3.6 Trigonometric Equations & Identities

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.6 Trigonometric Equations & Identities
Difficulty	Hard

Time allowed: 80

Score: /60

Percentage: /100

Question 1a

The value of $\tan\alpha = -\frac{3}{2}$ for $\frac{\pi}{2} \leq \alpha \leq \pi$.

(a) Find

(i) $\sin\alpha$

(ii) $\cos\alpha$

(iii) $\sin 2\alpha$

(iv) $\cos 2\alpha$

(v) $\tan 2\alpha$.

[7 marks]

Question 1b

(b) Use your results from part (a) to explain why $\pi < 2\alpha < \frac{3\pi}{2}$ must be true.

[2 marks]

Question 2a

The value of $\cos B = x$, for $\pi \leq B \leq \frac{3\pi}{2}$.

(a) Explain why

(i) $x \leq 0$

(ii) $\sin B = -\sqrt{1 - x^2}$.

[3 marks]

Question 2b

(b) Hence find the following in terms of x:

(i) $\cos 2B$

(ii) $\sin 2B$

(iii) $\tan 2B$.

[5 marks]

Question 3

An angle M is such that $\sin M = p$ and $\cos M = q$. Show that

(i) $\sin 4M = 4pq^3 - 4p^3q$

(ii) $\cos 4M = 8q^4 - 8q^2 + 1$.

[6 marks]

Question 4

Solve the equation $\sqrt{3}\cos 2\theta = 2\cos^2 2\theta$ in the interval $0 \leq \theta \leq 360°$.

[5 marks]

Question 5

Solve the equation $\sqrt{3}\tan x = \frac{1}{\sqrt{3}\tan x}$ for $0 \leq x \leq 540°$.

[5 marks]

Question 6a

(a) Use the fact that

$$16x^3 - 12x^2 - 4x + 3 = (4x - 3)(4x^2 - 1)$$

to fully factorise $16x^3 - 12x^2 - 4x + 3$.

[2 marks]

Question 6b

(b) Use your result from part (a) to solve the equation

$$16\sin^3 3\theta - 12\sin^2 3\theta - 4\sin 3\theta + 3 = 0$$

in the interval $0 \leq \theta \leq \frac{\pi}{2}$. You should give your answers as exact values where possible.

[7 marks]

Question 7

Solve the equation

$$\sin^2 x = \frac{\cos x + 4}{6}$$

in the interval $-\pi \leq x \leq \pi$. Give your answers as exact values where possible.

[6 marks]

Question 8a

Two functions, f and g, are defined by $f(x) = \sin x$ and $g(x) = \cos 2x$.

(a) Describe the single transformation of the graph of $y = \cos x$ that will produce the graph of $y = g(x)$.

[2 marks]

Question 8b

(b) On the same set of axes, sketch the graphs of $y = f(x)$ and $y = g(x)$ in the interval $-\pi \leq x \leq 3\pi$.

[4 marks]

Question 8c

(c) By using an appropriate trigonometric identity to solve the equation $\sin x = \cos 2x$ in the interval $-\pi \leq x \leq 3\pi$, determine the points of intersection of the two curves from your graph in part (b). Label those points on your graph.

[6 marks]