

1.4 Simple Proof & Reasoning

Question Paper

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| Course | DPIB Maths |
| Section | 1. Number & Algebra |
| Topic | 1.4 Simple Proof & Reasoning |
| Difficulty | Very Hard |

Time allowed: 60
Score: /44
Percentage: /100

Question 1a

(a) (i) Prove that

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

(ii) Specify any cases for which the relation in part (a)(i) is **not** valid.

[3 marks]

Question 1b

(b) Prove that $(p - q)^2 = (q - p)^2$ for all numbers p and q .

[2 marks]

Question 2

Prove that the product of two odd numbers is odd.

[4 marks]

Question 3

The sum of squares of two consecutive integers is 313. Find the possible values of the integers.

[5 marks]

Question 4

Prove that the sum of the cubes of any two consecutive odd integers is divisible by four.

[5 marks]

Question 5a

(a) Prove that $\frac{a^2 - a - 6}{a + 4} \times \frac{a^2 - 16}{a^2 + 2a} = a - 7 + \frac{12}{a}$.

[4 marks]

Question 5b

(b) State any values of a for which this mathematical statement does **not** hold true.

[1 mark]

Question 6

Prove that there are no integers p and q that satisfy the equation

$$4p^2 - q^2 = 49$$

[4 marks]

Question 7

Prove the binomial coefficient identity

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}.$$

[8 marks]

Question 8

Prove that the sum of all integers between 600 and 1400 (inclusive) that are not divisible by 7 is equal to 685885.

[8 marks]