IB Physics DP

3. Thermal Physics

CONTENTS

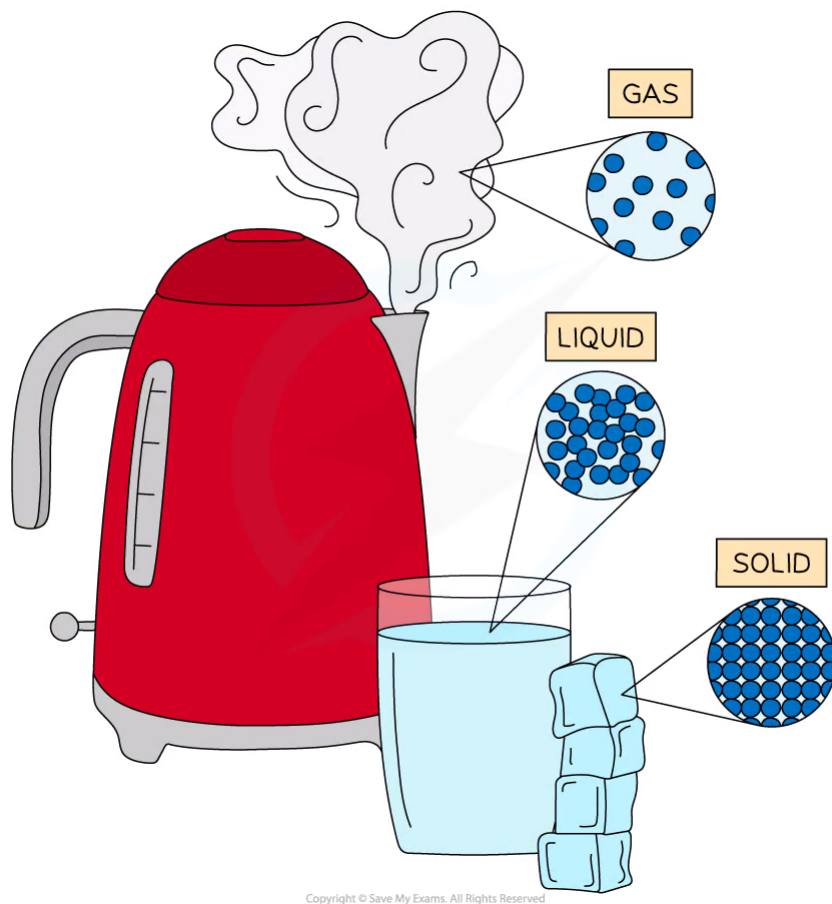## 3.1 Thermal Concepts

3.1.1 Solids, Liquids & Gases

Solids, Liquids & Gases

- The three states of matter are **solid**, **liquid** and **gas**
- The **kinetic theory of matter** is a model that attempts to explain the properties of the three states of matter
    - In this model, particles are assumed to be small solid spheres

Water has three states of matter; solid ice, liquid water and gaseous steam. The difference between each state is the arrangement of the particles

Solids

- Particles in solids:
    - Are held together by **strong intermolecular forces**
    - Are **closely packed**
    - Are arranged in a fixed pattern (**lattice structure**)
    - Can only **vibrate** about their fixed positions
   - Have low energies compared to particles in liquids and gases

SOLIDS

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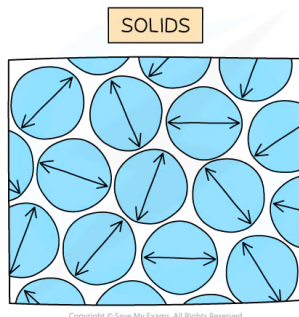
In a solid, particles are arranged in a fixed pattern, with no spaces between them, and are only able to vibrate about their fixed positions

- As a result of the arrangement and behaviour of their particles, solids:
  - Have a fixed shape (although some solids can be deformed when forces are applied)
  - Have a fixed volume
  - Are very difficult to compress
  - Have higher densities than liquids and gases

Liquids

- Particles in liquids:
  - Are held together by **weaker intermolecular forces** compared to the forces between particles in solids
  - Are **closely packed**
  - Are **randomly** arranged (i.e. there is no fixed pattern)
  - Can **flow** past each other
  - Have higher energies than particles in solids, but lower energies than gas particles

LIQUIDS

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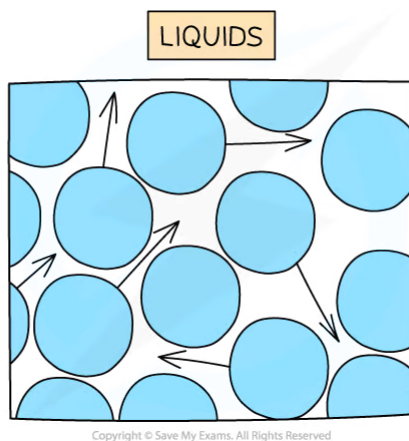
In a liquid, particles are arranged randomly and are able to flow past one another

- As a result of the arrangement and behaviour of their particles, liquids:
  - Do not have a fixed shape and take the shape of the container they are held in
  - Have a fixed volume
  - Are difficult to compress
  - Have lower densities than solids, but higher densities than gases

Gases

- Particles in gases:
  - Have **negligible intermolecular forces** between them
  - Are **far apart** (the average distance between the particles is ~10 times greater than the distance between the particles in solids and liquids)
  - Are **randomly** arranged
  - Move around in **all directions** at a variety of speeds, occasionally **colliding** with each other and with the walls of the container they are in
  - Are negligible in size compared to the volume occupied by the gas
  - Have higher energies than particles in solids and liquids
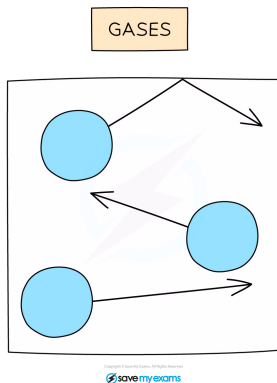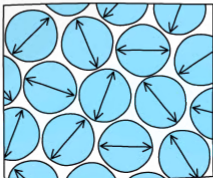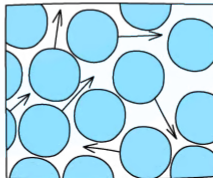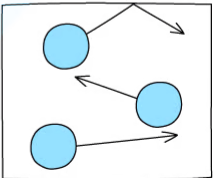
In a gas, particles can move around freely in all directions (shown by the arrows).

- As a result of the arrangement and behaviour of their particles, gases:
  - Do not have a fixed shape and take the shape of the container they are held in
  - Do not have a fixed volume and expand to completely fill the available volume
  - Can be compressed
  - Have the lowest densities (~1000 times smaller than the densities of solids and liquids)

State of Matter	Solid	Liquid	Gas
Particle arrangement	Fixed pattern (lattice structure)	Random	Random
Space between particles	No space	Some space	Large space
Intermolecular forces	Strong	Weak	Negligible
Particle movement	Vibrate about fixed position	Flow past each other	Move around at different speeds
Particle energy	Low	Medium	High
Substance shape	Fixed	Not fixed	Not fixed
Substance volume	Fixed	Fixed	Not fixed
Substance density	High	Medium	Low
2−D diagram of particle arrangement			

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? **Worked Example**

Liquids are about 1000 times denser than gases. Let *d* be the diameter of a molecule. Estimate the average intermolecular distance in a gas. Give your answer in terms of *d*.

Step 1: Recall the equation for density

$$\rho = \frac{m}{v}$$

Step 2: Write down the relationship between the density of a gas ρ_{gas} and the density of a liquid ρ_{liquid}

$$\rho_{liquid} = 1000 \times \rho_{gas}$$

Step 3: Substitute into the density equation to show the relationship between the masses and volumes of a liquid a gas

$$\frac{m_{liquid}}{v_{liquid}} = 1000 \times \left(\frac{m_{gas}}{v_{gas}}\right)$$

Step 4: Since the mass stays the same, the relationship between the densities translates into a relationship between volumes as mass cancels out

$$\frac{\cancel{m}_{liquid}}{v_{liquid}} = 1000 \times \left(\frac{\cancel{m}_{gas}}{v_{gas}}\right)$$

$$v_{gas} = 1000 \times v_{liquid}$$

Step 4: Relate the volume to the average distance between the molecules, x

- The average distance x between the molecules is related to the cube root of the volume

$$x = \sqrt[3]{1000\, v_{liquid}} = 10 \times d$$

3.1.2 Temperature

Temperature

- **Temperature** is a measure of how hot or cold objects are
- Temperature also determines the direction in which thermal energy will flow between two objects (or between an object and its surroundings)
- When thermal energy is exchanged, the objects (or systems) involved are said to have a **thermal interaction**
- The thermal energy exchanged during a thermal interaction is referred to as **heat**
- During a thermal interaction:
  - Thermal energy always flows from the hotter object to the colder object
  - The energy transfer continues until the two objects are in **thermal equilibrium** (i.e. they both have the same temperature)
  - Thermal energy can be transferred via **conduction**, **convection** or **radiation**

- Temperature is a **scalar** quantity and it is measured using a **thermometer**
- It is measured in **degrees Celsius (°C)** or **kelvin (K)**
  - The kelvin is the **SI base unit** for temperature

- The temperature of an object is a **macroscopic** measure of the **average kinetic energy** of the particles (atoms or molecules) that make up the object

Absolute temperature

- **Absolute temperature** is temperature measured in kelvin (K)
- **Absolute zero** is a temperature of zero kelvin (0 K) and corresponds to the temperature at which the **average kinetic energy of the molecules** is at its **minimum**
- The conversion between the Kelvin and the Celsius scale is given by:

$$T(K) = T(°C) + 273.15$$

- It is important to notice that **differences in absolute temperatures correspond to differences in Celsius temperatures**

$$\Delta T(K) = \Delta T(°C)$$

  - Where **ΔT** stands for **temperature change**

- The **absolute temperature** of a body is **directly proportional to the average kinetic energy of the molecules** within the body
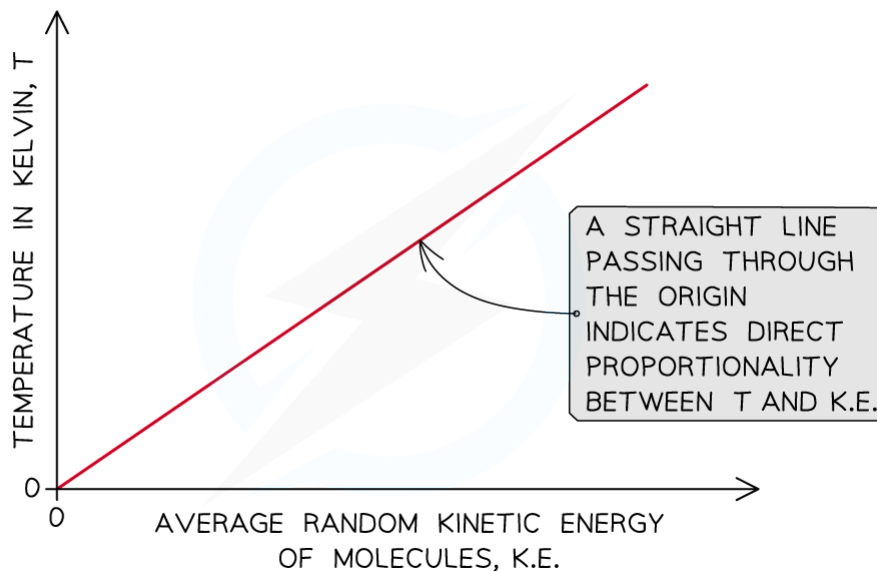
The ice point is determined by placing a thermometer in a beaker containing melting ice, while the steam point is determined by placing the thermometer in a beaker with boiling water

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? Worked Example

Give an estimate of room temperature in kelvin (K).

