3.6 Trigonometric Equations & Identities

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.6 Trigonometric Equations & Identities
Difficulty	Very Hard

Time allowed: 80

Score: /66

Percentage: /100

Question 1a

The value of $\tan 2\alpha = \frac{2}{5}$ for $\frac{\pi}{2} \leq 2\alpha \leq \frac{3\pi}{2}$.

(a) Find

(i) $\sin 2\alpha$

(ii) $\cos 2\alpha$

[3 marks]

Question 1b

(b) Hence show that

$$\sin \alpha = \sqrt{\frac{k + 5\sqrt{k}}{2k}} \quad \text{and} \quad \cos \alpha = -\sqrt{\frac{k - 5\sqrt{k}}{2k}}$$

where k is a positive integer to be determined, and use those results to find the exact value of $\tan \alpha$.

[6 marks]

Question 2

The value of $\sin B = x$, for $\frac{\pi}{2} < B < \pi$.

Find the following in terms of x:

(i) $\sin 2B$

(ii) $\cos 2B$

(iii) $\tan 2B$.

[7 marks]

Question 3a

It is given that $\cos 6x = p$.

(a) Show that

$$\tan 3x = \pm\sqrt{\frac{1-p}{1+p}}$$

[5 marks]

Question 3b

(b) For $0 < x < \frac{\pi}{3}$, determine the range of x values for which

(i) the 'plus' version of the part (a) result should be used

(ii) the 'minus' version of the part (a) result should be used

(iii) the value of $\tan 3x$ is not defined.

[3 marks]

Question 4

Solve the equation

$$\sin 2\theta - \sin \theta + \sqrt{3} \cos \theta = \frac{\sqrt{3}}{2}$$

in the interval $0 \leq \theta \leq 360°$.

[7 marks]

Question 5

Solve the equation

$$\sin 2x \tan 2x = \frac{1}{2\cos 2x}$$

in the interval $-90° \leq x \leq 90°$.

[5 marks]

Question 6

Solve the equation

$$\frac{5\sin^2 x - 2}{\cos x} = 3 - \cos x$$

in the interval $-\pi \leq x \leq 2\pi$.

[6 marks]

Question 7a

(a) Use the fact that

$$3p^3 - \left(7 - 3\sqrt{3}\right)p^2 - \left(20 + 7\sqrt{3}\right)p - 20\sqrt{3} = (3p + 5)\left(p^2 + \left(\sqrt{3} - 4\right)p - 4\sqrt{3}\right)$$

to fully factorise $3p^3 - \left(7 - 3\sqrt{3}\right)p^2 - \left(20 + 7\sqrt{3}\right)p - 20\sqrt{3}$.

[2 marks]

Question 7b

Two functions, f and g, are defined by

$$f(x) = 3\tan^3 3x - \left(20 + 7\sqrt{3}\right)\tan 3x \quad \text{and} \quad g(x) = \left(7 - 3\sqrt{3}\right)\tan^2 3x + 20\sqrt{3}$$

for $-\frac{\pi}{6} \leq x \leq \frac{\pi}{3}$.

(b) Use an algebraic method along with your result from part (a) to determine the x-coordinates of the points of intersection of the curves $y = f(x)$ and $y = g(x)$.

Your solution should show clear algebraic working, and your answers should be given as exact values where possible.

[8 marks]

Question 8a

Let OAB be an isosceles triangle with $\mathrm{OA} = \mathrm{OB}$ and $\mathrm{A\widehat{O}B} = \theta$.

(a) If the length of line segment AB is denoted by p, and the area of triangle OAB is denoted by q, show that

$$\cos\theta = \frac{1-m}{1+m}$$

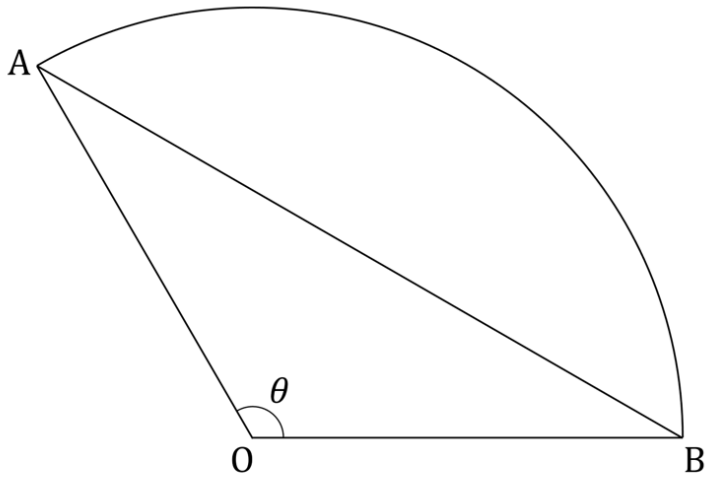
where

$$m = \frac{p^4}{16q^2}$$

[9 marks]

Question 8b

The diagram below shows circle sector OAB with centre O and angle at the centre $A\widehat{O}B = \theta$.



(b) Given that the length of chord AB is $2\sqrt{3}$ units, and that the area of triangle OAB is $\sqrt{3}$ units^2, find the area of sector OAB and the length of arc AB.

[5 marks]