5.4 Further Integration

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.4 Further Integration
Difficulty	Medium

Time allowed: 80

Score: /65

Percentage: /100

Question 1a

(a) Find the indefinite integral for

$$\int \sin x \; \mathrm{d}x$$

[1 mark]

Question 1b

(b) Find the exact value for

$$\int_1^4 \frac{1}{x} \; \mathrm{d}x$$

[3 marks]

Question 1c

(c) Find the indefinite integral for

$$\int 7e^{7x} \; \mathrm{d}x$$

[2 marks]

Question 2a

(a) Integrate

$$\int \cos 2x \ \mathrm{d}x$$

[2 marks]

Question 2b

(b) Find the definite integral

$$\int_0^2 (3x - 1)^3 \ \mathrm{d}x$$

[4 marks]

Question 2c

(c) Find an expression for y given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = e^{5x}$$

[2 marks]

Question 3

Using a suitable substitution, show that

$$\int_{1}^{2} \frac{x}{x+4} \, dx = 1 + 4\ln\frac{5}{6}$$

[7 marks]

Question 4

Given that $\cos 2\theta \equiv 2\cos^2\theta - 1$, find the exact value of

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2\theta \; d\theta$$

[6 marks]

Question 5a

(a) Given that $f(x) = 2x^3 + 4x$, find $f'(x)$.

[2 marks]

Question 5b
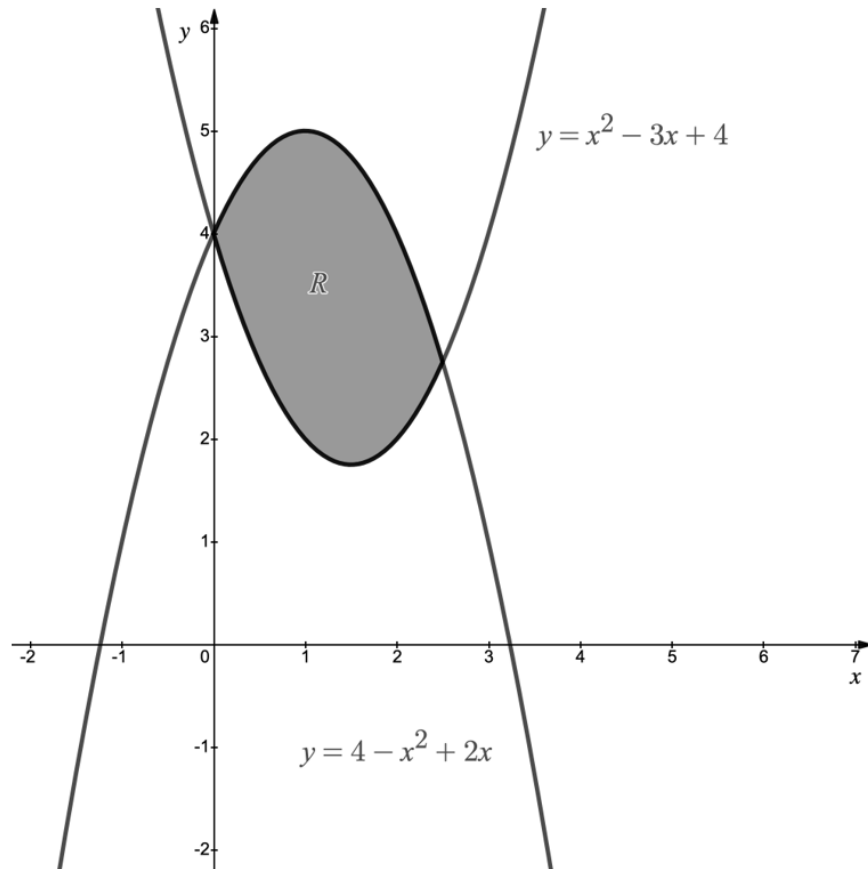
(b) Hence, or otherwise, find

$$\int \frac{3x^2 + 2}{2x^3 + 4x} \, \mathrm{d}x$$

[4 marks]

Question 6a

The diagram below shows a sketch of the curves with equations

$$y = x^2 - 3x + 4 \qquad \text{and} \qquad y = 4 - x^2 + 2x$$

(a) Find the x-coordinates of the intersections of the two graphs.