Step 1: State a reasonable value for room temperature in degree Celsius (°C)

room temperature (°C) ~ 20°C

Step 2: Write down the conversion between Celsius scale and Kelvin scale

$$T(K) = T(°C) + 273.15$$

Step 3: Convert the room temperature value and express it in kelvin (K)

room temperature (K) ~ 293 K

💡 Exam Tip

Remember that the lowest possible temperature on the Kelvin scale is absolute zero (0 K). Therefore, if you are calculating temperature in kelvin and you end up with a negative number, you need to check your work, since negative numbers do not exist on the Kelvin scale.

Internal Energy

- When a substance gains or loses thermal energy, its **internal energy** increases or decreases
- The internal energy of a substance is defined as:

   The sum of the total kinetic energy and the total intermolecular potential energy of the particles within the substance

- As thermal energy is transferred to a substance, two things can happen:
  - An increase in the average kinetic energy of the molecules within the substance – i.e. the molecules vibrate and move at higher speeds
  - An increase in the potential energy of the molecules within the substance – i.e. the particles get further away from each other or move closer to each other
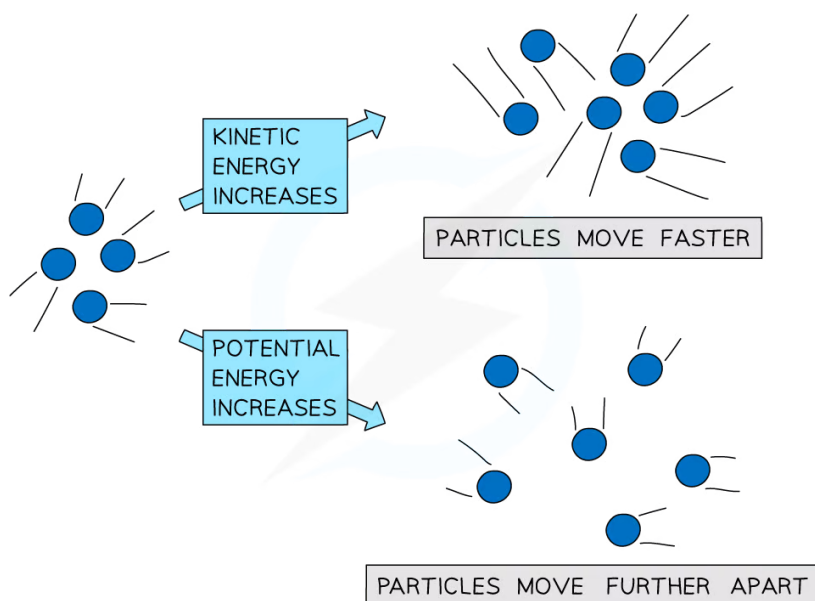
- Since temperature is a measure of the average kinetic energy of the molecules, only an **increase in the average kinetic energy of the molecules** will result in an **increase in temperature** of the substance
  - Due to thermal expansion, when the temperature of a substance increases, the potential energy of the molecules also increases

- When **only** the **potential energy of the molecules changes**, the **temperature** of the substance **does not change**
  - This is the case for all **state changes** (e.g. melting, boiling)

YOUR NOTES
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> **Exam Tip**
>
> Remember that a change in internal energy does not necessarily corresponds to a change in temperature.
>
> - A change in the average kinetic energy of the molecules corresponds to a change in temperature
> - A change in the average potential energy of the molecules does not affect temperature

Specific Heat Capacity

- The amount of thermal energy needed to change the temperature of an object depends on:
  - The change in temperature required ΔT – i.e. the larger the change in temperature the more energy is needed
  - The mass of the object m – i.e. the greater the mass the more energy is needed
  - The **specific heat capacity c** of the given substance – i.e. the higher the specific heat capacity the more energy is needed

Substance	Specific Heat Capacity (J kg⁻¹ K⁻¹)
Water	4200
Ice	2200
Aluminium	900
Copper	390
Gold	130

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- The equation for the thermal energy transferred, Q, is then given by:

$$Q = mc\Delta T$$

- Where:
  - m = mass of the substance in kilograms (kg)
  - ΔT = change in temperature in kelvin (K) or degrees Celsius (°C)
  - c = specific heat capacity of the substance (J kg⁻¹ K⁻¹)

- The specific heat capacity of a substance is defined as:

  The amount of energy required to change the temperature of 1 kg of a substance by 1 K (or 1°C)

- This definition can be explained when the above equation is rearranged for c:

$$c = \frac{Q}{m\Delta T}$$

- This means that, the higher the specific heat capacity of a substance the longer it takes for the substance to warm up or cool down
  - Note that the specific heat capacity is measured in J kg⁻¹ K⁻¹
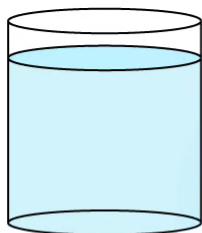
```
1 LITRE  OF  WATER
c = 4200 J/kg K
IT  TAKES  LONGER  TO
HEAT  UP OR COOL  DOWN
```

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1 KG  OF  COPPER
c = 390 J/kg K
IT  TAKES  LESS  TIME
TO  HEAT  UP  OR COOL  DOWN
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❓ Worked Example

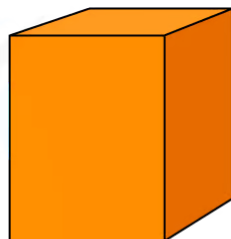A 2 kg piece of copper is kept inside a freezer at a temperature of –10°C. The copper is taken out of the freezer and placed into 5 litres of water at 20°C. A thermometer is placed into the water. After some time, the thermometer indicates that the water has cooled to 18°C. Determine the temperature of the copper at this time. Give your answer in degrees Celsius (°C).

- The specific heat capacity of water is 4200 $J\,kg^{-1}K^{-1}$
- The specific heat capacity of copper is 390 $J\,kg^{-1}K^{-1}$

Step 1: Write down the known quantities

- Mass of copper = 2 kg
- Mass of water = 5 L = 5 kg
- Initial temperature of copper = –10°C
- Initial temperature of water = 20°C
- Final temperature of water = 18°C
- Change in temperature of water = 18°C – 20°C = –2°C
- Specific heat capacity of water = 4200 $J\,kg^{-1}K^{-1}$
- Specific heat capacity of copper = 390 $J\,kg^{-1}K^{-1}$

Step 2: Write down the equation for thermal energy

$$Q = mc\Delta T$$

Step 3: Determine the energy transferred from the water to the copper

- The water is at a higher temperature than copper, hence thermal energy will flow from the water to the copper
- To quantify this energy, substitute numbers into the above equation
- In this case, the mass m is that of the water
- The specific heat capacity is that of water
- Since this is the energy lost by the water, it will be negative

$$Q = 5\ kg \times 4200\ J\,kg^{-1}K^{-1} \times (-2°C) = -42000\ J$$

YOUR NOTES
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Step 3: Determine the change in temperature ΔT of the copper

- The energy lost by the water is the same as the energy gained by the copper
- Since this is the energy gained by the copper, it is positive
- The equation for thermal energy can be rearranged to calculate the change in temperature ΔT of the copper
- In this case, the mass m is that of the copper
- The specific heat capacity is that of copper

$$\Delta T = \frac{Q}{mc} = \frac{42000}{2 \times 390} = 54°C$$

Step 4: Determine the final temperature of the copper

- Since the copper gains thermal energy, its final temperature will be higher than its initial temperature

final temperature of copper = ΔT + initial temperature of copper = 54°C – 10°C

final temperature of copper = 44°C

💡 **Exam Tip**

You should notice that changes in temperature ΔT can usually be written in degrees Celsius (although this is not the SI base unit for temperature) and do not need to be converted into kelvin (K). This is because differences in absolute temperatures always correspond to differences in Celsius temperature.If the question asks to determine the initial or final temperature of a substance, make sure you always check the unit of measure (°C or K) in which you are required to give your final answer.

YOUR NOTES
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Specific Latent Heat

- During a **phase change** (i.e. a change of state) thermal energy is transferred to a substance or removed from it, while the temperature of the substance does not change
- In this case, the thermal energy is calculated as follows:

$$Q = mL$$

- Where:
  - Q = heat energy transferred (J)
  - m = mass of the substance in kilograms (kg)
  - L = specific latent heat of the substance in J kg^{-1}

- The specific latent heat of a substance is defined as:

  The amount of energy required to change the state of 1 kg of a substance without changing its temperature

- This definition can be explained when the above equation is rearranged for L:

$$L = \frac{Q}{m}$$

- This means that the higher the specific latent heat of a substance, the greater the energy needed to change its state
  - Note that the specific latent heat is measured in J kg^{-1}

- The amount of energy required to melt (or solidify) a substance is **not** the same as the amount of energy required to evaporate (or condense) the same substance
- Hence, there are two types of specific heat:
  - **Specific latent heat of fusion, L_f**
  - **Specific latent heat of vaporisation, L_v**

- Specific latent heat of fusion is defined as:

  The energy released when 1 kg of liquid freezes to become solid at constant temperature

- This applies to the following phase changes:
  - Solid to liquid
  - Liquid to solid

- Therefore, the definition for specific latent heat of fusion could also be:

  The energy absorbed when 1 kg of solid melts to become liquid at constant temperature

- Specific latent heat of vaporisation is defined as:

  The energy released when 1 kg of gas condenses to become liquid at constant temperature

- This applies to the following phase changes:
  - Liquid to gas
  - Gas to liquid

- Therefore, the definition for specific latent heat of vaporisation could also be:

  The energy absorbed when 1 kg of liquid evaporates to become gas at constant temperature

- For the same substance, the value of the specific latent heat of vaporisation is always much higher than the value of the specific latent heat of fusion
  - In other words, $L_v > L_f$

- This is because much more energy is needed to **evaporate** (or condense) a substance than it is needed to **melt** it (or solidify it)
  - In melting, the intermolecular bonds only need to be weakened to turn from a solid to a liquid
  - When evaporating, the intermolecular bonds need to be completely broken to turn from liquid to gas. This requires a lot more energy.

