2.6 Further Modelling with Functions

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.6 Further Modelling with Functions
Difficulty	Hard

Time allowed: 100

Score: /79

Percentage: /100

Question 1a

Ben is planning an event, so he decides to investigate potential catering companies. Company A quotes a price of $1900 and says they will provide staff for food and drink service at an extra cost of $200 per hour, for a minimum of two hours. Company B quotes a price of $2100 and says they will provide staff for food and drink service for $160 per hour for a minimum of one hour and a maximum of six hours.

a)
Write down two functions, including their domains, for the total revenue generated by company A and company B from catering the event.

[2 marks]

Question 1b

Ben has already calculated other costs involving in running the event and has concluded that the event needs to be at least three hours long in order to make a profit.

b)
On the same diagram, sketch the graphs of both of the functions found in part (a).

[2 marks]

Question 1c

c)
Determine which catering company Ben should use to minimise the cost based on the duration of the event.

[3 marks]

Question 2a

A Ferris-wheel-type attraction at a wildlife park is partially submerged underwater to enable passengers to observe both land and aquatic creatures at the park. The wheel rotates at a constant speed and the height of a viewing capsule above the water level is modelled by the function

$$H(t) = 20\sin\left(\frac{\pi}{18}t\right) + 2, \qquad t \geq 0$$

where H is the height, in metres, of the viewing capsule, and t is the elapsed time, in minutes, since the start of the ride.

(a) Find

(i) the height above the water level at which passengers enter a viewing capsule,

(ii) the maximum height above the water level that a viewing capsule reaches,

(iii) the maximum depth below the water level that a viewing capsule reaches,

(iv) the amount of time spent on one full rotation on the Ferris-wheel.

[4 marks]

Question 2b

b)
Calculate, to one decimal place, the length of time for which a capsule is under water during one ride.

[2 marks]

Question 3a

The sound intensity level L, measured in decibels (dB), is given by $L = 10\log\left(\frac{I}{I_0}\right)$, where I is the sound intensity and I_0 is the reference sound intensity.

(a) Write down the sound intensity level, L dB, in the case where $I = I_0$.

[1 mark]

Question 3b

b)

i)

Using a reference sound intensity of 20 dB, find the sound intensity level of a sound intensity of 30 dB.

ii)

Find the sound intensity for a sound that has a sound intensity level of 2 dB using a reference sound intensity of 15 dB.

[2 marks]

Question 3c

c)

(i)

Show that for a sound intensity level of 70 dB, $I = 10^7 I_0$.

(ii)

Hence, or otherwise, write down an equation in the form $I = 10^x I_0$ for a sound intensity level of 50 dB.

(iii)

Hence show that a sound intensity (I) of 70 dB is 100 times more intense than a sound intensity of 50 dB.

[5 marks]

Question 4a

A manufacturing process takes place inside a sealed chamber and produces a pollutant that decays over time. After the process is completed, at time t = 0 seconds, the amount of pollutant, P ppm (parts per million) in the chamber is modelled by

$$P = P_B + P_A e^{kt}$$

a)
Write down an expression for
(i)
the background level of the pollutant in the chamber,
(ii)
the amount of pollutant in the chamber at the moment the process completes,
(iii)
the half-life of the pollutant.

[3 marks]

Question 4b

The half-life of the pollutant is $t_{0.5} = e^4$.

b)
Find the value of k.

[2 marks]

Question 4c

It is safe for the chamber to be opened once the amount of pollutant falls to an amount

that is less than 10% above the background level.

c)
Given that $P_A = 5P_B$, find the minimum number of complete minutes the chamber should remain sealed for after the manufacturing process completes.

[3 marks]

Question 5a

Once it has reached a certain height above ground level, the height, *h* centimetres, of a particular species of sunflower can be modelled by the logistic function

$$h(t) = \frac{300}{1 + 199e^{kt}}, \qquad t \geq 0.$$

where t is the number of days after the model becomes applicable and k is a constant.

a)
(i)
Write down the value of the carrying capacity and explain what this represents in terms of the species of sunflower.
(ii)
Write down the height above ground level the sunflower needs to reach so that the model becomes applicable.

[3 marks]

Question 5b

b)
Given that $k = 0.1$.

i)
Show that, according to the model, after 75 days the sunflower has grown to within 10% of its maximum possible height.

ii)
Find the number of days required for the sunflower to reach 60% of its maximum possible height.

[2 marks]

Question 5c

c)
According to horticulturists, this species of sunflower takes 80 – 120 days to reach its full height. Give one limitation of the model.

[1 mark]

Question 6a

A model of the form $V = A \times 3^r$, where $A > 0$, is proposed by an internet search engine to estimate the number of visits per day to websites.

Each website is given a rank, r, where $0 \le r \le 10$, based on how popular the search engine company decides that website is.

a)
Write down, in terms of A, the lowest and highest possible visits a website can achieve in a single day according to the model.

