2.4 Other Functions & Graphs

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.4 Other Functions & Graphs
Difficulty	Medium

Time allowed: 120

Score: /96

Percentage: /100

Question 1a

Let $f(x) = \frac{3x-2}{2x+1}$, for $x \neq -\frac{1}{2}$, and $g(x) = -x - 2$, for $x \in \mathbb{R}$.

The graphs of f and g intersect at points A and B.

(a) Find the coordinates of A and B.

[5 marks]

Question 1b

(b) Find the length of the line segment AB.

[3 marks]

Question 2a

Consider the functions $f(x) = -x^5 + 2020$ and $g(x) = \frac{1}{\sqrt{(1-x)^3}} - 2$.

(a) Find the coordinates of the y-intercepts for the graph of

(i) f

(ii) g.

[2 marks]

Question 2b

(b) Find the coordinates of the x-intercepts for the graph of

(i) f

(ii) g.

[3 marks]

Question 2c

(c) For the graph of g, find the equation of

(i) the vertical asymptote

(ii) the horizontal asymptote.

[2 marks]

Question 3a

Consider the function f defined by $f(x) = \frac{x+2}{2x-3}$, for $x \neq \frac{3}{2}$, and the line $x - 7y + 2 = 0$.

The graph of f and the line intersect at points A and B.

(a) Find the coordinates of A and B.

[5 marks]

Question 3b

(b) Find the midpoint of the line segment AB.

[2 marks]

Question 4a

Let $f(x) = \ln(x+2)$, $x > -2$.

(a) Find the coordinates of:

(i) the x −intercept

(ii) the y −intercept.

[2 marks]

Question 4b

(b) State the equation of the vertical asymptote to the graph of f.

[2 marks]

Question 4c

The graph of $y = f(x)$ intersects with its inverse, twice.

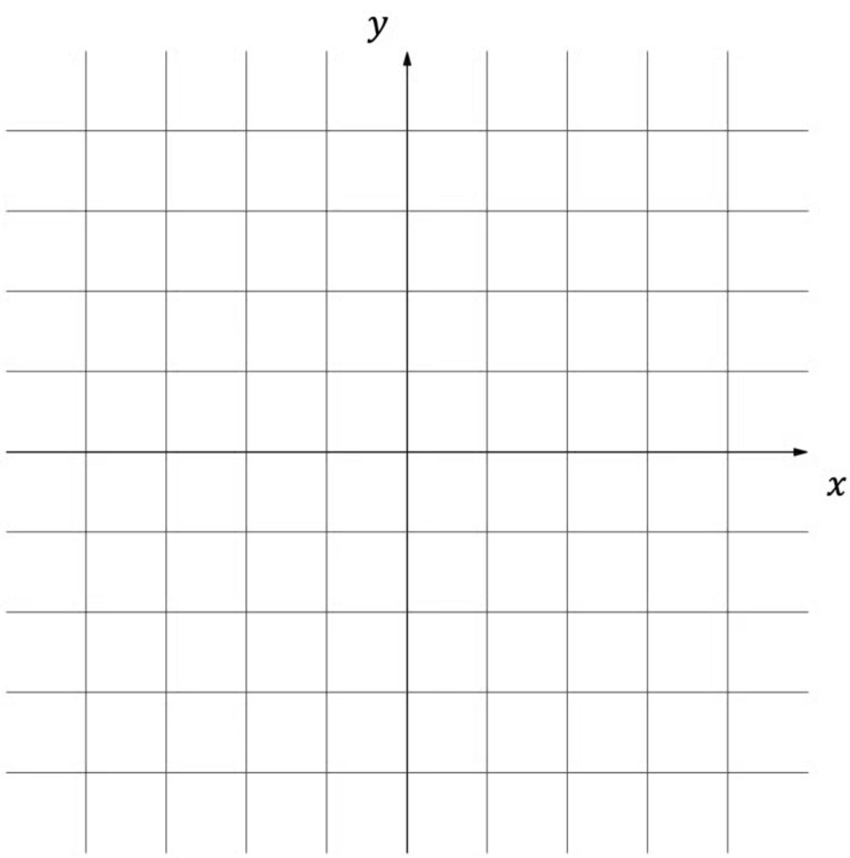
(c) Find the two coordinates where $f(x) = f^{-1}(x)$.

[2 marks]

Question 5a

Let $f(x) = 0.5e^{2x} + 1,\ \text{for} - 1 \leq x \leq 2.$

(a) On the following grid, sketch the graph of $y = f(x)$.



[3 marks]

Question 5b

(b) The inverse of f can be written in the form of $f^{-1}(x) = A \ln b(x - c)$.
Find the values of A, b and of c.

[4 marks]

Question 6a

Carbon-14 is a radioactive isotope of the element carbon.
Carbon-14 decays exponentially – as it decays it loses mass.
Carbon-14 is used in carbon dating to estimate the age of objects.
The time it takes the mass of carbon-14 to halve (called its half-life) is approximately 5700 years.

A model for the mass of carbon-14, m g, in an object of age t years is

$$m = m_0 e^{-kt}$$

where m_o and k are constants.

(a) For an object initially containing 100g of carbon-14, write down the value of m_0.

[1 mark]

Question 6b

(b) Briefly explain why, if $m_0 = 100$, m will equal 50g when $t = 5700$ years.

[2 marks]

Question 6c

(c) Using the values from part (b), show that the value of k is 1.22×10^{-4} to three significant figures.

[2 marks]

Question 6d

(d) A different object currently contains 60g of carbon-14.
In 2000 years' time how much carbon-14 will remain in the object?

[2 marks]

Question 7a

A small company makes a profit of £2500 in its first year of business and £3700 in the second year. The company decides they will use the model

$$P = P_0 y^k$$

to predict future years' profits.
£P is the profit in the y^{th} year of business.
P_0 and k are constants.

