IB Maths DP

YOUR NOTES

1. Number & Algebra

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1.1 Number Toolkit

1.1.1 Standard Form

Standard Form

Standard form (sometimes called **scientific notation** or **standard index form**) gives us a way of writing very big and very small numbers using powers of 10.

Why use standard form?

- Some numbers are too big or too small to write easily or for your calculator to display at all
  - Imagine the number 50^{50}, the answer would take 84 digits to write out
  - Try typing 50^{50} into your calculator, you will see it displayed in **standard form**
- Writing very big or very small numbers in standard form allows us to:
  - Write them more neatly
  - Compare them more easily
  - Carry out calculations more easily
- Exam questions could ask for your answer to be written in standard form

How is standard form written?

- In standard form numbers are always written in the form $a \times 10^k$ where a and k satisfy the following conditions:
  - $1 \le a < 10$
    - So there is one non – zero digit before the decimal point
  - $k \in \mathbb{Z}$
    - So k must be an integer
  - $k > 0$ for large numbers
    - How many times a is multiplied by 10
  - $k < 0$ for small numbers
    - How many times a is divided by 10

How are calculations carried out with standard form?

- Your GDC will display large and small numbers in standard form when it is in normal mode
  - Your GDC may display standard form as *aEn*
    - For example, 2.1×10^{-5} will be displayed as $2.1\text{E}-5$
    - If so, be careful to **rewrite the answer given in the correct form**, you will not get marks for copying directly from your GDC
- Your GDC will be able to carry out calculations in standard form
  - If you put your GDC into scientific mode it will automatically convert numbers into standard form
    - Beware that your GDC may have more than one mode when in scientific mode
    - This relates to the number of significant figures the answer will be displayed in
    - Your GDC may add extra zeros to fill spaces if working with a high number of significant figures, you do not need to write these in your answer

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- To add or subtract numbers written in the form $a \times 10^k$ without your GDC you will need to write them in full form first
- To multiply or divide numbers written in the form $a \times 10^k$ without your GDC you can either write them in full form first or use the laws of indices

Exam Tip

- Your GDC will give very big or very small answers in standard form and will have a setting which will allow you to carry out calculations in scientific notation
- Make sure you are familiar with the form that your GDC gives answers in as it may be different to the form you are required to use in the exam

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Worked Example

Calculate the following, giving your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

i)
3780×200

Using GDC: Choose scientific mode.

Input directly into GDC as ordinary numbers.

$3780 \times 200 = 7.56 \times 10^5$

GDC will automatically give answer in standard form.

Without GDC:

Calculate the value:

$3780 \times 200 = 756000$

Convert to standard form:

$756000 = 7.56 \times 10^5$

$\boxed{7.56 \times 10^5}$

ii) $(7 \times 10^5) - (5 \times 10^4)$

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Using GDC: Choose scientific mode.

Input directly into GDC

$7 \times 10^5 - 5 \times 10^4 = 6.5 \times 10^5$

This may be displayed as 6.5E5

Without GDC:

Convert to ordinary numbers:

$7 \times 10^5 = 700000$

$5 \times 10^4 = 50000$

Carry out the calculation:

$700000 - 50000 = 650000$

Convert to standard form:

$650000 = 6.5 \times 10^5$

$\boxed{6.5 \times 10^5}$

iii)

$(3.6 \times 10^{-3})(1.1 \times 10^{-5})$

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Input directly into GDC:
(Choose scientific mode).

$(3.6 \times 10^{-3})(1.1 \times 10^{-5}) = 3.96 \times 10^{-8}$

$$\boxed{3.96 \times 10^{-8}}$$

Note:

$$\overbrace{(\underbrace{3.6 \times 10^{-3})(1.1}_{3.6 \times 1.1 = 3.96} \times 10^{-5})}^{10^{-3} \times 10^{-5} = 10^{-8}} = 3.96 \times 10^{-8}$$

1.1.2 Exponents & Logarithms

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Laws of Indices

What are the laws of indices?

