2.2 Quadratic Functions & Graphs

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.2 Quadratic Functions & Graphs
Difficulty	Hard

Time allowed: 130

Score: /102

Percentage: /100

Question 1a

Consider $f(x) = -2x^2 + bx + c$, for $x \in \mathbb{R}$, where $b, c \in \mathbb{Z}$.

The graph of f has a local maximum at $x = 6$. The distance between the two x-intercepts of the graph of f is 10 units.

(a) Find the coordinates of the two x-intercepts.

[2 marks]

Question 1b

(b) Find the value of b and the value of c.

[4 marks]

Question 1c

(c) Find the coordinates of the local maximum.

[2 marks]

Question 2

For the equation $6kx^2 + 4kx + 2 = 0$, find the possible values of k, which will give

(i) two distinct real roots

(ii) two equal real roots

(iii) no real roots.

[6 marks]

Question 3a

Let $f(x) = 3x^2 - 3x + 2$.

(a) Find the coordinates of the vertex of f.

[3 marks]

Question 3b

Let $g(x) = 3x + 2$. The graph of f and g intersect at points A and B.

(b) Find the coordinates of A and B.

[3 marks]

Question 3c

(c) Find the exact length of the line AB.

[2 marks]

Question 4a

The function $f(x) = ax^2 + bx + c$ intersects the y-axis at -8 and has an x-intercept at $x = -4$. The function can be obtained by an appropriate shift of the graph $y = -2x^2$.

(a) Find the values of a, b and c.

[3 marks]

Question 4b

(b) Find the other x-intercept of $f(x)$.

[2 marks]

Question 4c

(c) Determine the coordinates of the maximum value of $f(x)$.

[3 marks]

Question 5a

A fence of length L is made to go around the perimeter of a rectangular paddock that borders a straight river. The cost of the fence along the river is \$15 per metre, while on the other three sides the cost is \$10 per metre. The total cost of the fence is \$2000.

(a) Calculate the maximum area of the paddock.

[3 marks]

Question 5b

(b) Using the value for the area from part (a), calculate

(i) the side lengths.

(ii) the total length L of the fence.

[3 marks]

Question 6a

A factory produces cardboard boxes in the shape of a cuboid, with a fixed height of 25 cm and a base of varying area. The area, A, of each base can be modelled by the function

$$A(x) = x(50 - x), \qquad 10 \leq x \leq 40,$$

where x is the width of the base of the cardboard box in centimetres.

Cardboard box M has a width of 12 cm.

(a) Find the volume of cardboard box M.

[2 marks]

Question 6b

(b) Find the possible dimensions of a cardboard box with volume of 15 400 cm^3.

[3 marks]

Question 6c

(c) (i) Find the value of x that makes the volume of the cardboard box a maximum.

(ii) Write down the maximum volume of the cardboard box.

(iii) State the mathematical shape of the carboard box when its volume is a maximum.

[3 marks]

Question 7a

Consider $f(x) = m(x - n)(x - 2)$. The graph of $y = f(x)$ has axis of symmetry $x = 4$ and y-intercept at $(0, -6)$.

(a) Find the value of n.

[3 marks]

Question 7b

(b) Find the value of m.

[3 marks]

Question 7c

(c) Write $f(x)$ in the form $f(x) = ax^2 + bx + c$.

[2 marks]

Question 8a

Let $f(x) = 2x^2 + 16x + 29$.

(a) Write down the coordinates of the y-intercept.

[1 mark]

Question 8b

The function f can be written in the form $f(x) = a(x - h)^2 + k$.

(b) (i) Find the values a, h and k.

(ii) Hence write down the coordinates of the vertex and state whether it is a maximum or minimum point.

[5 marks]

Question 8c

(c) Sketch the graph of $y = f(x)$, clearly labelling the vertex and any points where the graph intersects coordinate axes.

[3 marks]

Question 9a

Let $f(x) = -x^2 + 7x - 10$.

(a) Write down the coordinates of the y-intercept.

[1 mark]

Question 9b

The function f can be written in the form $f(x) = a(x-p)(x-q)$.

(b) (i) Find the values a, p and q.