(a) Write down two equations connecting P_0 and k.

[2 marks]

Question 7b

(b) Find the values of P_0 and k.

[2 marks]

Question 7c

(c) Find the predicted profit for years 3 and 4.

[2 marks]

Question 7d

(d) Show that

$$P = P_0 y^k$$

can be written in the form

$$\log P = \log P_0 + k \log y.$$

[2 marks]

Question 8a

In an effort to prevent extinction scientists released some rare birds into a newly constructed nature reserve.
The population of birds, within the reserve, is modelled by

$$B = 16e^{0.85t}$$

B is the number of birds after t years of being released into the reserve.

(a) Write down the number of birds the scientists released into the nature reserve.

[1 mark]

Question 8b

(b) According to this model, how many birds will be in the reserve after 3 years?

[2 marks]

Question 8c

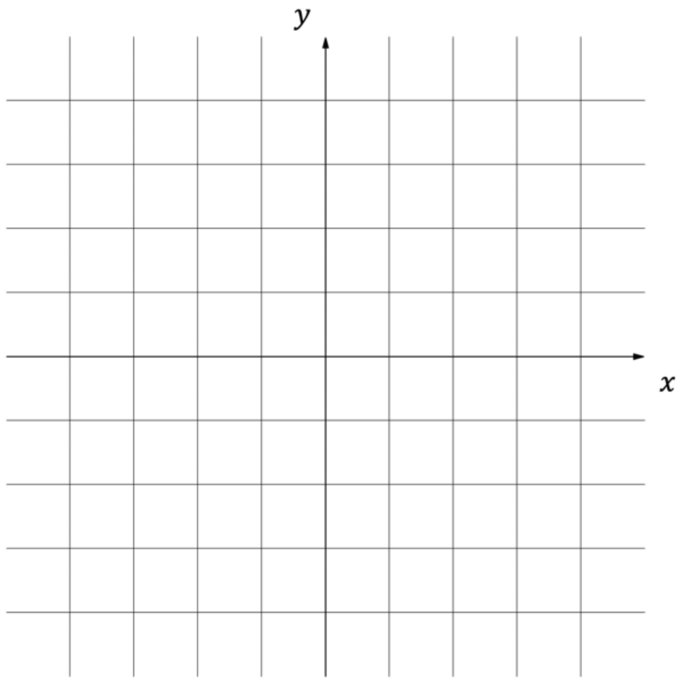
(c) How long will it take for the population of birds within the reserve to reach 500?

[2 marks]

Question 9a

Rebecca recently had the COVID-19 vaccine. The volume, V, of the vaccine in her blood over time can be modelled by an equation of the form $V_1(t) = 1.7te^{-1.25t}$, where V is the concentration (in mg) of the vaccine in the bloodstream and t is time measured in days after 9am on Monday.

(a) On the following grid, sketch the graph of $y = V_1(t)$.



[3 marks]

Question 9b

(b) Find, to the nearest minute, the time when the vaccine volume, V_1 reaches a maximum value.

[2 marks]

Question 9c

(c) Rebecca experienced side-effects from the vaccine between the times when the volume reached its maximum value until it had dropped to half of its maximum value. Find, to the nearest minute, the length of time that Rebecca experienced side-effects from taking the vaccine.

[3 marks]

Question 9d

(d) The vaccine is medically determined to be no longer in Rebecca's bloodstream when it drops down to 1% of its maximum value. Find the time that the vaccine is no longer in Rebecca's bloodstream.

[2 marks]

Question 9e

(e) Rebecca's friend, Zara, also had the vaccine on the same day. The volume in Zara's bloodstream can be modelled by an equation of the form of $V_2(t) = 1.766te^{-1.3t}$. Calculate, to the nearest minute, how much faster V_2 took to reach a maximum volume compared to V_1.

[1 mark]

Question 10a

Let $f(x) = e^x + 1$ and $g(x) = 4x + a$, where $x \in \mathbb{R}$ and a is a constant.

(a) Find $(g \circ f)(x)$.

[2 marks]

Question 10b

(b) Given that $(g \circ f)(0) = 2$, find the value of a.

[2 marks]

Question 10c

(c) Solve the equation $(g \circ f)(x) = 0$.

[3 marks]

Question 11a

Let $f(x) = ab^x$, where $x, a, b \in \mathbb{R}$ and $x \geq 0$, $a, b > 1$.

The graph of f contains the points $(0, 3)$ and $(2, 75)$.

(a) Find the values of a and b.

[3 marks]

Question 11b

(b) Find an expression for $f^{-1}(x)$.

[3 marks]

Question 11c

(c) Find the value of $f^{-1}(375)$.

[2 marks]

Question 12a

Consider $f(x) = \ln\left(\sqrt{x^2 - 16}\right)$.

(a) Find the largest possible domain D_f for f to be a function.

[2 marks]

Question 12b

Let $f(x) = \ln\left(\sqrt{x^2 - 16}\right)$, for $x \in D_f$.

(b) Explain why

(i) f is an even function

(ii) the inverse function f^{-1} does not exist.

[3 marks]

Question 13a

Let $f(x) = \frac{2(x+1)}{x-1}$, for $x \neq 1$, and $g(x) = x + 1$, for $x \in \mathbb{R}$.

The graphs of f and g intersect at points A and B.

(a) Find the coordinates of A and B.

[5 marks]

Question 13b

(b) Find the equation of the straight line that passes through A and B, giving your answer in the form $ax + by + d = 0$.

[3 marks]

Question 13c

(c) Write down the gradient of the line that is perpendicular to the line passing through A and B.

[2 marks]