- Laws of indices (or index laws) allow you to simplify and manipulate expressions involving exponents
  - An exponent is a power that a number (called the base) is raised to
  - Laws of indices can be used when the numbers are written with the same base
- The index laws you need to know are:
  - $(xy)^m = x^m y^m$
  - $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$
  - $x^m \times x^n = x^{m+n}$
  - $x^m \div x^n = x^{m-n}$
  - $(x^m)^n = x^{mn}$
  - $x^1 = x$
  - $x^0 = 1$
  - $\frac{1}{x^m} = x^{-m}$
- These laws are **not in the formula booklet** so you must remember them

How are laws of indices used?

- You will need to be able to carry out multiple calculations with the laws of indices
  - Take your time and apply each law individually
  - Work with numbers first and then with algebra
- Index laws only work with terms that have the same base, make sure you **change the base** of the term before using any of the index laws
  - Changing the base means rewriting the number as an exponent with the base you need
  - For example, $9^4 = (3^2)^4 = 3^{2\times4} = 3^8$
  - Using the above can them help with problems like $9^4 \div 3^7 = 3^8 \div 3^7 = 3^1 = 3$

Exam Tip

- Index laws are rarely a question on their own in the exam but are often needed to help you solve other problems, especially when working with logarithms or polynomials
- Look out for times when the laws of indices can be applied to help you solve a problem algebraically

Worked Example

Simplify the following equations:

i)
$$\frac{(3x^2)(2x^3y^2)}{(6x^2y)}.$$

Apply each law separately:

$3 \times 2 = 6$

$$\frac{(3x^2)(2x^3y^2)}{6x^2y}$$

expand numerator

$x^2 \times x^3 = x^5$

$$\frac{(6x^2)(x^3y^2)}{6x^2y}$$

cancelling

$$\frac{\cancel{6}x^5y^2}{\cancel{6}x^2y}$$

$x^5 \div x^2 = x^{5-2} = x^3$

$y^2 \div y = y^{2-1} = y$

$$x^3y$$

$$\boxed{\frac{(3x^2)(2x^3y^2)}{6x^2y} = x^3y}$$

ii)
$(4x^2y^{-4})^3(2x^3y^{-1})^{-2}$.

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$$(4x^2y^{-4})^3(2x^3y^{-1})^{-2}$$

Rewrite as a fraction

$$\frac{(4x^2y^{-4})^3}{(2x^3y^{-1})^2}$$

expand numerator and denominator

$$\frac{64x^6y^{-12}}{4x^6y^{-2}}$$

cancelling

$$\frac{\cancel{64}\,\cancel{x^6}\,y^2}{\cancel{4}\,\cancel{x^6}\,y^{12}}$$

The negative exponents can be rewritten as their reciprocals

$$16y^{-10}$$

$$\boxed{\frac{16}{y^{10}}}$$

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Introduction to Logarithms

What are logarithms?

- A logarithm is the inverse of an exponent
  - If $a^x = b$ then $\log_a(b) = x$ where a > 0, b > 0, a ≠ 1
    - This is in the formula booklet
    - Your GDC will be able to use this function to solve equations involving exponents
- Try to get used to 'reading' logarithm statements to yourself
  - $\log_a(b) = x$ would be read as "the power that you raise a to, to get b, is x"
  - So $\log_5 125 = 3$ would be read as "the power that you raise 5 to, to get 125, is 3"
- Two important cases are:
  - $\ln x = \log_e(x)$
    - Where e is the mathematical constant 2.718…
    - This is called the natural logarithm and will have its own button on your GDC
  - $\log x = \log_{10}(x)$
    - Logarithms of base 10 are used often and so abbreviated to log *x*

Why use logarithms?

- Logarithms allow us to solve equations where the exponent is the unknown value
  - We can solve some of these by inspection
    - For example, for the equation $2^x = 8$ we know that *x* must be 3
  - Logarithms allow use to solve more complicated problems
    - For example, the equation $2^x = 10$ does not have a clear answer
    - Instead, we can use our GDCs to find the value of $\log_2 10$

Exam Tip

- Before going into the exam, make sure you are completely familiar with your GDC and know how to use its logarithm functions

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Worked Example

Solve the following equations:

i)
$x = \log_3 27$,

$x = \log_3 27 \iff 3^x = 27$

We can see from inspection:

$3^3 = 27 \iff x = 3$

$\boxed{x = 3}$

OR: use GDC to find answer directly.

ii)
$2^x = 21.4$, giving your answer to 3 s.f.

