3.8 Further Trigonometry

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.8 Further Trigonometry
Difficulty	Very Hard

Time allowed: 90

Score: /68

Percentage: /100

Question 1

Find the value of $\arctan\left(\frac{7}{3}\right) - \arctan\left(\frac{2}{5}\right)$.

[5 marks]

Question 2

Solve $2\tan 2x = \frac{2\tan x}{12 + 5\tan x}$ algebraically for $-\frac{3}{2}\pi < x < \frac{\pi}{2}$.

[6 marks]

Question 3

By writing $\sqrt{3}\cos\theta + \sin\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, or otherwise, solve

$5\sqrt{3}\cos\left(-\frac{\pi}{12}\right) = 2x - 5\sin\left(-\frac{\pi}{12}\right)$.

[8 marks]

Question 4

Let $f(x) = \tan(2x + \pi)\sin(x - \pi)$ where $0 < x < \frac{\pi}{2}$.

Express $f(x)$ in terms of $\sin x$.

[6 marks]

Question 5

Given that $\sin A = -\frac{1}{\sqrt{3}}$ for $\pi \leq A \leq \frac{3\pi}{2}$ and $\cos B = \frac{\sqrt{2}}{3}$ for $\frac{\pi}{2} \leq B \leq \pi$, find the value of $\tan(A+B)$. Give the answer in the form $\sqrt{p}(\sqrt{q}-r)$ where p, q and r are prime numbers.

[8 marks]

Question 6a

(a)

By writing $A = \frac{A+B}{2} + \frac{A-B}{2}$ and finding a similar expression for B, or otherwise, show that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

.

[4 marks]

Question 6b

(b)

Hence show that $\sin\frac{7\pi}{12} + \sin\frac{\pi}{12} = \frac{\sqrt{6}}{2}$.

[3 marks]

Question 7

Prove that $\frac{\cos 3\theta - \cos \theta}{\sin 3\theta - \sin \theta} \equiv -\tan 2\theta$.

[8 marks]

Question 8a

(a)

By writing $A = \frac{A+B}{2} + \frac{A-B}{2}$ and finding a similar expression for B, or otherwise, show that

$$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

[4 marks]

Question 8b

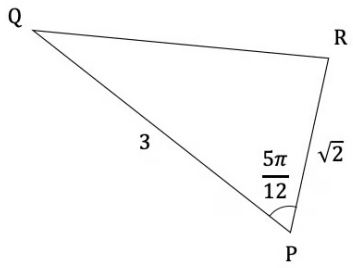
(b)

Hence solve $4\sin\left(\frac{75+\theta}{2}\right)\sin\left(\frac{75-\theta}{2}\right)=1-\frac{\sqrt{6}-\sqrt{2}}{2}$ for $-315° \leq \theta \leq 45°$.

[6 marks]

Question 9a

The following diagram shows the triangle PQR where $PQ = 3$, $PR = \sqrt{2}$ and $R\hat{P}Q = \frac{5\pi}{12}$.



(a)

Expand and simplify $\frac{1}{2}(3\sqrt{3}-1)^2$.

[2 marks]

Question 9b

(b)

Show that the length of QR is $\frac{3\sqrt{6}-\sqrt{2}}{2}$ units.

[8 marks]