3.8 Vector Equations of Lines

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.8 Vector Equations of Lines
Difficulty	Hard

Time allowed: 100

Score: /77

Percentage: /100

Question 1a

Point A has coordinates $(7,\ -1,\ 20)$ and the line l is defined by the equations:

$$l:\begin{cases} x = 3 + \lambda \\ y = 2\lambda - 1 \\ z = \lambda \end{cases}$$

Point B lies on the line l such that $[\mathrm{AB}]$ is perpendicular to l.

(a)
Find the coordinates of point B.

[5 marks]

Question 1b

(b)
Hence find the shortest distance from A to the line l.

[2 marks]

Question 2a

a)
Find the vector equation of the line l_1 with Parametric equations

$$l_1: \begin{cases} x = 4\lambda - 3 \\ y = 2 + 5\lambda \\ z = 4\lambda + 3 \end{cases}$$

[1 mark]

Question 2b

A second line l_2 runs parallel to l_1 and passes through the points $\mathrm{X}(t, 2, -3)$ and $\mathrm{Y}\ (23, 22, q)$.

b)
Find the value of t and q .

[4 marks]

Question 2c

c)
Hence write down the equation of line l_2 in Parametric form.

[1 mark]

Question 3

A line l passes through the points $\mathrm{P}(6,\ 5,\ -2)$ and $\mathrm{Q}(2x + 2,\ x - 5,\ x)$ and lies perpendicular to the vector $3i + 4j - k$.

Find the vector equation of l .

[6 marks]

Question 4

Find the obtuse angle formed by the two lines l_1 and l_2 defined by the equations:

$$l_1:\begin{cases} x = 4 - 2\lambda \\ y = 1 + 5\lambda \\ z = \lambda - 1 \end{cases}$$

$$l_2:\begin{cases} x = 4 + 3\mu \\ y = 18 + \mu \\ z = 6 + 2\mu \end{cases}$$

[6 marks]

Question 5a

Consider the two lines l_1 and l_2 as defined by:

$$l_1 : \begin{cases} x = 5 + \mu \\ y = 3 - \mu \\ z = 2\mu - 8 \end{cases}$$

$$l_2 : r = \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -5 \\ 2 \end{pmatrix}$$

a)
Find a vector that is perpendicular to both lines.

[3 marks]

Question 5b

b)
Hence find the shortest distance between the two lines.

[5 marks]

Question 6

Consider the lines l_1 and l_2 defined by the equations:

$$l_1: \begin{cases} x = 2 + 6\lambda \\ y = 2 + q\lambda \\ z = -8 - 5\lambda \end{cases}$$

$$l_2: r = \begin{pmatrix} -4 \\ 5 \\ p \end{pmatrix} + \lambda \begin{pmatrix} -24 \\ 12 \\ 20 \end{pmatrix}$$

Given that l_1 and l_2 are identical, find the value of p and q.

[6 marks]

Question 7a

Consider the two lines l_1 and l_2 defined by the equations:

$$l_1 : r = \begin{pmatrix} 4 \\ 4 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

$$l_2 : r = \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix}.$$

a)
Show that the lines are not parallel and do not intersect.

[3 marks]

Question 7b

b)
Calculate the exact value of the acute angle between the lines.

[4 marks]

Question 8a

A helicopter is hovering in the sky at coordinates (4.5, 8, 2.7) relative to a helipad positioned on the ground at the origin, O.

The x direction is due east, the y direction is due north and the z direction is vertically upwards. The distances are measured in kilometres.

a)
Write down the equation of a line the helicopter should travel along for it to travel directly to the helipad.

[2 marks]

Question 8b

b)
Assuming the helicopter travels directly towards the helipad, but stops to hover at a point, P, 0.54 km vertically above the ground, find

i)
the coordinates of the point P,

ii)
the distance the helicopter has left to travel on its final descent to the helipad, given that it continues along the most direct route.

[4 marks]

Question 8c

c)
Assuming instead the helicopter travels directly towards a point, Q, 0.04 km vertically above the helipad, and then descends vertically downwards to the ground, find
i)
The coordinates of the point Q,
ii)
The component of the direction the helicopter actually travelled in that is perpendicular to the direction vector found in part (a),
iii)
The distance the helicopter has travelled from its starting position, including the vertical descent from Q to the helipad.

[5 marks]

Question 9a

Consider the triangle ABC. The points A, B and C have coordinates $(-6,\ 3,\ 13)$, $(4,\ 5,\ -8)$ and $(3,\ -4,\ t)$ respectively. A vector equation of the line that passes through point A and the midpoint of $[\mathrm{BC}]$ is $r = \begin{pmatrix} -6 \\ 3 \\ 13 \end{pmatrix} + \lambda \begin{pmatrix} 19 \\ -5 \\ -27 \end{pmatrix}$

(a)
Find the value of t.

[3 marks]

Question 9b

(b)
Find the vector equation of the line that passes through point B and the midpoint of [AC].

[3 marks]

Question 9c

The two lines intersect inside the triangle at point X.

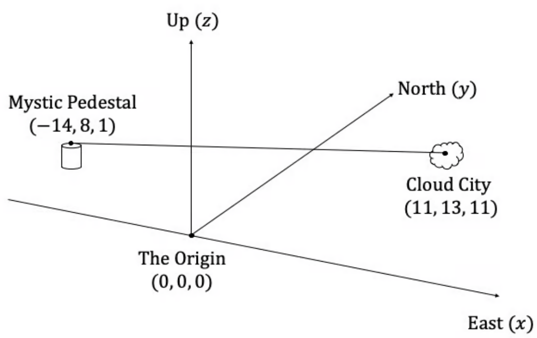
(c)

Show that the area of AXC is $\frac{1}{3}$ the area of triangle ABC .

[7 marks]

Question 10

In the magical kingdom of Cartesia, all positions are measured relative to the ancient stone of power known as the Origin. This reference system corresponds to the standard x, y, z coordinate system used in mathematics, as shown in the diagram below.



Prince Vector, son of the King Prime of Cartesia, needs to fly on his magical unicorn from the top of the Mystic Pedestal all the way to Cloud City, on an urgent rescue mission.

The Mystic Pedestal is 14 kilometres west and 8 kilometres north of the Origin, and its top is one kilometre up from the level of the Origin. Cloud City is 11 kilometres east and 13 kilometres north of the Origin, and it is 11 kilometres up from the level of the Origin.

Since there is not much time, the prince must fly directly from the top of the Mystic Pedestal to Cloud City. Unfortunately, the unicorn's magic levels are low. In order for the unicorn to recharge it must pass within 12 kilometres of the Origin during the flight, and must do this before reaching the halfway point between the Mystic Pedestal and Cloud City. If the unicorn does not recharge before this point then it and the prince will crash into the barren wastes and the kingdom will perish.

Using a vector method, determine whether or not the prince will reach Cloud City successfully. Use clear mathematical workings to justify your answer.

[7 marks]