$2^x = 21.4$ This cannot be seen from inspection:

$2^x = 21.4 \iff x = \log_2 21.4$

Use GDC to find answer directly.

$\log_2 21.4 = 4.4195\ldots$

$\boxed{x = 4.42 \text{ (3 s.f.)}}$

1.1.3 Approximation & Estimation

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Approximating Values

How do I know what to round my answer to?

- Unless otherwise told, always round your answers to **3 significant figures** (3 s.f.)
  - The first non-zero digit is the first **significant** digit
  - The first digit after the third significant digit determines whether to 'round up' (5) or 'leave it alone' (<5)
    - where the 'it' we are rounding up or leaving alone is the third significant figure
  - Your final answer will have three **significant digits** and the rest will be zero
    - Any zero **after** the first significant digit is still significant
    - For large numbers be careful not to change the **place value** of the significant digits, you will have to fill in any zeros after the third significant figure
    - If your GDC is in **scientific mode** it may display unnecessary zeros after the decimal point, you do not need to copy these
- Look out for any questions that ask you to round your answer in a different way
  - Questions often ask for **2 decimal places** (2 d.p.)
    - Your final answer will only have 2 digits after the decimal point
    - For 2 d.p. it is the third digit after the decimal place that determines whether to 'round up' (5) or 'leave it alone' (<5)
- If you are working with a **currency** you must choose the appropriate degree of accuracy
  - For most this will be a **whole number**
    - E.g. yen, yuan, peso
  - For others this will be to **2 decimal places**
    - E.g. dollars, euro, pounds
  - It will be clear from the question which currency you are using and how you should round your answer
    - The question will state the name of the currency and the symbol you should use as a unit
    - E.g. YEN, ¥

Are there cases when I always have to round up?

- Yes - there are cases when it makes sense to always round up (or down)
- These normally involve finding the **minimum** or **maximum number** of objects
  - For example consider the scenario: There are 26 people and 5 people can fit in a single vehicle, how many vehicles are needed?
    - $\frac{26}{5} = 5.2$ and normally we'd round to 5
    - However 5 vehicles wouldn't be enough as there would only be room for 25 people
    - In this case we would round up to find the **minimum** number needed
- These kind of problems can be solved by inequalities
  - For $x > k$ take the **smallest value** of x at the appropriate degree of accuracy that is **greater than k**
    - For example: Using 3sf the smallest solution to $x > 2.5731...$ is $x = 2.58$

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- For $x < k$ take the **biggest value** of x at the appropriate degree of accuracy that is **less than *k***
  - For example: The biggest integer solution to $x < 10.901...$ is $x = 10$

Exam Tip

- In the exam you should always give non exact answers correct to 3 significant figures unless otherwise told
  - This means you must round using a higher degree of accuracy within your working to ensure that your final answer is rounded correctly
  - Where possible always use exact values within your working rather than rounding mid way through a question

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Worked Example

Let $T = \dfrac{b\sin(3a)}{5}$, where $a = 15°$ and $b = 20$.

a)
Calculate the exact value of T.

Substitute a and b into T :

$$T = \frac{20\sin(3 \times 15)}{5}$$

$$\boxed{T = 2\sqrt{2}}$$

b)
Give your answer from part a) correct to two decimal places.

≥ 5 so round up

$$2\sqrt{2} = 2.82|842\ldots$$

2nd digit after decimal point

$$\boxed{T = 2.83 \text{ (2d.p.)}}$$

c)
Give your answer from part a) correct to two significant figures.

first significant figure

< 5 so don't round up

$$2\sqrt{2} = 2.8|2842\ldots$$

2nd significant figure

$$\boxed{T = 2.8 \ (2 \text{ s.f.})}$$

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Upper & Lower Bounds

What are bounds?

- Bounds are the smallest **(lower bound, LB)** and largest **(upper bound, UB)** numbers that a **rounded number** can lie between
  - It simply means how low or high the number could have been before it was rounded
- The bounds for a number, x, can be written as $\mathrm{LB} \leq x < \mathrm{UB}$
  - Note that the lower bound is included in the range of values x could have taken but the upper bound is not

How do we find bounds?