Substance	Specific Latent Heat of Fusion (J kg⁻¹)	Specific Latent Heat of Vaporisation (J kg⁻¹)
Aluminium	4.0×10^5	1.1×10^7
Water	3.3×10^5	2.3×10^6
Copper	2.1×10^5	4.7×10^6
Gold	6.3×10^4	1.7×10^6

? Worked Example

Determine the energy needed to melt 200 g of ice at 0°C.

- The specific latent heat of fusion of water is 3.3×10^5 J kg⁻¹
- The specific latent heat of vaporisation of water is 2.3×10^6 J kg⁻¹

Step 1: Determine whether to use latent heat of fusion or vaporisation

- We need to use the specific latent heat of **fusion** because the phase change occurring is from **solid** to **liquid**

Step 2: List the known quantities

- Mass of the ice, m = 200 g = 0.2 kg
- Specific latent heat of fusion of water, $L_f = 3.3 \times 10^5$ J kg⁻¹

Step 3: Write down the equation for the thermal energy

$$Q = mL_f$$

Step 4: Substitute numbers into the equation

$$Q = 0.2 \, kg \times (3.3 \times 10^5) \, J \, kg^{-1}$$

$$Q = 6.6 \times 10^4 \, J = 66 \, kJ$$

> **?** **Worked Example**
>
> Energy is supplied to a heater at a rate of 2500 W. Determine the time taken to boil 0.50 kg of water at 100°C. Ignore energy losses.
>
> - The specific latent heat of fusion of water is $3.3 \times 10^5 \, J \, kg^{-1}$
> - The specific latent heat of vaporisation of water is $2.3 \times 10^6 \, J \, kg^{-1}$

Step 1: Determine whether to use latent heat of fusion or vaporisation

- We need to use the specific latent heat of **vaporisation** because the phase change occurring is from **liquid** to **gas**

Step 2: Write down the known quantities

- Power, $P = 2500 \, W$
- Mass, $m = 0.50 \, kg$
- Specific latent heat of vaporisation of water, $L_v = 2.3 \times 10^6 \, J \, kg^{-1}$

Step 3: Recall the equation linking power P, energy E and time t

$$E = Pt$$

Step 4: Write down the equation for the thermal energy E

- The energy E in the previous equation is the thermal energy Q transferred by the heater to the water

$$Q = mL_f$$

Step 5: Equate the two expressions for energy

$$Pt = mL_f$$

Step 6: Solve for the time t

$$t = \frac{mL_f}{P}$$

$$t = \frac{0.50 \, kg \times \left(2.3 \times 10^6\right) J \, kg^{-1}}{2500 \, W}$$

$$t = 460 \, s$$
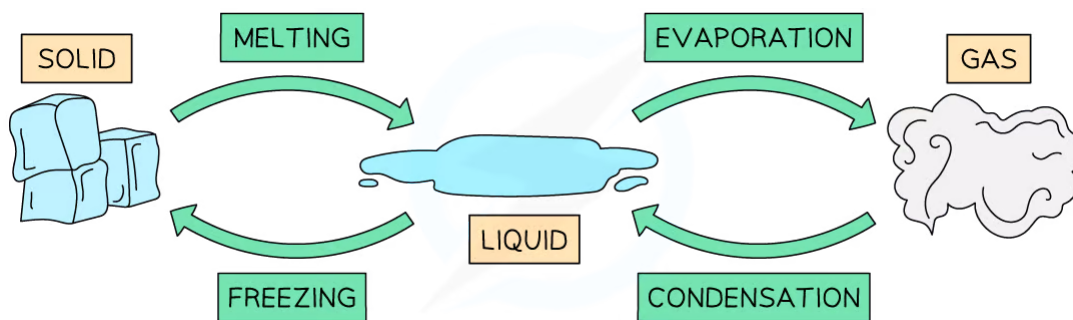
3.1.6 Phase Change

Phase Change

- A **phase change** happens whenever matter changes its state
- During a phase change, thermal energy is transferred to or from a substance
- This energy transfer does **not change the temperature** of the substance undergoing the phase change
- This means:
  - The thermal energy provided (or removed) does not affect the kinetic energy of the molecules within the substance
  - Only the potential energy (i.e. the spacing between the atoms or molecules) is affected

- The four main phase changes are:
  - **Melting** - i.e. when a substance changes from solid to liquid as it absorbs thermal energy
  - **Freezing** - i.e. when a substance changes from liquid to solid as it releases thermal energy
  - **Vaporisation** (or **boiling**) - i.e. when a substance changes from liquid to gas as it absorbs thermal energy
  - **Condensation** - i.e. when a substance changes from gas to liquid as it releases thermal energy

Water

- Each substance has its own melting (or freezing) and boiling points
  - For example, the freezing point of water is 0°C and its boiling point is 100°C

- Possible phase changes of water include:
  - Solid ice **melting** into liquid water at 0°C
  - Liquid water **boiling** and changing into gaseous water vapour at 100°C

- Both these changes happen when **thermal energy** is absorbed
  - If thermal energy is released from water vapour at 100°C, it condenses back into water
  - If water continues to release thermal energy, it cools down until it reaches 0°C and freezes into ice

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Phase changes for water

- Melting and freezing happen at the **melting / freezing point** of a substance
- Vaporisation and condensation happen at the **boiling point** of a substance
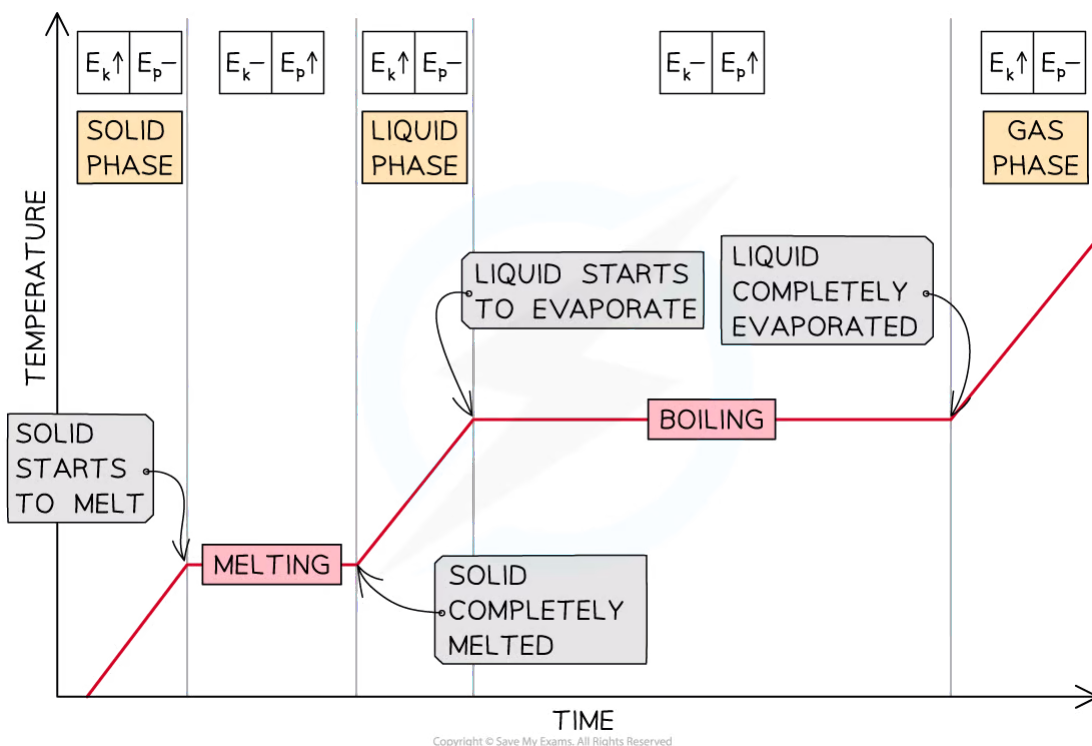
YOUR NOTES
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Phase Change Graphs

- A **heating** or **cooling curve** shows how the temperature of a substance changes with time
- The 'flat' sections of the graph indicate that there is **no change in temperature** over time, hence the substance is undergoing a **phase change**
  - The thermal energy supplied to or removed from the substance only affects the **potential energy of the particles**

- The regions of the graph that are not flat indicate that the substance is being heated or cooled down
  - The thermal energy supplied to or removed from the substance changes the **average kinetic energy of the particles**, hence resulting in an overall change in temperature of the substance

Heating Curves

- As energy is being supplied to a solid substance, its temperature increases until it reaches its melting point
- The temperature remains constant until the substance has melted completely
- If energy continues to be supplied, the liquid substance warms up until the boiling point is reached, and the substance vaporises
- Then, the temperature of the gas increases

Cooling Curves

- As energy is being removed from a gaseous substance, its temperature decreases until the boiling point is reached
- The temperature remains constant until the substance has condensed completely

- If energy continues to be removed, the liquid substance cools down until its freezing point and changes into a solid
- Then, the temperature of the solid decreases
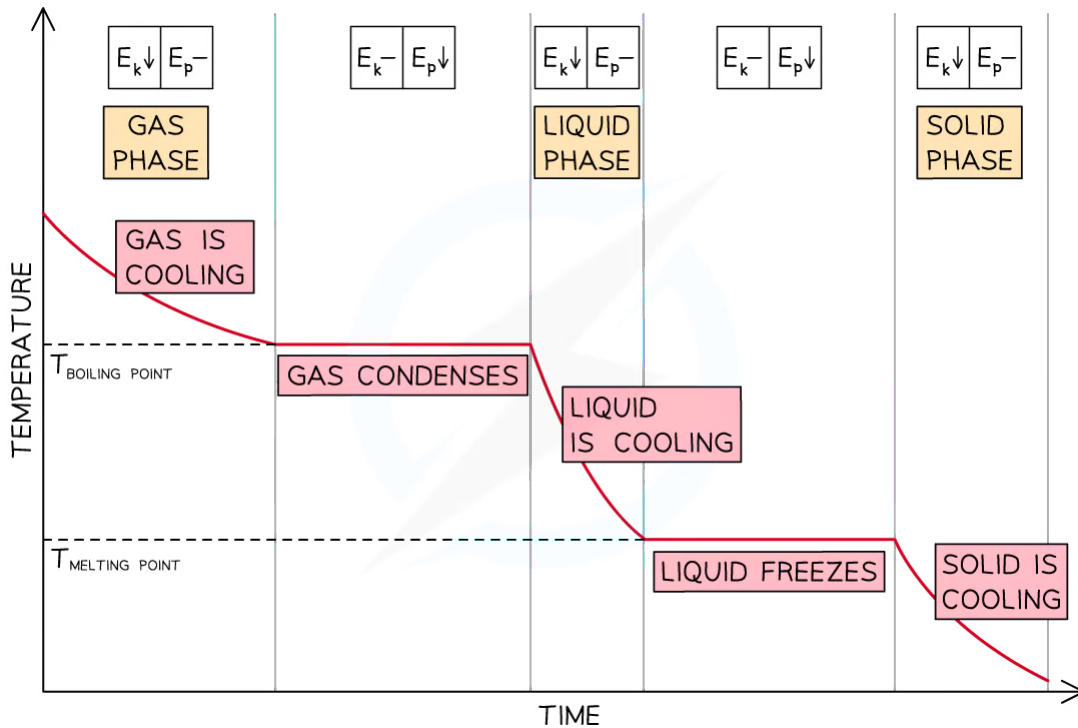
- Heating or cooling curves can also display how the temperature of a substance changes with energy
  - In the following worked example, **energy** (in J) is plotted on the **x-axis** instead of time

