1.6 Binomial Theorem

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.6 Binomial Theorem
Difficulty	Very Hard

Time allowed: 130

Score: /103

Percentage: /100

Question 1

Given that $(2 + nx)^2(1 - 2x)^n = 4 - 24x + \ldots$

Find the value of n.

[7 marks]

Question 2

Given that $(1 + nx)^2 \left(1 + \frac{2x}{3}\right)^n = 1 + 40x \ldots$

Find the value of n.

[7 marks]

Question 3a

Consider the expansion $(5 + x)^5$.

(a) Write down and simplify the expansion in descending powers of x.

[3 marks]

Question 3b

(b) Hence, find the exact value of $(5.1)^5$.

[3 marks]

Question 4a

Consider the expansion $(2-x)^3$.

(a) Write down and simplify the expansion in descending powers of x.

[3 marks]

Question 4b

(b) Hence find the exact value of $(1.8)^3$.

[3 marks]

Question 5a

Given that $(1-2x)^2(1+yx)^3 = 1+zx+32x^2+\cdots+ky^3x^5$.

(a) Determine the value of k.

[2 marks]

Question 5b

(b) Find the possible values of y and z.

[7 marks]

Question 6a

Given that $(1-2ax)^3(1+3x)^3 = 1+bx-27x^2+\cdots+ka^3x^6$.

(a) Determine the value of k.

[2 marks]

Question 6b

(b) Find the possible values of a and b.

[7 marks]

Question 7

In the expansion of $2x^2(3 + kx)^7$, the coefficient of the term in x^5 is 210.

Find the value of k.

[6 marks]

Question 8

Consider the expansion of $\left(\frac{x^3}{a} + 3x^5\right)^9$, $a > 0$. The coefficient of the x^{39} term is five times the coefficient of the x^{31} term.

Find a, giving your answer to 3 significant figures.

[7 marks]

Question 9

Consider the expansion of $\left(2x^3 - \frac{k}{x^2}\right)^{12}$, where $k > 0$. The coefficient of the term in x^6 is equal to the coefficient of the term in x^{16}.

Find k.

[6 marks]

Question 10

The coefficient of the x^5 term in the expansion of $(1 + 2x)^4(1 - px)^3$ is -120.

Find the value of p.

[8 marks]

Question 11

Consider the binomial expansion of $\frac{1+x^2}{1-x^2}$.

(a) Find the first four terms, in ascending powers of x, of the expansion.

[8 marks]

Question 12

Find the coefficient of the term in x^3 in the expansion of $(-2x^2 + 7x - 3)^{-1}$.

[8 marks]

Question 13a

Consider the identity $\frac{1-7x}{(x+2)(3-x)} = \frac{A}{x+2} + \frac{B}{3-x}$, where A and B are constants to be determined.

(a) Find the values of A and B.

[3 marks]

Question 13b

(b) Hence, or otherwise, find the binomial expansion of $\frac{1-7x}{(x+2)(3-x)}$, in ascending powers of x, up to and including the term in x^2.

[3 marks]

Question 13c

(c) State the interval of convergence for the expansion found in part (b).

[2 marks]

Question 14a

Consider the binomial expansion of $\sqrt{2^n(1-x)^{n+1}}$, where $n \in \mathbb{Z}$.

(a) Given that the coefficient in x^2 is $\frac{3}{16}$, show that

$$2^{\frac{n+2}{2}}(n^2-1) = 3$$

[4 marks]

Question 14b

(b) Given also that the constant term is $\frac{1}{2}$, find

(i) the value of n

(ii) the first three terms of the expansion $\sqrt{2^n(1-x)^{n+1}}$, in ascending powers of x.

[4 marks]