- The basic rule is "half up, half down"
  - To find the upper bound add on half the degree of accuracy
  - To find the lower bound take off half the degree of accuracy
- Remember that the upper bound is the cut off point for the greatest value that the number could have been rounded from but will not actually round to the number itself

How do we calculate using bounds?

- Find bounds before carrying out the calculation and then use the rules:
  - To add or multiply **UB = UB + UB** and **LB = LB x LB** etc
  - To divide **UB = UB / LB** and **LB = LB / UB**
  - To subtract **UB = UB – LB** and **LB = LB – UB**
- Use logic to decide which bound to use within the calculation
  - For example if you are finding the **maximum** volume of a sphere with the radius given correct to 1 decimal place substitute the **upper bound** of the radius into your calculation for the volume

Exam Tip

- When in an exam environment it can be easy to make silly errors in questions like this, read the question carefully to determine which parts bounds need to be found for
  - This will normally be any part in the question that has been rounded

Worked Example

A rectangular field has length, L, of 14.3 m correct to 1 decimal place and width, W, of 9.61 m correct to 2 decimal places.

a)
Calculate the lower and upper bound for L and W.

$L = 14.3$ m (1 d.p.) the degree of accuracy is 1 d.p (0.1)

Find half the degree of accuracy :

$$\frac{0.1}{2} = 0.05$$

The upper bound is

$$14.3 + 0.05 = 14.35$$

The lower bound is

$$14.3 - 0.05 = 14.25$$

$$\boxed{14.25 \leq L < 14.35}$$

$W = 9.61$ m (2 d.p.) the degree of accuracy is 2 d.p (0.01)

Find half the degree of accuracy :

$$\frac{0.01}{2} = 0.005$$

The upper bound is

$$9.61 + 0.005 = 9.615$$

The lower bound is

$$9.61 - 0.005 = 9.605$$

$$\boxed{9.605 \leq W < 9.615}$$

b)
Calculate the lower and upper bound for the perimeter, P, and area, A, of the field.

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For the lower bound use :

$L = 14.25 \quad W = 9.605$

$P = 2(14.25) + 2(9.605)$

$P = 47.71\ m$

$A = (14.25)(9.605)$

$A = 136.87125$

For the upper bound use :

$L = 14.35 \quad W = 9.615$

$P = 2(14.35) + 2(9.615)$

$P = 47.93\ m$

$A = (14.35)(9.615)$

$A = 137.97525$

$\boxed{47.7\ m \leq P < 47.9\ m \quad (3\ s.f.)}$

$\boxed{137\ m^2 \leq A < 138\ m^2 \quad (3\ s.f.)}$

Percentage Error

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What is percentage error?

- Percentage error is how far away from the actual value an estimated or rounded answer is
  - Percentage error can be calculated using the formula

$$\varepsilon = \left|\frac{v_A - v_E}{v_E}\right| \times 100\%$$

    - where v_E is the exact value and v_A is the approximate value of v
    - The | | is the **absolute value** meaning if you get a negative value within these straight brackets, you should take the **positive** value
  - This formula is **in the formula booklet** so you do not need to remember it
- The further away the estimated answer is from the true answer the greater the percentage error
- If the exact value is given as a surd or a multiple of π make sure you enter it into the formula exactly as you see it
- Percentage error should always be a positive number

Exam Tip

- In the exam percentage error will usually be a part of a bigger question on another topic, make sure you know how to find the formula for it in the formula book so that you are prepared to answer these questions

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Worked Example

Let $P = x\cos(2y)$, where $y = 15°$ and $x = 4$.

a)
Calculate the exact value of P.

$x = 4$

$P = x\cos 2y = 4\cos(2 \times 15°)$ $y = 15°$

$= 4\cos(30°)$

$= 2\sqrt{3}$ ← leave answer as exact value

$$\boxed{P = 2\sqrt{3}}$$

b)
Calculate the percentage error if an estimate for P was 3.5.

Percentage error formula:

$$\varepsilon = \left| \frac{V_A - V_E}{V_E} \right| \times 100\%$$

$V_A = 3.5$ (approximated value)
$V_E = 2\sqrt{3}$ (exact value)

Sub V_A and V_E into the formula:

$$\varepsilon = \left| \frac{3.5 - 2\sqrt{3}}{2\sqrt{3}} \right| \times 100\%$$

$= 1.03629...\ \%$

$$\boxed{\varepsilon = 1.04\%\ (3\text{s.f.})}$$

Accuracy & Estimation

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What are exact values?