? Worked Example

The graph below is the heating curve for a 25 g cube of ice being heated at a constant rate. Calculate:

- The specific heat capacity of water in its liquid phase
- The specific latent heat of vaporisation

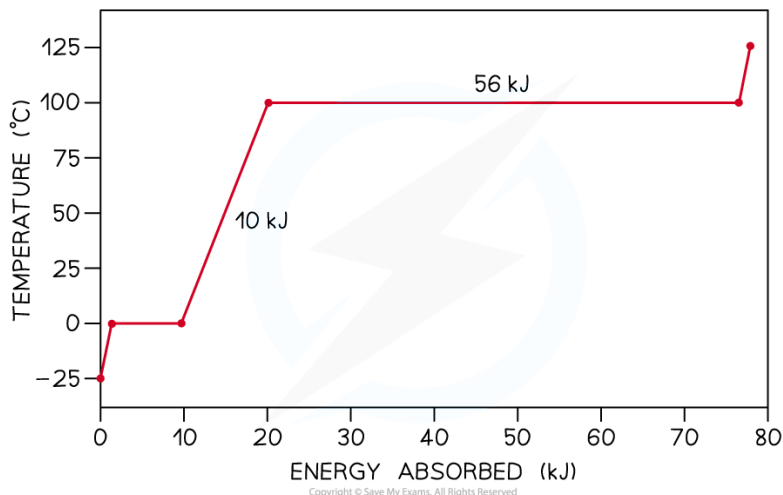**Step 1: Write down the mass m of ice in kilograms (kg)**

$$m = 25 \, g = 0.025 \, kg$$

Step 2: Read from the graph the amount of energy E_1 being supplied to the water in its liquid phase as it warms up

- The first flat section of the graph indicates the change of phase of ice into water
- The non-flat region that follows is the one relating to water in its liquid phase being heated up

$$E_1 = 20 \, kJ - 10 \, kJ = 10 \, kJ$$

Step 3: Convert this energy from kilojoules into joules

$$E_1 = 10 \, kJ = 10000 \, J$$

Step 4: Read from the graph the change in temperature ΔT_1 of the water as it warms up

$$\Delta T_1 = 100°C - 0°C = 100°C$$

Step 5: Write down the equation linking thermal energy E_1 to mass m, specific heat capacity c and change in temperature ΔT_1

$$E_1 = mc\Delta T_1$$

Step 6: Solve for the specific heat capacity c

$$c = \frac{E_1}{m \Delta T_1}$$

$$c = \frac{10000 \text{ J}}{0.025 \text{ kg} \times 100°C}$$

$$c = 4000 \text{ J kg}^{-1} °C^{-1}$$

Step 7: Read from the graph the amount of energy E_2 being supplied to the water as it changes into a gas at 100°C

- The second flat section of the graph indicates the change of phase of water into water vapour
- This energy must be converted from kilojoules (kJ) into joules (J)

$$E_2 = 56 \text{ kJ} = 56000 \text{ J}$$

Step 8: Write down the equation linking thermal energy E_2 to mass m and specific latent heat of vaporisation L_v

$$E_2 = mL_v$$

Step 9: Solve for the specific latent heat of fusion L_v

$$L_v = \frac{E_2}{m}$$

$$L_v = \frac{56000 \text{ J}}{0.025 \text{ kg}}$$

$$L_v = 2.2 \times 10^6 \text{ J kg}^{-1}$$

YOUR NOTES
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Investigating Thermal Energy

Estimating the Specific Heat Capacity of a Metal

Aim of the Experiment

- The aim of the experiment is to determine the specific heat capacity of a metal block

Variables

- Independent variable = Time, t (s)
- Dependent variable = Temperature, T (°C)
- Control variables:
  - Mass of the metal block
  - Voltage of the power supply

Equipment List

| Apparatus	Purpose
Power Supply	To generate voltage for the circuit
Immersion Heater	To heat up the metal block
Block of Metal	Sample to be heated
Digital Scale	To measure the mass of the metal block
Insulating Material	To reduce exchange of thermal energy between the metal block and the surroundings
Liquid-in-glass Thermometer	To measure the temperature of the metal block
Stop-clock	To measure the time
Oil or water	To seal the air gap between the metal and the thermometer, improving the accuracy of the temperature reading
Voltmeter	To measure the voltage across the immersion heater
Ammeter	To measure the current flowing through the immersion heater
Cables and Crocodile Clips	To connect all parts of the circuit

- Resolution of measuring equipment:

- Digital scale = 0.1 g
- Liquid-in-glass thermometer = 1°C
- Stop-clock = 0.01 s
- Voltmeter = 1 mV
- Ammeter = 1 mA

Method

Part 1: Measuring the temperature change
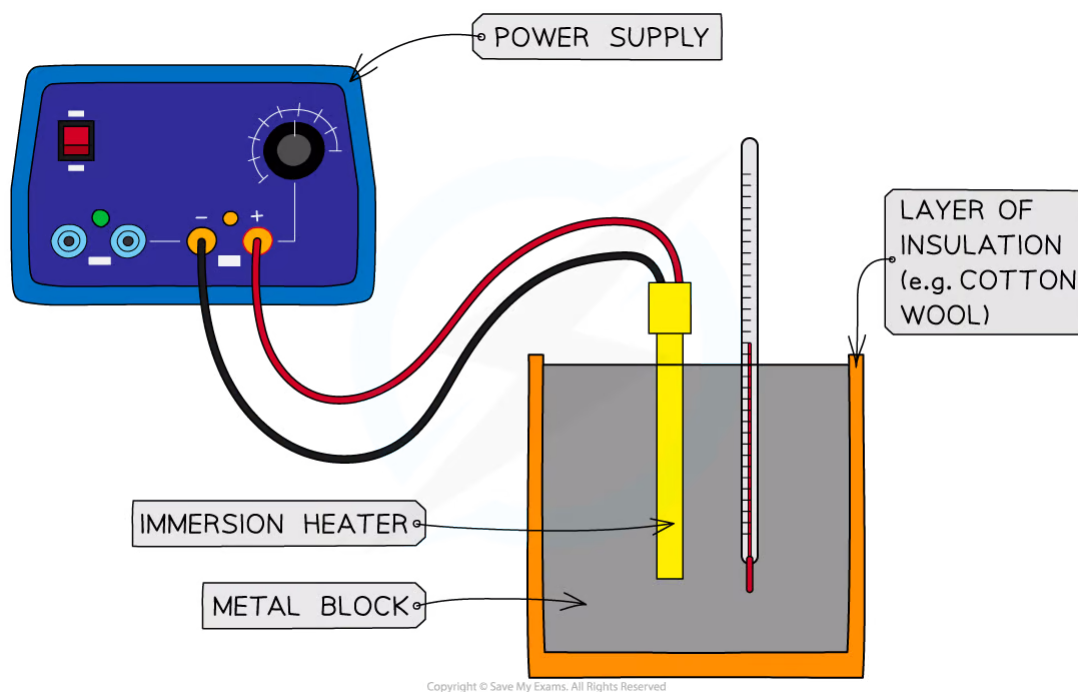
Apparatus used to investigate the specific heat capacity of a metal block

1. Use the digital scale to measure the mass *m* of the metal block. Record this value with its uncertainty *Δm* (this is simply the smallest division on the digital scale - e.g. ± 0.0001 kg)
2. Wrap the insulating material (e.g. cotton wool) around the block of metal to minimise energy transfers with the surroundings
3. Insert the immersion heater into the central hole of the metal block
4. Place the thermometer into the smaller hole in the metal block
5. Put a few drops of oil or water into the hole where the thermometer is, to seal the air gap between the metal and the thermometer
6. Measure the temperature *T* of the metal block and record this value in the first row of the table (*t* = 0), together with its uncertainty *ΔT*
7. Connect the immersion heater to the power supply
8. Turn on the power supply and start the stop-clock at the same time
9. Measure the temperature of the block every minute (i.e. 60 seconds) and record the temperature values in the table (for a total of 10 readings)
10. Turn the immersion heater off

- An example of a results table might look like this:

THIS IS EQUAL TO THE SUM OF THE SMALLEST DIVISION ON THE STOP-CLOCK (e.g. 0.01s) AND THE AVERAGE HUMAN REACTION TIME (e.g. 0.3 s)

READ FROM THE THERMOMETER

THIS IS EQUAL TO THE SMALLEST DIVISION ON THE THERMOMETER (e.g. 1°C)

TEMPERATURE OF THE METAL BLOCK WHEN THE POWER SUPPLY IS SWITCHED OFF

Time, t (s)	Δt (s) ±	Temperature, T (°C)	T (°C) ±
0.0	0.3		1
60.0	0.3		1
120.0	0.3		1
180.0	0.3		1
240.0	0.3		1
300.0	0.3		1
360.0	0.3		1
420.0	0.3		1
480.0	0.3		1
540.0	0.3		1
600.0	0.3		1

Part 2: Calculating the energy transferred

POWER SUPPLY
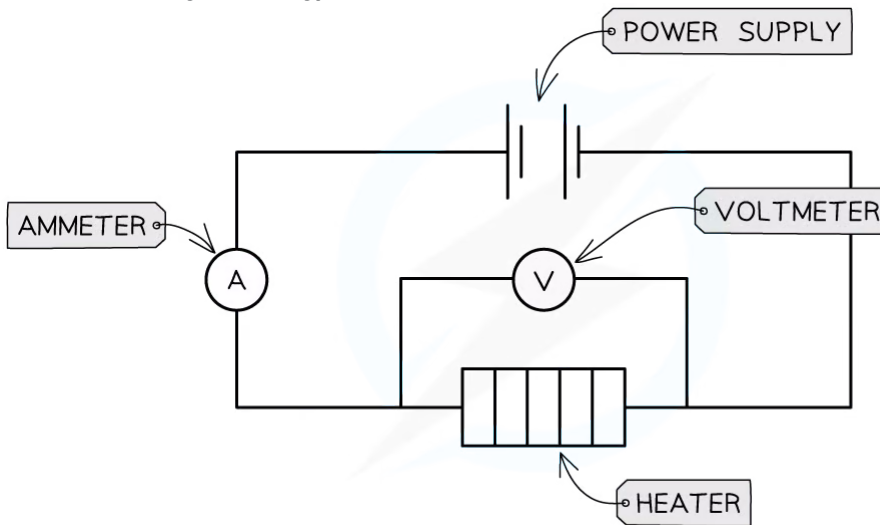
AMMETER

VOLTMETER

HEATER

Apparatus used to measure the energy transferred to the metal block

1. Connect the ammeter in series with the immersion heater and the voltmeter across the immersion heater (i.e. in parallel)
2. Turn on the power supply using the same value of voltage used to take the readings of temperature
3. Read the value of the current I from the ammeter and record this value with its uncertainty ΔI
4. Read the value of the voltage V from the voltmeter (not from the power supply) and record this value with its uncertainty ΔV