[2 marks]

Question 6b

b)
Sketch the graph of $\log\ V$ against r. Label the points corresponding to the answers found in part (a).

[4 marks]

Question 6c

c)
Given that the graph of $\log V$ against r passes through the point $(5, 5.08)$, find the value of A. Give your answer correct to one significant figure.

[2 marks]

Question 6d

d)
Show that a website with a rank of 7 would have approximately 80 times as many visits per day as a website with a rank of 3 would have.

[2 marks]

Question 7a

The water depth, d m (metres), at a port can be modelled by the function

$$d(t) = A\sin\left(\frac{\pi}{12}(2t-1)\right) + B, \qquad 0 \le t \le 24$$

where t is the elapsed time in hours since midnight. A and B are constants.

a)
With regards to the function $d(t)$, and giving your answers in terms of A and/or B as appropriate, write down

(i) the phase shift,

(ii) the period,

(iii) the amplitude,

(iv) the equation of the principal axis.

[4 marks]

Question 7b

b)
Explain what the period and the amplitude of $d(t)$ mean in terms of the depth of water in the port.

[2 marks]

Question 7c

c)
Find the time of day at which the water depth first reaches its minimum value.

[2 marks]

Question 7d

All water vehicles are prohibited from entering the port whilst the depth of water is below 3 m.

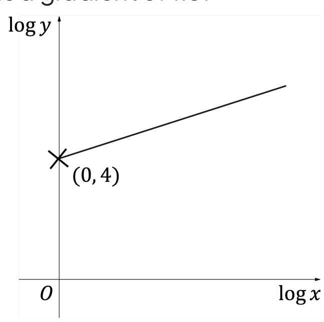
It is given that $A = 3$ and $B = 5$.

d)
Find the times of day between which all water vehicles are prohibited from entering the port.

[3 marks]

Question 8a

a)
The sketch below shows the graph of $\log y$ against $\log x$.
The graph passes through the point (0,4) and has a gradient of 1.5.

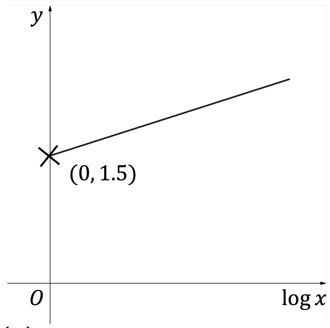


Write down a function for y, in terms of x.

[3 marks]

Question 8b

b)
The sketch below shows the graph of y against $\log x$.
The graph passes through the point $(0,\ 1.5)$ and has a gradient of 0.5.



Write down a function for y in the form $y = \log f(x)$..

[3 marks]

Question 9a

The profile of a non-symmetrical skate ramp is modelled by the piecewise function

$$h(w) = \begin{cases} \frac{3}{w+1} - \frac{1}{2} & 0 \le w \le 5 \\ 0 & 5 \le w \le 7 \\ \frac{5}{128}(w-7)^2 & 7 \le w \le 15 \end{cases}$$

where h m is the height of the ramp above ground level at the point w m from the southernmost end of the skate ramp.

a)
Verify that both the southernmost and northernmost ends of the skate ramp are at a height of 2.5 m above ground level.

[2 marks]

Question 9b

b)
Find the height of the ramp when it is a distance of $3\ \text{m}$ from the southernmost end of the skate ramp.

[2 marks]

Question 9c

c)
Find the other distance from the southernmost end of the skate ramp when the ramp is at the same height as the answer to part (b). Give your answer to a sensible degree of accuracy.

[2 marks]

Question 9d

d)
Briefly explain, why, in this context, negative values of w could be considered.

[1 mark]

Question 10a

The magnitude of an earthquake, R, on the Richter scale can be modelled by the function

$$R(E) = \frac{2\ln E}{3\ \ln 10} - 3.2$$

where E is the amount of energy, in joules (J), released by the earthquake.

a)
Find the magnitude, to one decimal place, of an earthquake which releases 7.2×10^{10} J of energy.

[2 marks]

Question 10b

In February of 2011 an earthquake with magnitude 6.3 struck the region of Canterbury in the South Island of New Zealand.

b)
Find the amount of energy, in joules, released by this earthquake, giving your answer in standard index form, correct to 2 significant figures.

[2 marks]

Question 11a

The average time, T in minutes, it takes for people to choose a movie to watch at the cinema can be modelled by the function

$$T(n) = 0.72\log_5(n+1), \qquad n \geq 3$$

where n is the number of movies being shown at the cinema.

a)
Calculate the number of minutes it would take someone to choose a movie at a cinema that is playing seven movies.

[2 marks]

Question 11b

b)
Find an expression for the inverse function $T^{-1}(n)$.

[2 marks]

Question 11c

c)
It takes 1.073 minutes for someone to choose a movie at a cinema. Find the number of movies being shown at the cinema.

[2 marks]