- Exact values forms that represent the full value of a number
    - For example, π is an exact value and 3.14 is an approximation using 3 significant figures
- If a number has an infinite number of non-zero digits after the decimal point then you can use three dots to signal that the decimal representation goes on for example
    - For example, √2 = 1.414...
- Exact values can involve
    - Fractions: $\frac{2}{7}$
    - Roots: $\sqrt{3}$, $\sqrt[5]{7}$
    - Logarithms: $\ln 2$, $\log_{10} 5$
    - Mathematical constants: π, e
- Your GDC might automatically give your answer as an exact answer
- If your GDC does not do this then you may need to evaluate parts of the expression separately and use algebra
    - For example: If $f(x) = \mathrm{e}^x(2 + \sqrt{x})$ then your GDC will probably not give you the exact value of $f(2)$
    - You would insist evaluate it without a GDC to get the exact value: $f(2) = \mathrm{e}^2(2 + \sqrt{2})$

Why use estimation?

- We **estimate** to find approximate answers to difficult sums
- Or to check our answers are about the right size (order of magnitude)
    - For example, if the question is to find a length the answer cannot be negative
    - or if we are looking for the mean age of some people an answer of 150 must be incorrect
- Estimating an answer before carrying out a calculation will help you know what you are looking for and determine if your answer is likely to be correct or not
- In real life estimation skills are used every day in many activities

How do I choose the correct answer?

- Sometimes a mathematical argument will lead to more than one answer
    - This is common with problems involving quadratics, you will usually have two solutions
    - If you have more than one solution after you have solved a problem, **always** check to see if they are both valid
- Most of the time you can simply use logic to choose the correct answer
    - If the problem involves length or area and one of the answers is negative, the true solution will be the positive answer
- Occasionally you will need to see if an answer can be valid
    - If one of your answers is $\cos x > 1$ for example, x will not give a true solution

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Exam Tip

- Be aware that your GDC will not always give you an answer as an exact value, this means that you will need to find the exact value by hand

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Worked Example

A rectangular floor has an area of 40 m^2 to the nearest square metre. It is going to be tiled using square tiles with side length 39.8 cm.

a)
Use estimation to find the number of tiles needed to cover the whole area.

Each tile is approximately:

$$40 \times 40 \text{ cm} = 1600 \text{ cm}^2$$

Area of the rectangle is approximately:

$$40\text{m}^2 = 400\,000 \text{ cm}^2$$

$$400\,000 \text{ cm}^2 \div 1600 \text{ cm}^2 = \frac{4000\cancel{00}}{16\cancel{00}}$$

$$= \frac{4000}{16}$$

$$= 250 \text{ tiles}$$

$$\boxed{\approx 250 \text{ tiles}}$$

b)
Given that there are 15 more tiles places length-wise than width-wise, find the approximate length and width of the floor.

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Let the number of tiles covering the width of the floor be x, then the number of tiles covering the length will be $x + 15$.

number of tiles placed widthways

number of tiles placed lengthways

$$x(x + 15) = 250$$

$$x^2 + 15x - 250 = 0$$

$$x = 10 \quad \text{or} \quad x = -25$$

not possible as x cannot be a negative number

Width of floor $\approx 10 \times 40$ cm

$= 400$ cm $= 4$ m

Length of floor $\approx 25 \times 40$ cm

$= 1000$ cm $= 10$ m

$$\boxed{\text{Length} \approx 10\text{m},\ \text{Width} \approx 4\text{m}}$$

1.1.4 GDC: Solving Equations

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Systems of Linear Equations

What are systems of linear equations?

- A linear equation is an equation of the first order (degree 1)
  - It is usually written in the form $ax + by + c = 0$ where a, b, and c are constants
- A system of linear equations is where two or more linear equations work together
  - Usually there will be two equations with the variables x and y
  - The **variables** x and y will satisfy all equations
  - They are usually known as **simultaneous equations**
  - Occasionally there may be three equations with the variables x, y and z
- They can be complicated to solve but your GDC has a function allowing you to solve them
  - The question may say 'using technology, solve…'
    - This means you do not need to show a method of solving the system of equations, you can use your GDC

How do I use my GDC to solve a system of linear equations?