- An example of a results table might look like this:

Voltage, V (V)	ΔV (V) ±	Current, I (A)	ΔI (A) ±

Analysis of Results

- The equation linking the thermal energy E transferred to the metal block, the mass m of the block, the specific heat capacity c of the block and the temperature change ΔT is:

$$E = mc\Delta T$$

- Rearrange this to calculate the specific heat capacity c:

$$c = \frac{E}{m\Delta T}$$

- Write the energy E in terms of power P and time Δt:

$$E = P\Delta t$$

- Substitute this expression for E into the equation for the specific heat capacity:
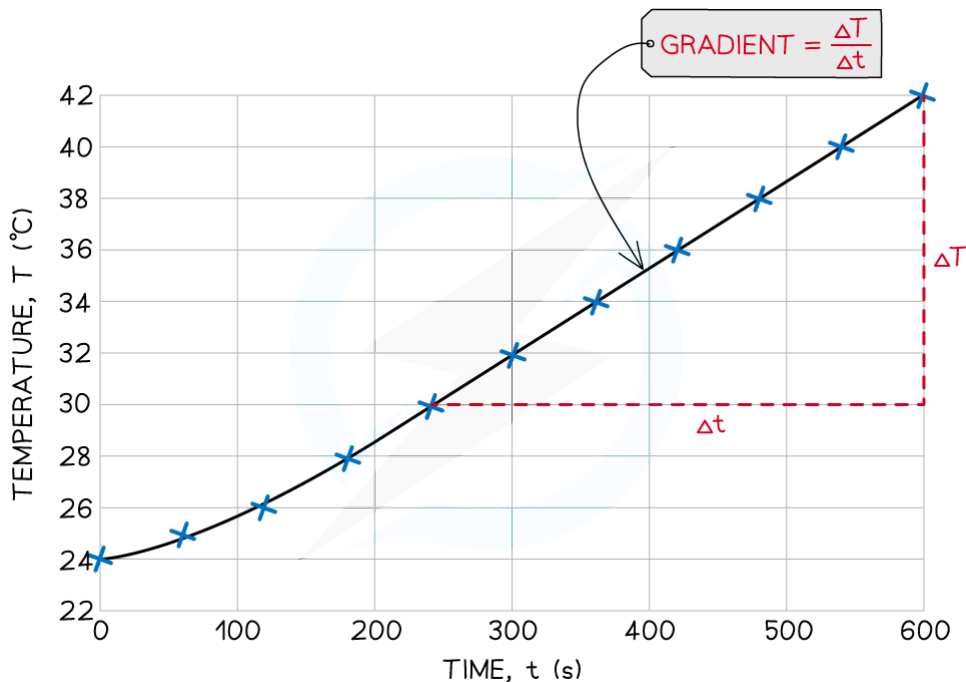
$$c = \frac{P\Delta t}{m\Delta T}$$

- Write the power P as the product of voltage V and current I:

$$c = \frac{VI\Delta t}{m\Delta T}$$

- Plot a graph of temperature T (°C) against time t (s)
- Calculate the gradient of the linear portion of the graph
  - *Gradient = $\Delta T/\Delta t$*

- Calculate the specific heat capacity c of the metal (in J kg^{-1} K^{-1} or, equivalently, in J kg^{-1} °C^{-1}) as follows:

$$c = \frac{VI}{m} \times gradient^{-1}$$

- Compare this result with the accepted value of the specific heat capacity for the metal used

Evaluating the Experiment

Systematic errors:

- Some energy is lost to the surroundings
  - The layer of insulation around the metal block and the drops of oil or water in the hole where the thermometer is placed all help reducing energy losses, but some energy is inevitably transferred to the surroundings, causing a systematic error

- The thermometer might not be calibrated correctly
  - Place the thermometer in a beaker with a mixture of ice and water for 30 s, of the thermometer reads 0°C, then it is reading correctly and can be used

Random errors:

- There might be parallax error when reading the values of temperature from the thermometer
  - Make sure you hold the thermometer at eye level when taking the temperature readings to reduce random error

- The temperature readings might not all be taken exactly 60 s apart from each other
  - Make sure you work with a partner, so you can read out the temperature values to them and they can write or type these in the table, while you are only looking at the stop-clock and thermometer

- Delays in the responsiveness of the thermometer might still cause random errors, using a thermometer with a smaller bulb and thinner glass walls will improve its responsiveness

Safety Considerations

- Position all equipment away from the edge of the desk
- Do not touch the metal block or the immersion heater when the power supply is switched on
- Allow time to cool before touching hot parts of the apparatus

---

? Worked Example

Use error analysis to obtain expressions for the relative error δc and the absolute error Δc on the specific heat capacity.

Step 1: Determine the relative errors on voltage V, current I, temperature T, mass m and time t

- $\delta V = \Delta V / V$
- $\delta I = \Delta I / I$
- $\delta T = \Delta T / T$ (where T is the increase in temperature for the straight portion of the graph)
- $\delta m = \Delta m / m$
- $\delta t = \Delta t / t$ (where t is the increase in time for the straight portion of the graph)

Step 2: Sum all relative errors in Step 1 to obtain the relative error δc on the specific heat capacity

$$\delta c = \delta V + \delta I + \delta T + \delta m + \delta t$$

Step 3: Recall the relationship between absolute and relative errors to determine the absolute error on the specific heat capacity Δc

- Remember that absolute errors have units (in this case, $J\,kg^{-1}\,K^{-1}$ or in $J\,kg^{-1}\,°C^{-1}$)
- c in the equation below is the experimental value of the specific heat capacity

$$\Delta c = c \times \delta c$$

3.2 Modelling a Gas

3.2.1 Ideal Gas Laws

Ideal Gas Laws

Boyle's Law

- Boyle's Law states:

  For a fixed mass of gas at a constant temperature, the pressure _p_ is inversely proportional to the volume _V_

- This can be expressed in equation form as:

$$p \propto \frac{1}{V}$$

- Plotting the pressure against the volume for a gas at a constant temperature on a graph (i.e. _p-V_ graph) gives the so-called **isothermal curve** for the gas

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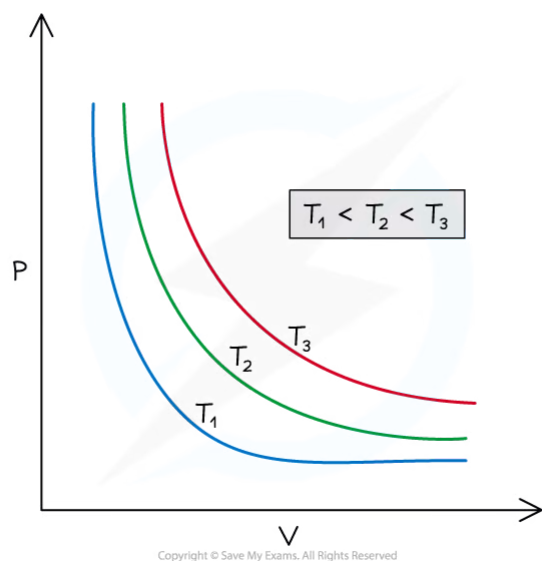
Graph of pressure against volume for a fixed mass of gas at three different temperatures, with T1 < T2 < T3. The curves are called isotherms (i.e. the temperature along each curve is constant)

- Plotting the pressure against the reciprocal of the volume (i.e. 1/_V_) for a gas at constant temperature still gives an isothermal, but this time, the line is straight

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Graph of pressure against reciprocal of the volume for a fixed mass of gas at three different temperatures, with T1 < T2 < T3. In this case, the isotherms are straight lines

- Boyle's law can be rewritten as follows:

$$pV = constant$$

- Which means that:

$$p_1V_1 = p_2V_2$$

- Where:
  - p_1 = initial pressure in pascals (Pa) or atmospheres (atm)
  - V_1 = initial volume in metres cubed (m^3) or litres (L)
  - p_2 = the final pressure in pascals (Pa) or atmospheres (atm)
  - V_2 = the final volume in metres cubed (m^3) or litres (L)
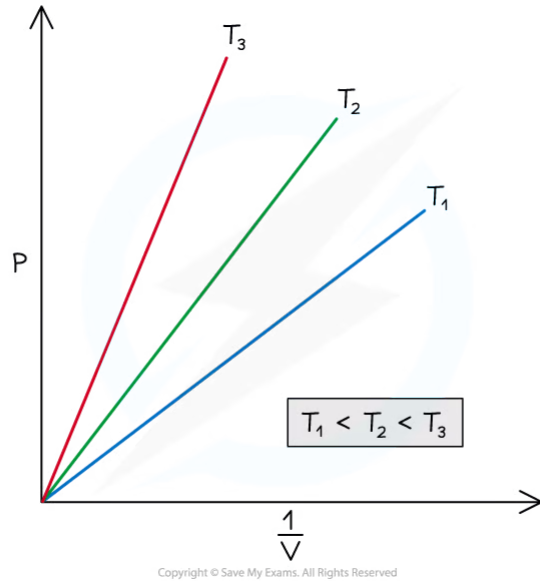
Charles's Law

- Charle's Law states:

  For a fixed mass of gas at constant pressure, the volume *V* is directly proportional to the absolute temperature *T*

- This can be expressed in equation form as:

$$V \propto T$$

- The direct proportionality relationship is only valid if the gas temperature is measured in kelvin (K)
- Plotting the volume against the temperature for a gas at constant pressure gives a straight line along which the gas pressure does not change

Graph of volume against temperature for a fixed mass of gas at three different pressures, with p1 < p2 < p3

- Charles's law can be rewritten as follows:

$$\frac{V}{T} = constant$$

- Which means that:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

- Where:
    - V_1 = initial volume in metres cubed (m^3) or litres (L)
    - T_1 = initial temperature in kelvin (K)
    - V_2 = final volume in metres cubed (m^3) or litres (L)
    - T_2 = final temperature in kelvin (K)
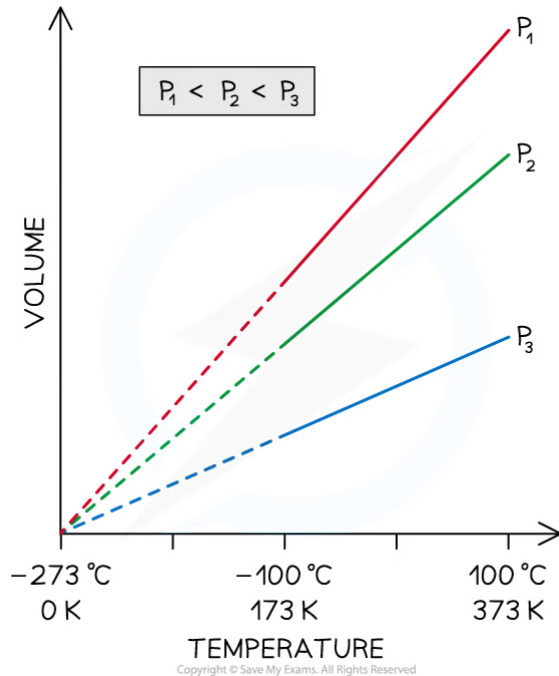
Gay Lussac's Law

