sale price of each litre sold, in dollars, at and $N$ is the linear function for the number of litres the company can sell per month at each given sale price.

(a) In the context of the question, explain the significance of the 0.45.

**[1 mark]**

## Question 9b

It is given that $N(0.5) = 400$ and $N(1.25) = 250$.

(b) Write down the function of $N$, in the form $N(x) = mx + c$, where $m$ and $c$ are constants.

**[2 marks]**

## Question 9c

(c) Find the values of $x$ when $\mathrm{P}(x) = 0$ and explain their significance in the context of the question.

**[2 marks]**

## Question 9d

(d) Calculate

(i) the maximum monthly profit.

(ii) the sale price needed to generate the maximum monthly profit.

(iii) the number of litres sold to generate the maximum monthly profit.

**[3 marks]**

## Question 10

Let $f(x) = -(x-a)^2 + b$ and $g(x) = 4(x-c)^2 + d$, where $a, b, c, d \in \mathbb{R}$. The vertex of the graph of $f$ is at $(2k, 8k^2)$ and the vertex of the graph of $g$ is at $(k, 2k)$, where $0 < k < 1$. The graphs of $f$ and $g$ intersect at exactly one point. Find the value of $k$.

**[7 marks]**

## Question 11

Consider $f(x) = r(x - s)(x + 3)$. The graph of $f$ has an axis of symmetry at $x = -1$ and $y$-intercept at $(0, -6)$. The line $y = mx - 14$ is a tangent to the graph of $f$.

Find the possible values of $m$.

**[8 marks]**