- Your GDC will have a function within the algebra menu to solve a system of linear equations
- You will need to choose the number of equations
  - For two equations the variables will be x and y
  - For three equations the variables will be x, y and z
- Enter the equations into your calculator as you see them written
- Your GDC will display the values of x and y (or x, y, and z)

How do I set up a system of linear equations?

- Not all questions will have the equations written out for you
- There will be two bits of information given about two variables
  - Look out for clues such as 'assuming a linear relationship'
- Choose to assign x to one of the given variables and y to the other
  - Or you can choose to use more meaningful variables if you prefer
  - Such as c for cats and d for dogs
- Write your system of equations in the form

$$ax + by = e$$
$$cx + dy = f$$

- Use your GDC to solve the system of equations
- This function on the GDC can also be used to find the points of intersection of two straight line graphs
  - You may wish to use the graphing section on your GDC to see the points of intersection

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Exam Tip

- Be sure to write down what you are putting into your GDC
  - If you have had to set up the system of equations as well make sure you write them down clearly before typing into your GD

Worked Example

A theme park has set ticket prices for adults and children. A group of three adults and nine children costs $153 and a group of five adults and eleven children costs $211.

i)
Set up a system of linear equations for the cost of adult and child tickets.

Set up variables:

Let the cost of an adult ticket be 'a'

Let the cost of a child ticket be 'c'

Set up equations: $3a + 9c = 153$
$5a + 11c = 211$

$$\boxed{\begin{aligned} 3a + 9c &= 153 \\ 5a + 11c &= 211 \end{aligned}}$$

ii)
Find the price of one adult and one child ticket.

Enter into GDC:

Let a be x and c be y, then GDC gives

$x = 18$
$y = 11$

$$\boxed{a = \$18, \quad c = \$11}$$

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Polynomial Equations

What is a polynomial equation?

- A polynomial is an algebraic expression consisting of a finite number of terms, with non-negative integer indices only
  - It is in the form $ax^n + bx^{n-1} + cx^{n-2} + \ldots, n \in \mathbb{N}$
- A **polynomial equation** is an equation where a polynomial is equal to zero
- The number of **solutions (roots or zeros)** depend on the **order** of the polynomial equation
  - A polynomial equation of order two can have up to two solutions
  - A polynomial equation of order five can have up to five solutions
- A polynomial equation of an odd degree will always have at least one solution
- A polynomial equation of an even degree could have no solutions

How do I use my GDC to solve polynomial equations?

- You should use your GDC's graphing mode to look at the shape of the polynomial
  - You will be able to see the number of solutions
  - This will be the number of times the graph cuts through or touches the x-axis
  - When entering a function into the graphing section you may need to adjust your zoom settings to be able to see the full graph on your display
  - Whilst in this mode you can then choose to **analyse** the graph
  - This will give you the option to see the solutions of the polynomial equation
    - This may be written as the **zeros** (points where the graph meets the x-axis)
- Your GDC will also have a function within the algebra menu to solve polynomial equations
  - You will need to enter the **order (highest degree)** of the polynomial
  - This is the highest power (or exponent) in the equation
  - Enter the equation into your calculator
  - Your GDC will display the solutions (roots) of the equation
    - Be aware that your GDC may either show all solutions or only the first solution, it is always worth plotting a graph of the function to check how many solutions there should be

Exam Tip

- Be sure to write down what you are putting into your GDC
- If you are using a graphical method it is often a good idea to sketch the graph that your GDC display shows
  - Don't spend too much time on this, a very quick sketch is fine

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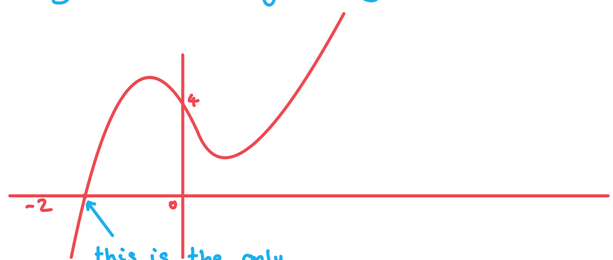
Worked Example

For the polynomial equation $2x^3 - 2x^2 - 3x + 4 = 0$:

i)
Use your GDC's graphing function to sketch the graph of $y = 2x^3 - 2x^2 - 3x + 4$ and determine the number of solutions to the polynomial equation.