(ii) Hence write down the coordinates of the x-intercepts.

[5 marks]

Question 9c

(c) Sketch the graph of $y = f(x)$, clearly labelling the vertex and any points where the graph intersects coordinate axes.

[3 marks]

Question 10a

Let $f(x) = 2x^2 - 12x + c$, for $x \in \mathbb{R}$, where $c \in \mathbb{Z}$. The graph of f intersects the x-axis at $x = 6$.

(a) Find the equation of the axis of symmetry of the graph of f.

[2 marks]

Question 10b

(b) Find the coordinates of the other point where the graph of f intersects the x-axis.

[2 marks]

Question 10c

(c) Find the value of c.

[1 mark]

Question 11a

(a) Solve the equation $4\sqrt{x} = 21 - x$.

[3 marks]

Question 11b

(b) Solve the equation $13x^2 = x^4 + 36$.

[3 marks]

Question 12a

The function f is a quadratic in the form $f(x) = ax^2 + bx - 2$.

The graph of f has x-intercepts $\left(\frac{1+\sqrt{5}}{2}, 0\right)$ and $\left(\frac{1-\sqrt{5}}{2}, 0\right)$.

(a) Find the values of a and b.

[4 marks]

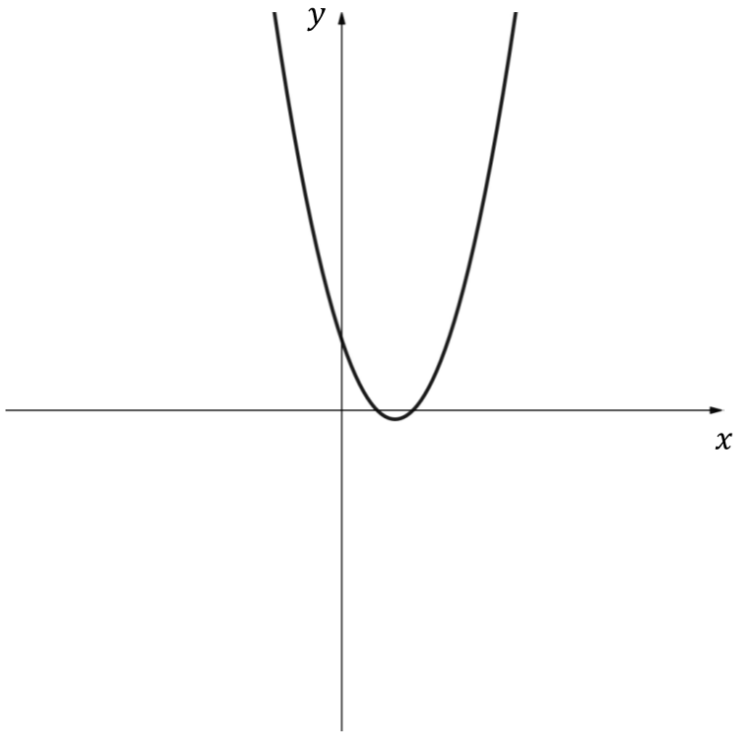
Question 12b

(b) Sketch the graph of $y = f(x)$, clearly labelling the vertex and any points where the graph intersects the coordinate axes.

[4 marks]

Question 13a

Let $f(x) = x^2 - 3x + 2$. The diagram below shows part of the graph of f.



Another function is defined by $g(x) = 2 - x$.

(a) Sketch the graph of g on the axes above.

[1 mark]

Question 13b

The graph of f and g intersect at points A and B.

(b) Find the coordinates of A and B and label them on the diagram above.

[3 marks]

Question 13c

(c) Find the length of the line AB.

[2 marks]

Question 14a

The function $f(x) = ax^2 + bx + c$ intercepts the y-axis at -12 and has an x-intercept at $x = 3$. The function can be obtained by an appropriate shift of the graph $y = -4x^2$.

(a) Find the values of a, b and c.

[4 marks]

Question 14b

(b) Find the other x-intercept of $f(x)$.

[1 mark]

Question 14c

(c) Determine the coordinates of the maximum value of $f(x)$.

[2 marks]