[2 marks]

Question 6b

(b) Show that the area of the shaded region labelled R is given by

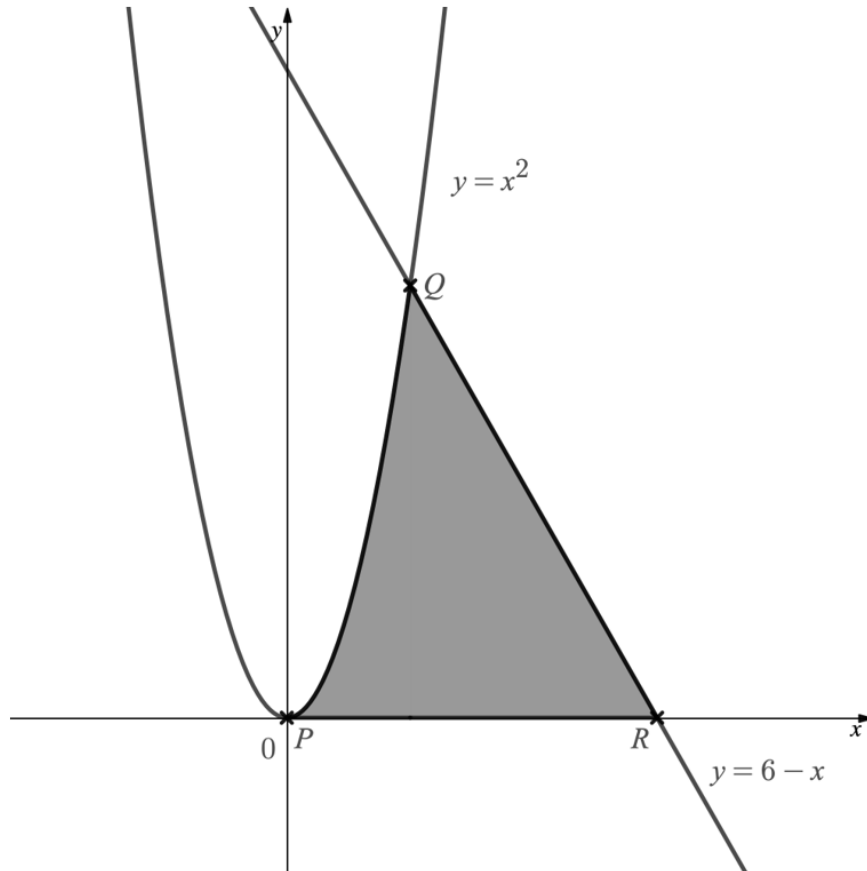$$\int_0^{\frac{5}{2}} (5x - 2x^2) \, dx$$

[2 marks]

Question 6c

(c) Find the area of the shaded region labelled R.

[2 marks]

Question 7a

The diagram below shows the graphs of the line $y = 6 - x$ and the curve $y = x^2$.

Point P is the point of intersection of the curve $y = x^2$ with the x-axis. Point Q is the point of intersection of the curve $y = x^2$ with the line $y = 6 - x$ for which $x > 0$. Point R is the point of intersection of the line $y = 6 - x$ with the x-axis.

(a) Work out the x-coordinates of points P, Q and R.

[3 marks]

Question 7b

(b) Work out the area of the shaded region.

[4 marks]

Question 8a

Consider the function $h(x)$ such that

$$\int_1^5 h(x)\,\mathrm{d}x = 2.$$

(a) Find

$$\int_5^1 h(x)\,\mathrm{d}x$$

[2 marks]

Question 8b

(b) Find

$$\int_1^5 \frac{h(x)+1}{2} \, dx$$

[3 marks]

Question 8c

(c) Find

$$\int_1^5 (h(x) + 2x) \, dx$$

[3 marks]

Question 9a

Consider the function $f(x) = \ln(2x^2 + 1)$.

(a) Find $f'(x)$.

[3 marks]

Question 9b

(b) Hence, find

$$\int \frac{x}{2x^2 + 1} \, dx$$

[3 marks]

Question 10

Let $f'(x) = x^2 \cos(x^3 + 1)$.

Find $f(x)$ given that $f(-1) = 1$.

[5 marks]