- Gay Lussac's Law states:

    For a fixed mass of gas at constant volume, the pressure p is directly proportional to the absolute temperature T

- This can be expressed in equation form as:

$$p \propto T$$

- The direct proportionality relationship is only valid if the gas temperature is measured in kelvin (K)
- Plotting the pressure against the temperature for a gas at constant volume gives a straight line along which the gas volume is the same

YOUR NOTES
↓
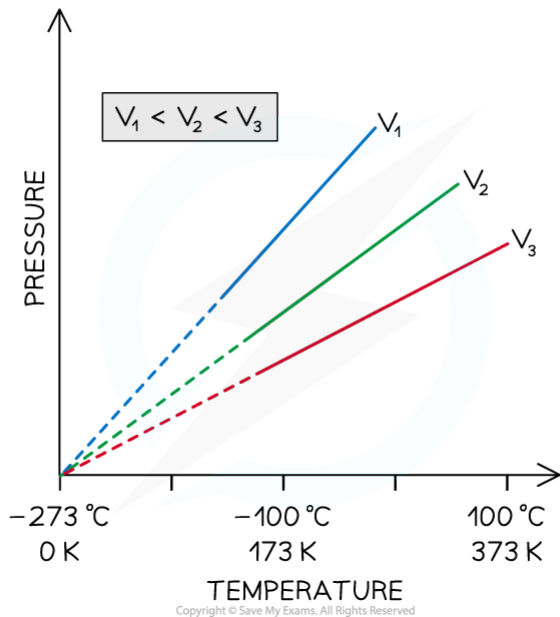
Graph of pressure against temperature for a fixed mass of gas at three different volumes,
with V1 < V2 < V3

- Gay Lussac's law can be rewritten as follows:

$$\frac{p}{T} = constant$$

- Which means that:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

- Where:
  - p_1 = initial pressure in pascals (Pa) or atmospheres (atm)
  - T_1 = initial temperature in kelvin (K)
  - p_2 = final pressure in pascals (Pa) or atmospheres (atm)
  - T_2 = final temperature in kelvin (K)

Gas Laws Combined

- The three gas laws can be combined into one
- For a fixed mass of gas, the following holds:

$$\frac{pV}{T} = constant$$

- Which means that:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

- Where:
  - p_1 = initial pressure in pascals (Pa) or atmospheres (atm)
  - V_1 = initial volume in metres cubed (m^3) or litres (L)
  - T_1 = initial temperature in kelvin (K)
  - p_2 = final pressure in pascals (Pa) or atmospheres (atm)

- V_2 = final volume in metres cubed (m^3) or litres (L)
- T_2 = final temperature in kelvin (K)

> ? **Worked Example**
>
> An ideal gas occupies a volume equal to $5.0 \times 10^{-4}\ m^3$. Its pressure is $2.0 \times 10^6\ Pa$ and its temperature is 40°C. The gas is then heated and reaches a temperature of 80°C. It also expands to a new volume of $6.0 \times 10^{-4}\ m^3$. Determine the new pressure of the gas.

Step 1: Write down the given quantities

- Initial volume, $V_1 = 5.0 \times 10^{-4}\ m^3$
- Initial pressure, $p_1 = 2.0 \times 10^6\ Pa$
- Initial temperature, $T_1 = 40°C = 313\ K$
- Final volume, $V_2 = 6.0 \times 10^{-4}\ m^3$
- Final temperature, $T_2 = 80°C = 353\ K$

Remember to:

- Use the appropriate subscripts for initial (i.e. 1) and final (i.e. 2) values
- Convert the temperature from degrees Celsius into Kelvin (K)

Step 2: Write down the equation for the three gas laws combined

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

Step 3: Rearrange the equation to calculate the unknown final pressure p_2

$$p_2 = \frac{p_1 V_1 T_2}{V_2 T_1}$$

Step 4: Substitute numbers into the equation

$$p_2 = \frac{(2.0 \times 10^6)\ Pa \times (5.0 \times 10^{-4})\ m^3 \times 353\ K}{(6.0 \times 10^{-4})\ m^3 \times 313\ K}$$

$$p_2 = 1.9 \times 10^6\ Pa$$

> 💡 **Exam Tip**
>
> When dealing with gas laws problems, always remember to convert temperatures from degrees Celsius (°C) to kelvin (K). After you solve a problem using any of the gas laws (or all of them combined), always check whether your final result makes physically sense - e.g. if you are asked to calculate the final pressure of a fixed mass of gas being heated at constant volume, your result must be greater than the initial pressure given in the problem (since Gay-Lussac's law states that pressure and absolute temperature are directly proportional at constant volume).

3.2.2 Ideal Gas Equation

Ideal Gas Equation

Avogadro's Law

- Avogadro's Law states:

  For a gas at constant temperature and pressure, the number of moles *n* is directly proportional to the volume *V* of the gas

- This can be expressed in equation form as:

$$n \propto V$$

- This means that two different gases of equal temperatures, pressures and volumes have the same number of particles *N*
  - Note that the number of particles *N* is directly proportional to the number of **moles** *n*

Equation of State for an Ideal Gas

- Boyle's Law, Charles's Law and Gay-Lussac's law can be combined with Avogadro's law to give a single constant, known as the **ideal gas constant**, *R*
- Combining the four equations leads to the **equation of state of an ideal gas**

$$\frac{pV}{nT} = R$$

$$pV = nRT$$

- Where:
  - p = pressure in pascals (Pa)
  - V = volume in metres cubed (m^3)
  - T = temperature in kelvin (K)
  - n = number of moles in the gas (mol)
  - $R = 8.31\ J\ K^{-1}\ mol^{-1}$ (ideal gas constant)

> **?** ## Worked Example
>
> A gas has a temperature of –55°C and a pressure of 0.5 MPa. It occupies a volume of 0.02 m^3. Calculate the number of gas particles.

Step 1: Write down the known quantities

- Temperature, T = –55°C = 218 K
- Pressure, p = 0.5 MPa = 0.5 × 10^6 Pa
- Volume, V = 0.02 m^3

Note the conversions:

- The pressure p must be converted from megapascals (MPa) into pascals (Pa)
- The temperature must be converted from degrees Celsius (°C) into kelvin (K)

YOUR NOTES
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Step 2: Write down the equation of state of ideal gases

$$pV = nRT$$

Step 3: Rearrange the above equation to calculate the number of moles *n*

$$n = \frac{pV}{RT}$$

Step 4: Substitute numbers into the equation

- From the data booklet, $R = 8.31 \ \text{J K}^{-1} \text{mol}^{-1}$

$$n = \frac{(0.5 \times 10^6) \ \text{Pa} \times 0.02 \ \text{m}^3}{8.31 \ \text{J K}^{-1} \text{mol}^{-1} \times 218 \ \text{K}}$$

$$n = 5.5 \ \text{mol}$$

Step 5: Calculate the number of particles *N*

- Write down the relationship between number of particles and number of moles

$$N = nN_A$$

- From the data booklet, $N_A = 6.02 \times 10^{23} \ \text{mol}^{-1}$ (Avogadro constant)

$$N = 5.5 \ \text{mol} \times (6.02 \times 10^{23}) \ \text{mol}^{-1}$$

$$N = 3.3 \times 10^{24}$$

💡 Exam Tip

When using the equation of state of ideal gases, always remember to convert temperatures from degrees Celsius (°C) to kelvin (K). Note that the number of moles *n* is not the same as the number of particles *N*:

- When a question asks to calculate the number of particles in a sample of gas, you should first use the equation of state to determine the number of moles *n* of the gas, and then calculate the number of particles using $N = nN_A$
- If a question gives the number of particles in a sample of gas instead of the number of moles, you should first use $n = N/N_A$ to calculate the number of moles of the gas, and then use the equation of state to perform any further calculation (e.g. volume, pressure, etc.)

YOUR NOTES
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# Kinetic Model of an Ideal Gas

Gas Pressure

- A gas is made of a large number of particles
- Gas particles have **mass** and move **randomly** at high speeds
- Pressure in a gas is due to the collisions of the gas particles with the walls of the container that holds the gas
- When a gas particle hits a wall of the container, it undergoes a change in momentum due to the force exerted by the wall on the particle (as stated by Newton's Second Law)
    - Final momentum = $-mv$
    - Initial momentum = mv
- Therefore, the change in momentum Δp can be written as:

$$\Delta p = \text{final momentum} - \text{initial momentum}$$

$$\Delta p = -mv - mv = -2mv$$

$$-F = \frac{\Delta p}{\Delta t} = -\frac{2mv}{\Delta t}$$

- According to Newton's Third Law, there is an **equal** and **opposite** force exerted by the particle on the wall (i.e. $F = 2mv/\Delta t$)
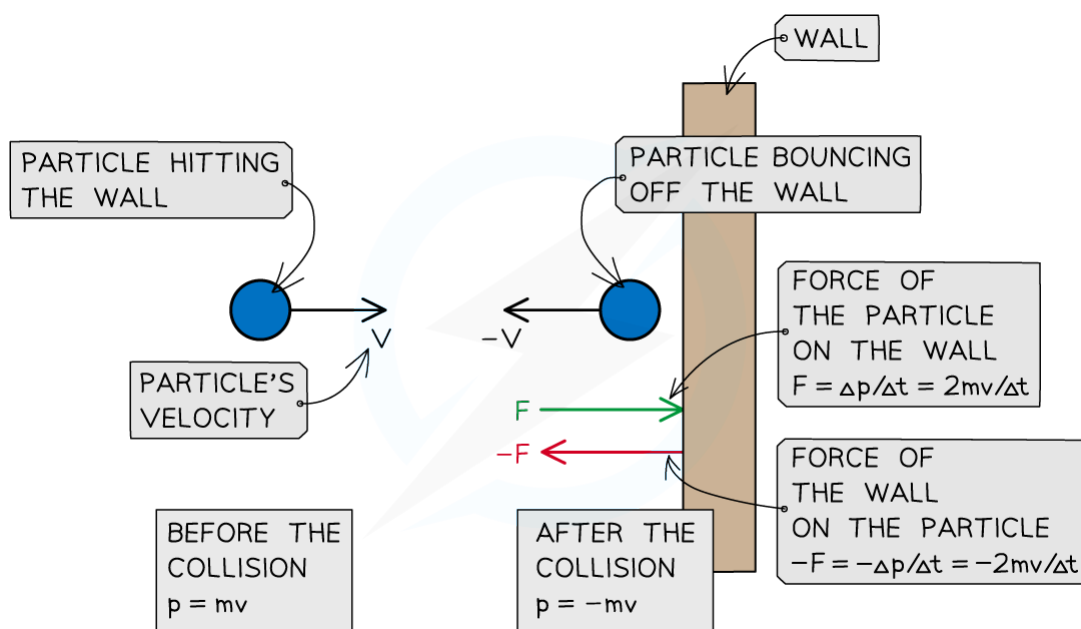
A particle hitting a wall of the container in which the gas is held experiences a force from the wall and a change in momentum. The particle exerts an equal and opposite force on the wall

