1.6 Binomial Theorem

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.6 Binomial Theorem
Difficulty	Hard

Time allowed: 130

Score: /104

Percentage: /100

Question 1

Find the coefficient of the x^{16} term in the expansion $(2x^2 - x^3)^7$.

[4 marks]

Question 2a

Consider the expansion of $(5x^3 - x)^6$.

(a) Write down the number of terms in this expansion.

[1 mark]

Question 2b

(b) Find the first three terms, in descending powers of x, of the expansion.

[4 marks]

Question 3a

Consider the expansion of $\left(\frac{ax}{2} + \frac{3}{x^2}\right)^5$.

(a) Find an expression, in terms of a, for the coefficient of the x^{-1} term.

[3 marks]

Question 3b

The coefficient of the x^{-1} term is 90.

(b) Find the value of a.

[4 marks]

Question 4a

Consider the quadratic expression $5x^2 - 15x + 10$.

(a) Write down the quadratic expression in the form $(px - q)(x - r)$.

[2 marks]

Question 4b

(b) Find the coefficient of the x^8 term in the expansion of $(5x^2 - 15x + 10)^5$. Give your answer in the form $a \times 10^k$, where $1 \leq a < 10, k \in \mathbb{Z}$.

[5 marks]

Question 5a

The coefficient of x^7 in the expansion of $\left(\frac{x}{3}\right)^5 (ax + 5)^2$ is $\frac{1}{3}$.

(a) Find the possible values of a.

[4 marks]

Question 5b

The sum of the coefficients of the expansion is $\frac{196}{243}$.

(b) Determine which value of a found in part (a) is correct.

[4 marks]

Question 6

Consider the expansion $(1-3x)^4(1-2kx)^2$.

The coefficient of the x^6 term is 36. Find the possible values of k.

[6 marks]

Question 7a

Consider the expansion of $\left(\frac{x^3}{3}+\frac{k}{x}\right)^4$. The constant term is $-\frac{500}{3}$.

(a) Find the value of k.

[4 marks]

Question 7b

(b) Find the coefficient of the x^4 term.

[4 marks]

Question 8

In the expansion of $\left(\frac{1}{2}x + 1\right)^n$, the coefficient of the x^2 term is $8n$, where $n \in \mathbb{Z}^+$.

Find n.

[5 marks]

Question 9a

Consider the expansion of $(4x - 2)^4$.

(a) Find the term in x^4 in the expansion.

[2 marks]

Question 9b

(b) Hence find the term in x^6 in the expansion of $(3x-5)^2(4x-2)^4$.

[3 marks]

Question 10a

Consider the expansion of $\left(\frac{3}{2}x-5\right)^6$.

(a) Find the term in x^3 in the expansion.

[3 marks]

Question 10b

(b) Hence find the term in x^4 in the expansion of $(x-2)\left(\frac{3}{2}x-5\right)^6$.

[5 marks]

Question 11a

Consider the binomial expansion of $\frac{1+x^2}{1-x^2}$.

(a) Find the first four terms, in ascending powers of x, of the expansion.

[4 marks]

Question 11b

(b) State the interval of convergence for the complete expansion.

[1 mark]

Question 11c

(c) By substituting an appropriate value into the expression found in part (a), find an approximation for the value of $\frac{101}{99}$.

[3 marks]

Question 12a

Consider the binomial expansion of $\sqrt{\frac{x}{2(x+3)}}$.

(a) Write down the first three terms, in ascending powers of x, of the expansion.

[5 marks]

Question 12b

(b) State the interval of convergence for the complete expansion.

[1 mark]

Question 12c

(c) Using the expansion found in part (a), find an approximation for the value of $\frac{1}{\sqrt{8}}$, giving your answer as an exact value in as simple a form as possible.

[3 marks]

Question 13

Consider the binomial expansion of $(1+2x)(1-ax)^{\frac{2}{3}}$, $a \in \mathbb{N}$.

Given that the coefficient of the term in x^2 is -5, find

(i) the value of a

(ii) the coefficient of the term in x

(iii) the first three terms of the expansion, in ascending powers of x.

[8 marks]

Question 14a

Consider the binomial expansion of $\frac{1}{(2x+3)^n}$, where $n \in \mathbb{Z}^+$.

(a) For the case where the coefficient in x is $-\frac{2}{27}$, show that $n = 3^{n-2}$.

[4 marks]

Question 14b

(b) For the value of n found in part (a), find the coefficient of x^2.

[4 marks]

Question 15a

Consider the binomial expansion of $\frac{1}{(2x^2+x-3)^2}$.

(a) Show that $\frac{1}{(2x^2+x-3)^2}$ can be written in the form $(2x+a)^{-2}(x+b)^{-2}$, and find the values of a and b.

[2 marks]

Question 15b

(b) Hence, or otherwise, find the first three terms of the expansion, in ascending powers of x.

[6 marks]