3.11 Vector Planes

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.11 Vector Planes
Difficulty	Very Hard

Time allowed: 130

Score: /105

Percentage: /100

Question 1

Determine whether the points A(1, -1, 8) , B(0, 10, 15) , C (-2, -6. 10) and D(3, -5, 3) can lie in the same plane.

[9 marks]

Question 2a

The plane Π has vector equation $\mathrm{r} = \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 1 \\ 5 \end{pmatrix}$

The line L has vector equation $\mathrm{r} = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix} + s \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$

The plane Π and the line L intersect at the point X.

(a)
Find the coordinates of X.

[3 marks]

Question 2b

(b)
Find the acute angle, in degrees, between the line L and the plane Π.

[5 marks]

Question 2c

The point P(2, 6, -1) lies on the line L.

(c)
Find the shortest distance between the point P and the plane Π. Fully justify your answer.

[4 marks]

Question 3

Find the acute angle, in radians, between the two planes Π_1 and Π_2 which can be defined by the equations:

$$\Pi_1 : 7x + 3y - 2z = 84,$$

$$\Pi_2 : \mathrm{r} = \begin{pmatrix} 11 \\ -7 \\ 9 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 5 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 6 \\ -4 \end{pmatrix}.$$

[7 marks]

Question 4a

The plane Π_1 is defined by the equation $x - 2y - 2z + 15 = 0$ and the line L is defined by the vector equation

$$\mathrm{r} = \begin{pmatrix} -5 \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix}.$$

(a)
Show that the line L lies on the plane Π_1.

[2 marks]

Question 4b

The plane Π_2 is defined by the equation $\mathrm{r} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + s \begin{pmatrix} 12 \\ -1 \\ 7 \end{pmatrix} + t \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix},$

Show that the plane Π_2 is parallel to the plane Π_1.

[3 marks]

Question 4c

(c)
Find a vector equation of the line that is perpendicular to both planes and passes through the point P(3, 1, 4).

[2 marks]

Question 4d

(d)
Hence find the shortest distance between Π_1 and Π_2.

[4 marks]

Question 5a

The plane Π has the Cartesian equation $x+4y+2z+25=0$.

The line L has the Cartesian equation $\frac{3-x}{2}=k(y+2)=z+\frac{1}{5}$, where $k\in\mathbb{R}$.

(a)
Show that the L is not parallel to the plane Π.

[3 marks]

Question 5b

(b)
Given that the acute angle between the line L and the plane Π is 60°, find the possible values of k.

[7 marks]

Question 6a

Consider the two planes defined by the Cartesian equations:

$$\Pi_1 : 3x - 5y + 2z = 9$$

$$\Pi_2 : 4x + 2y - z = 13.$$

The line L is the intersection of the planes Π_1 and Π_2.

(a)

Find a vector equation of the line L. Give your answer in the form $\mathrm{r} = \begin{pmatrix} 1 \\ a \\ b \end{pmatrix} + \lambda \begin{pmatrix} c \\ d \\ e \end{pmatrix}$ where $a,\ b,\ c,\ d,\ e \in \mathbb{Z}$.

[5 marks]

Question 6b

A third plane Π_3 has the Cartesian equation $x + 3y + kz = 10$ where $k \in \mathbb{R}$. The three planes do not meet at a unique point.

(b)
Find the exact value of k and determine the geometrical relationship between the three planes.

[5 marks]

Question 7a

Consider the four planes with Cartesian equations:

$$\Pi_1 : 6x - y + 3z = 16$$

$$\Pi_2 : 4x + ky + 2z = 4$$

$$\Pi_3 : 2x - 5y + 2z = 7$$

$$\Pi_4 : x + 3y - z = m$$

where k and m are real constants.

(a)
In the case where there is no unique point of intersection of the three planes Π_1, Π_2 and Π_3, find the value of k and give a geometric interpretation of the three planes.

[2 marks]

Question 7b

(b)
In the case where $k = 6$, find the coordinates of the point of intersection between the three planes Π_1, Π_2 and Π_3.

[4 marks]

Question 7c

(c)
In the case where there is a common line of intersection between the three planes Π_2, Π_3 *and* Π_4, find the values of k and m.

[5 marks]

Question 8a

The point P(2, 0, -1) is reflected in the plane Π which has equation $\mathrm{r}.\begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix} = 78.$

(a)
Find the coordinates of the reflection of P in the plane Π.

[7 marks]

Question 8b

The line L_1 passes through the point P and intersects the plane Π at the point Q(8, 3, 11) . The line L_1 is reflected in the plane Π to form line L_2.

(b)
Find a vector equation of the line L_2 .

[2 marks]

Question 8c

(c)
Find the acute angle, in degrees, between the lines L_1 and L_2.

[4 marks]

Question 9a

Two planes are defined by the equations:

$$\Pi_1 : x + 2y - z = 5,$$

$$\Pi_2 : 2x + 5y + 2z = 7.$$

(a)
Find the exact value of $\cos \theta$ where θ is the acute angle between Π_1 and Π_2.

[3 marks]

Question 9b

Π_1 and Π_2 intersect at the line $L_{1,2}$. A third plane Π_3 is defined by the equation $x + ky + 11z = m$ where $k, m \in \mathbb{R}$ and Π_3 is perpendicular to Π_1. When $m = a$ the line $L_{1,2}$ lies on all three planes.

(b)
Find the values of k and a.

[4 marks]

Question 9c

Given that $m \neq a$, Π_1 and Π_3 intersect at the line $L_{1,3}$, Π_2 and Π_3 intersect at the line $L_{2,3}$. The shortest distance between the lines $L_{1,2}$ and $L_{1,3}$ is $\sqrt{11}$.

(c)
Find the shortest distance between the lines $L_{1,2}$ and $L_{2,3}$. Give your answer as an exact value.

[3 marks]

Question 10

The plane Π is defined by the Cartesian equation $4x - 5y + 3z = 59$.

The line L is defined by the Cartesian equation $\frac{4-x}{2} = y + 1 = 2(z-3)$.

Determine whether the point P(5, 8, 15) is closer to the plane Π or the line L.

Fully justify your answer.

[12 marks]