- Since there is a large number of particles, their collisions with the walls of the container give rise to gas pressure, which is calculated as follows:

$$p = \frac{F}{A}$$

- Where:
  - p = pressure in pascals (Pa)
  - F = force in newtons (N)
  - A = area in metres squared (m^2)

Average Random Kinetic Energy of Gas Particles

- Particles in gases have a variety of different speeds
- The average random kinetic energy **of the particles E_K**, which can be written as follows:

$$E_K = \frac{3}{2} k_B T$$

- Where:
  - E_K = average random kinetic energy of the particles in joules (J)
  - k_B = 1.38×10^{-23} J K^{-1} (Boltzmann's constant)
  - T = absolute temperature in kelvin (K)

- **k_B** is known as **Boltzmann's constant**, and it can be written as follows:

$$k_B = \frac{R}{N_A}$$

- Where:
  - R = 8.31 J K^{-1} mol^{-1} (ideal gas constant)
  - N_A = 6.02×10^{23} mol^{-1} (Avogadro constant)

Internal Energy of the Gas

- Using the equation of state of ideal gases, the internal energy can be written as follows:

$$U = \frac{3}{2} pV$$

- Where:
  - U = internal energy of the gas in joules (J)
  - p = gas pressure in pascals (Pa)
  - V = gas volume in metres cubed (m^3)

> **?** **Worked Example**
>
> 2 mol of gas is sealed in a container, at a temperature of 47°C.
>
> Determine:
>
> a. The average random kinetic energy of the particles in the gas
> b. The internal energy of the gas

Part (a)

Step 1: Write down the temperature T of the gas in kelvin (K)

$T = 47°C = 320 \text{ K}$

Step 2: Write down the equation linking the absolute temperature T of the gas to the average random kinetic energy E_K of the gas particles

$$E_K = \frac{3}{2} k_B T$$

Step 3: Substitute numbers into the equation

- From the data booklet, $k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$

$$E_K = \frac{3}{2} \times \left(1.38 \times 10^{-23}\right) \text{ J K}^{-1} \times 320 \text{ K}$$

$$E_K = 6.6 \times 10^{-21} \text{ J}$$

Part (b)

Step 1: Write down the equation linking the internal energy U of the gas to the number of moles n and the absolute temperature T

$$U = \frac{3}{2} nRT$$

Step 2: Substitute numbers into the equation

- From the data booklet, $R = 8.31 \text{ J K}^{-1} \text{mol}^{-1}$

$$U = \frac{3}{2} \times 2 \text{ mol} \times 8.31 \text{ J K}^{-1} \text{mol}^{-1} \times 320 \text{ K}$$

$$U = 8000 \text{ J} = 8 \text{kJ}$$

- Note that, alternatively, the internal energy can be calculated using the following equation:

$$U = NE_K = 8000 \text{ J} = 8 \text{kJ}$$

$$N = nN_A = 2 \text{ mol} \times (6.02 \times 10^{23}) \text{ mol}^{-1} = 1.2 \times 10^{24}$$

$$E_K = 6.6 \times 10^{-21} \text{ J (calculated in Step 3)}$$

💡 **Exam Tip**

Momentum is a Mechanics topic that should have been covered in a previous unit. The above derivation of change in momentum and resultant force should have already been studied – if you're not comfortable with it then make sure you go back to revise this!

Real & Ideal Gases

- The equation of state of ideal gases and the equations for the kinetic energy of the particles and the internal energy of the gas derived previously, only apply to **ideal gases**
- An ideal gas is defined as one to which all the statements below apply:

1. It contains a very large number of **identical particles** (atoms or molecules)
2. Each gas particle occupies a negligible volume compared to the volume of the gas
   - This means they can be considered as **point particles**
3. The gas particles move **randomly** at **high speeds**
4. The gas particles obey **Newton's laws of motion**
5. There are **no intermolecular forces** between the gas particles
   - Therefore, the internal energy of the gas is equal to the total kinetic energy of the particles ($U = NE_K$)
6. The gas particles undergo elastic collisions with each other and with the walls of the container in which the gas is held
   - Hence, the total kinetic energy of the particles and the temperature of the gas do not change as a result of these collisions
7. The duration of the collisions is negligible compared with the time interval between collisions
8. Each particle exerts a force on the wall of the container with which it collides
   - This means the average of the forces produced by all gas particles results in a **uniform gas pressure**

- **Real gases** are not ideal gases, however, under certain conditions, they can be considered as ideal gases. An ideal gas is a good approximation of a real gas when:
  - Pressure and density are low
  - Temperature is moderate

3.2.4 Mole Calculations

Mole Calculations

The Mole

- The **mole** is one of the seven SI base units
- It is used to measure the **amount of substance**
- One mole is defined as follows:

> **The amount of substance that contains as many elementary entities as the number of atoms in 12 g of carbon-12**

- This amount of substance is exactly $6.02214076 \times 10^{23}$ elementary entities (i.e. particles, atoms, molecules)
  - At IB level, this number can be rounded to 6.02×10^{23}
- One mole of gas contains a number of particles (atoms or molecules) equal to the **Avogadro constant**
- Therefore, to calculate the number of particles N in a gas, knowing the number of moles n, the following relationship must be used:

$$N = nN_A$$

- Where:
  - N = number of gas particles
  - n = number of moles of gas (mol)
  - $N_A = 6.02 \times 10^{23}$ mol^{-1} (Avogadro constant)

Molar Mass

- The **molar mass M** of a substance is defined as the mass m of the substance divided by the amount (in moles) of that substance
- The molar mass is calculated as follows:

$$M = \frac{m}{n}$$

- Where:
  - M = molar mass in g mol^{-1}
  - m = mass in grams (g)
  - n = number of moles (mol)

> ❓ Worked Example
>
> Nitrogen is normally found in nature as a diatomic molecule, N_2. The molar mass of nitrogen is 28.02 g mol^{-1}.
>
> Determine the mass of a nitrogen atom.

Step 1: Write down the relationship between the molar mass M and the mass m

$$M = \frac{m}{n}$$

Step 2: Rearrange the equation to find the mass m of n = 1 mol of nitrogen

$$m = Mn$$

$$m = 28.02 \, \text{g mol}^{-1} \times 1 \, \text{mol}$$

$$m = 28.02 \, \text{g}$$

Step 3: Identify the number of nitrogen atoms in 1 mol of nitrogen

- n = 1 mol of nitrogen contains a number of **molecules** equal to the Avogadro constant
- From the data booklet, $N_A = 6.02 \times 10^{23}$

$$N_{molecules} = nN_A$$

$$N_{molecules} = 6.02 \times 10^{23}$$

- Each nitrogen molecule contains 2 atoms of nitrogen
- n = 1 mol of nitrogen contains a number of **atoms** equal to twice the number of molecules

$$N_{atoms} = 2 \times N_{molecules} = 2 \times (6.02 \times 10^{23})$$

$$N_{atoms} = 1.2 \times 10^{24}$$

Step 4: Divide the mass of 1 mol of nitrogen m by the number of atoms in 1 mol of nitrogen N_{atoms} in order to find the mass of a nitrogen atom

$$\text{Mass of a nitrogen atom} = \frac{m}{N_{atoms}}$$

$$\text{Mass of a nitrogen atom} = 2.3 \times 10^{-23} \, \text{g}$$

YOUR NOTES
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Investigating Gas Laws

Investigating Gay-Lussac's Law

Aim of the Experiment

- The aim of the experiment is to investigate the relationship between the pressure and the temperature of a gas at constant volume

Variables

- Independent variable = pressure, p (Pa)
- Dependent variable = temperature, T (°C)
- Control variables:
 - Volume of the gas
  - Temperature of the room

Equipment List

Apparatus	Purpose
Retort Stand and Clamp	To hold the flask above the surface of the table
Gas	Sample to be investigated
Sealed Flask	To contain the sample of gas to be investigated, and ensure the gas is not allowed to expand
Beaker	To contain the water and ice (or the water only)
Stirring Rod	To stir the water and ice (or the water only), and ensure the temperature is homogeneous
Tripod Stand and Gauze	To support the beaker
Liquid-in-glass Thermometer	To measure the temperature of the water
Bunsen Burner	To heat up the water in the beaker, and hence the gas in the flask
Water and Ice	To provide a water bath for the flask
Pressure Meter	To measure the pressure of the gas in the flask

- Resolution of measuring equipment:
  - Beaker = 50 ml
  - Liquid-in-glass thermometer = 1°C
  - Pressure meter = 0.001×10^5 Pa

Method
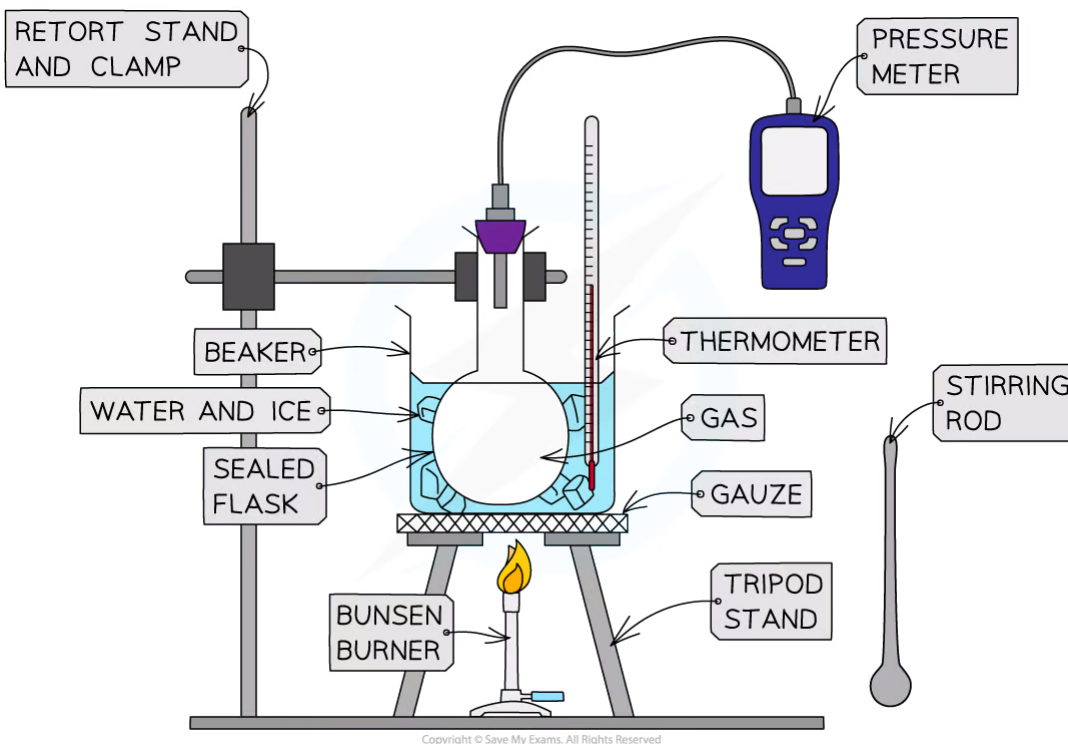
YOUR NOTES
↓

Apparatus used to investigate the relationship between the pressure and the temperature of a gas at constant volume

