4.6 Random Variables

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.6 Random Variables
Difficulty	Hard

Time allowed: 70

Score: /55

Percentage: /100

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Question 1

The random variable X has mean 8 and variance 15. Given that

$$\mathrm{E}(aX+b)=23$$
$$\mathrm{Var}(aX+b)=135$$

find the two possible values of b.

[4 marks]

Question 2a

The random variable X has mean of μ and standard deviation of σ. The random variable has mean of 3 and standard deviation of 4. Given that

$$E(2X+5Y)=25$$
$$\mathrm{Var}(2X+5Y)=724$$

a)
Find the value of μ and the value of σ.

[5 marks]

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Question 2b

b)
State the assumption that has been made about the random variables X and Y.

[1 mark]

Question 3a

Frank has a variable tariff for his electricity and gas bills. His monthly electricity bill is $\$E$ and his monthly gas bill is $\$G$. E and G are independent random variables with distributions $\mathrm{N}(85, 9.4^2)$ and $\mathrm{N}(53,\ 12.45)$ respectively.

a)
Find the probability that the total electricity and gas bill in a month exceeds \$150.

[4 marks]

Question 3b

Frank has a part-time job tutoring college students. His monthly income from this job can be modelled as a Normal distribution with mean \$504 and standard deviation \$41. Frank uses this income to pay for his gas and electricity bills, he puts the remaining money into his partner's bank account each month.

b)
i)
Find the probability that Frank puts between \$350 and \$450 into his partner's bank account in a month.
ii)
State the assumption needed in part (b)(i) regarding his income and his bills.

[5 marks]

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Question 4a

Veronica, a taxi driver in London, charges her customers a fixed fee of £5 plus £1.20 per mile. The lengths of her customers' journeys are normally distributed with mean
16.7 miles and standard deviation 4.1 miles.

a)
Find the standard deviation of the prices of Veronica's taxi rides.

[1 mark]

Question 4b

b)
Find the probability that a taxi ride will cost less than £30.

[2 marks]

Question 4c

c)
Find the probability that the total cost of two independent taxi rides is more than £60.

[3 marks]

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Question 4d

On a bank holiday, Veronica doubles her prices.

d)
Find the variance of the prices of Veronica's taxi ride on a bank holiday.

[2 marks]

Question 4e

e)
Find the probability that a taxi ride on a bank holiday will cost more than £60.

[1 mark]

Question 5a

The random variable X has the distribution $\mathrm{N}(5.9,\ 2.1^2)$.

a)
Find the probability that the sum of 50 independent observations of X exceeds 300.

[3 marks]

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Question 5b

b)
Hence find the probability that the mean of 50 independent observations of X is less than 6.

[1 mark]

Question 6a

Dinah's Diner is famous for its triple burger which is made up of three beef patties, two rashers of bacon and a toasted bread bun. The mass, in grams, of a beef patty follows the distribution $\mathrm{N}(110,\ 6^2)$. The mass, in grams, of a rasher of bacon follows the distribution $\mathrm{N}(30,\ 5^2)$. The mass, in grams, of a toasted bread bun follows the distribution $\mathrm{N}(50,\ 3^2)$.

a)
Estimate the proportion of triple burgers at Dinah's Diner that have a mass of more than 450 g.

[4 marks]

Question 6b

b)
State, with a reason, whether the probability that the total mass of two triple burgers exceeding 900 g is equal to your answer in part (a).

[1 mark]

Question 7

Ella buys some fruit from the grocery store. The average mass of an apple at the store is 220 grams with standard deviation 15 grams. The average mass of an orange at the store is 120 grams with standard deviation 8 grams. Ella buys 5 random apples and 8 random oranges and packs them in her grocery bag which weighs 135 grams when empty.

Find the expectation and standard deviation of the total mass of the grocery bag and the 13 pieces of fruit. State any assumptions that are needed and where they are needed.

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[5 marks]

Question 8a

A football coach ends each training session with a penalty shoot-out competition. Each player takes 15 penalty shots and scores 6 points for each goal. The coach does not want anybody to get zero points so gives all players 10 points just for participating. Raquel takes part in the challenge each session and it is known that on average 65% of her shots go in the goal.

a)
Find the mean and standard deviation for the number of points Raquel achieves in the competition. State any assumptions that are needed.

[4 marks]

Question 8b

b)
Find the probability that Raquel scores more than 80 points in the competition.

[3 marks]

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Question 9a

Harietta has a summer job during her break from college. The random variable X represents the amount of money ($) Harietta earns each day. Each day she gets a guaranteed $50 plus an extra $10 for every extra hour she works. The number of extra hours she works each day can be modelled by the random variable H. The probability distribution of H is shown below.

h	0	1	2	3	4
$P(H=h)$	0.35	p	0.07	q	0.18

a)
Given that the expected amount of money that Harietta earns in a day is $65.20, find the values of p and q.

[4 marks]

Question 9b

b)
Given that $\text{Var}(X) = 228.96,$, find $\text{Var}(H)$.

[2 marks]

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