1. Set up the apparatus as shown in the diagram
2. Pour 500 ml of ice and water into the beaker, and stir it with the stirring rod
3. Measure the temperature *T* of the water and record this value in the table, together with its uncertainty Δ*T*
4. Measure the pressure *p* of the gas and record this value in the table, together with its uncertainty Δ*p*
5. Light the Bunsen burner and start to heat the water
6. Wait until the temperature reading on the thermometer reaches 10°C and measure the pressure of the gas. Record these new values of temperature and pressure in the table
7. Keep heating the water and measure the pressure of the gas at increments of 10°C between 0°C and 100°C

- An example of a results table might look like this:

READ FROM THE THERMOMETER

THIS IS EQUAL TO THE SMALLEST DIVISION ON THE THERMOMETER (e.g. 1°C)
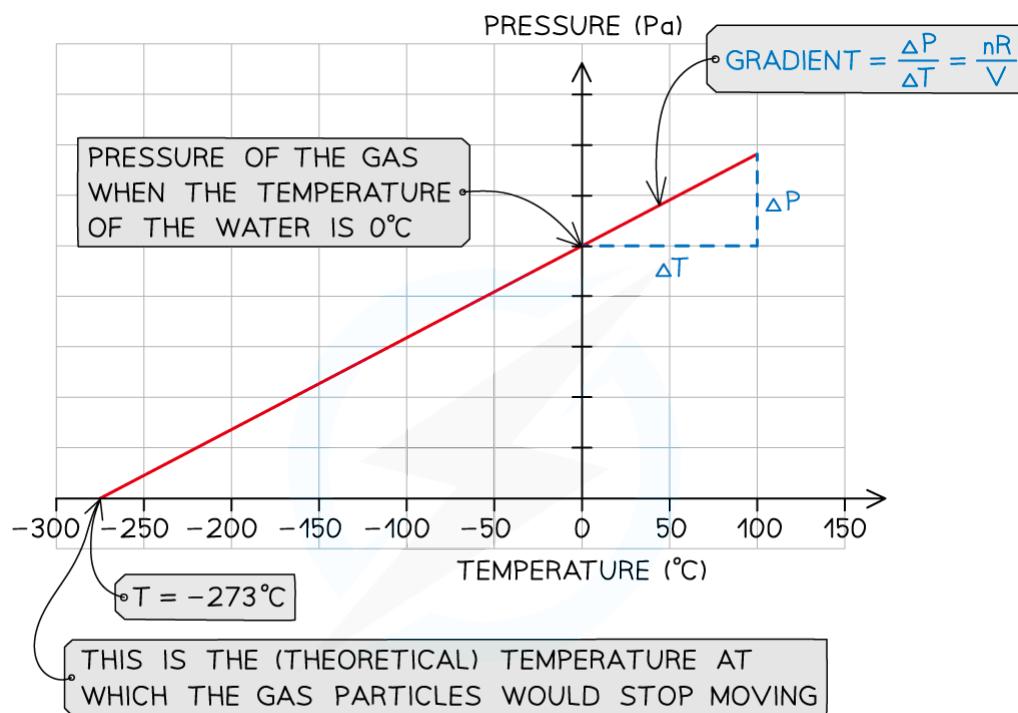
READ FROM THE PRESSURE METER

THIS IS EQUAL TO THE SMALLEST DIVISION ON THE PRESSURE METER (e.g. 0.001 × 10⁵Pa)

PRESSURE OF THE GAS WHEN THE WATER AND ICE ARE NOT HEATED

Temperature, T (°C)	ΔT (°C) ±	Pressure, p (×10⁵ Pa)	Δp (×10⁵ Pa) ±
0	1		0.001
10	1		0.001
20	1		0.001
30	1		0.001
40	1		0.001
50	1		0.001
60	1		0.001
70	1		0.001
80	1		0.001
90	1		0.001
100	1		0.001

Analysis of Results

- Plot a graph of pressure p (Pa) against temperature T (°C)
  - The y-intercept is the value of the pressure of the gas when the water and ice are not heated ($T = 0$°C)
- Extrapolate the graph backwards
  - The x-intercept is the value of the temperature of the gas when the pressure is zero
  - This refers to the (theoretical) temperature of the gas when the particles would stop moving, corresponding to the absolute zero on the kelvin scale: $T = 0$ K = -273°C
  - Verify that the x-intercept corresponds to this value

PRESSURE (Pa)

GRADIENT = $\frac{\Delta P}{\Delta T} = \frac{nR}{V}$

PRESSURE OF THE GAS WHEN THE TEMPERATURE OF THE WATER IS 0°C

ΔP

ΔT

−300 −250 −200 −150 −100 −50 0 50 100 150

TEMPERATURE (°C)

T = −273°C

THIS IS THE (THEORETICAL) TEMPERATURE AT WHICH THE GAS PARTICLES WOULD STOP MOVING

- Calculate the gradient of the graph
  - Gradient = $\Delta p / \Delta T$
  - Recall the equation of state of ideal gases
  - Use this to write the gradient as follows:

$$Gradient = \frac{nR}{V}$$

- Where:
  - n = number of moles of gas (mol)
  - V = volume of the gas (m^3)
  - R = 8.31 J K^{-1} mol^{-1} (ideal gas constant)
- Using the **volume V** of the gas sample, and the **number of moles n**, the value of the **ideal gas constant R** can be calculated

Evaluation

- Parallax error might affect the temperature readings
  - Make sure to hold the thermometer at eye level when taking the temperature readings to reduce random error

- There might be a delay in the responsiveness of the thermometer
  - Use a thermometer with a smaller bulb and thinner glass walls to improve its responsiveness

- In this experiment, the temperature of the gas inside the flask is assumed to be the same as the temperature of the water bath, but this might not always be true, as sometimes the reading of pressure might be taken when the gas is not yet in thermal equilibrium with the

water in the beaker. The two improvements below ensure the temperature of the gas is much closer to that of the water:

- Cover the beaker with a lid and wrap it in tin foil to reduce thermal energy transfers with the surroundings due to evaporation and convection
- Use a flask with thinner glass walls to improve conduction

- The volume of the gas inside the flask might not be exactly constant due to gaps in the flask's aperture
  - Make sure there is no gap between the stopper used to seal the flask and the pressure sensor inserted into it. Seal any small gap with Blu Tack if necessary

Safety Considerations

- Position all equipment away from the edge of the desk
- Tie any long hair back to avoid it catching fire
- Make sure not to spill hot water onto your skin
- Make sure you turn off the gas tap as soon as you finish collecting your data
- Allow time to cool before touching hot parts of the apparatus

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? ## Worked Example

A student used an online simulation to investigate Gay-Lussac's law for an ideal gas made of 100 particles, kept at a constant volume.

She obtained the following graph of pressure (kPa) against absolute temperature (K). The gradient of the graph is 4.239 kPa K^{-1}

PRESSURE (kPa) VS ABSOLUTE TEMPERATURE (K)
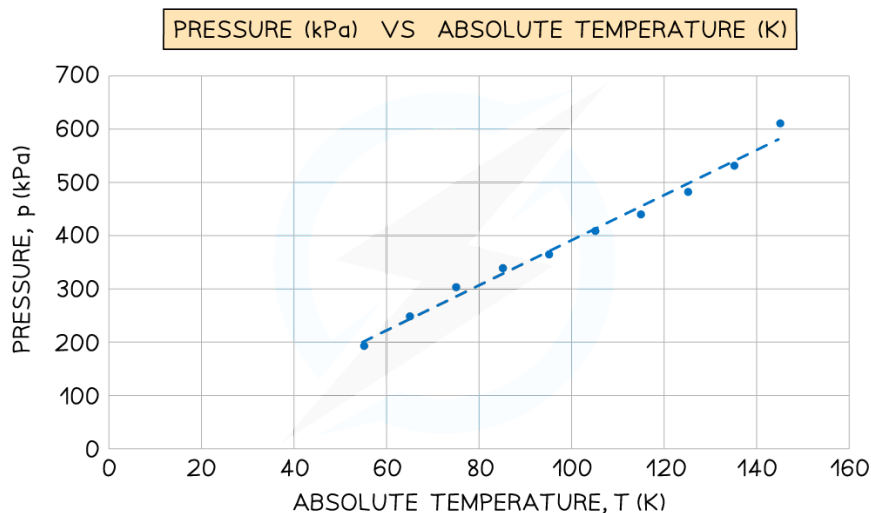
Determine the constant volume of the gas.

Step 1: Write down the known quantities

- *Gradient* = 4.239 kPa K^{-1} = 4239 Pa K^{-1}
- Number of gas particles, N = 100

Note that you must convert the gradient from kPa K^{-1} into Pa K^{-1}

Step 2: Calculate the number of moles n

- Write down the relationship between the number of particles N and the number of moles, $N = nN_A$
- Rearrange this to calculate the number of moles n

$$n = \frac{N}{N_A}$$

- Substitute the numbers into the equation for n
  - From the data booklet, $N_A = 6.02 \times 10^{23}$ mol^{-1}

$$n = \frac{100}{(6.02 \times 10^{23}) \text{ mol}^{-1}}$$

$$n = 1.7 \times 10^{-22} \text{ mol}$$

Step 3: Recall the relationship between the gradient of the graph and the volume of the gas V

$$Gradient = \frac{nR}{V}$$

Step 4: Rearrange the above equation to calculate the volume V

$$V = \frac{nR}{Gradient}$$

Step 5: Substitute the numbers into the above equation

- From the data booklet, $R = 8.31$ J K^{-1} mol^{-1}

$$V = \frac{(1.7 \times 10^{-22}) \text{ mol} \times 8.31 \text{ J K}^{-1}\text{mol}^{-1}}{4239 \text{ Pa K}^{-1}}$$

$$V = 3.3 \times 10^{-25} \text{ m}^3$$