Enter the equation $y = 2x^3 - 2x^2 - 3x + 4$ into your GDC's graphing software :



this is the only place the graph cuts the x-axis.

The polynomial equation $2x^3 - 2x^2 - 3x + 4 = 0$ has 1 solution

ii)
Use your GDC to find the solution(s) of the polynomial equation.

Use your GDC's graph analysis tool to find the 'zeros'.

$x = -1.3101...$

$\boxed{x = -1.31 \ (3sf)}$

Alternative method:
Enter the equation $2x^3 - 2x^2 - 3x + 4 = 0$ into your GDC's equation solving mode.

YOUR NOTES
⬇

1.2 Sequences & Series

1.2.1 Language of Sequences & Series

Language of Sequences & Series

What is a sequence?

- A **sequence** is an ordered set of numbers with a rule for finding all of the numbers in the sequence
  - For example 1, 3, 5, 7, 9, … is a sequence with the rule *'start at one and add two to each number'*
- The numbers in a sequence are often called **terms**
- The terms of a sequence are often referred to by letters with a subscript
  - In IB this will be the letter u
  - So in the sequence above, $u_1 = 1$, $u_2 = 3$, $u_3 = 5$ and so on
- Each term in a sequence can be found by **substituting** the term number into **formula for the nth term**

What is a series?

- You get a **series** by summing up the terms in a sequence
  - E.g. For the sequence 1, 3, 5, 7, … the associated series is 1 + 3 + 5 + 7 + …
- We use the notation S_n to refer to the sum of the first n terms in the series
  - $S_n = u_1 + u_2 + u_3 + \ldots + u_n$
  - So for the series above $S_5 = 1 + 3 + 5 + 7 + 9 = 25$

What are increasing, decreasing and periodic sequences?

- A sequence is **increasing** if $u_{n+1} > u_n$ for all positive integers
  - i.e. every term is greater than the term before it
- A sequence is **decreasing** if $u_{n+1} < u_n$ for all positive integers
  - i.e. every term is less than the term before it
- A sequence is **periodic** if the terms repeat in a cycle

YOUR NOTES

Worked Example

Determine the first five terms and the value of S_5 in the sequence with terms defined by $u_n = 5 - 2n$.

$u_n = 5 - 2n$

find the term you want by replacing n with it's value.

term number

first term

$u_1 = 5 - 2(1) = 3$

$u_2 = 5 - 2(2) = 1$

$u_3 = 5 - 2(3) = -1$

$u_4 = 5 - 2(4) = -3$

$u_5 = 5 - 2(5) = -5$

recognise the pattern.

-2

-2 rule is subtract 2

'start with 3 and subtract 2 from each number'.

$S_5 = 3 + 1 + (-1) + (-3) + (-5) = -5$

the sum of the first 5 terms

$3, 1, -1, -3, -5$

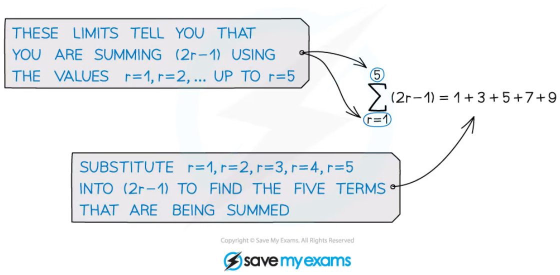
$S_5 = -5$

Sigma Notation

YOUR NOTES

What is sigma notation?

- Sigma notation is used to show the sum of a certain number of terms in a sequence
- The symbol Σ is the capital Greek letter sigma
- Σ stands for 'sum'
  - The expression to the right of the Σ tells you what is being summed, and the limits above and below tell you which terms you are summing



- Be careful, the limits don't have to start with 1
  - For example $\sum_{k=0}^{4}(2k+1)$ or $\sum_{k=7}^{14}(2k-13)$
  - r and k are commonly used variables within sigma notation

Exam Tip

- Your GDC will be able to use sigma notation, familiarise yourself with it and practice using it to check your work

YOUR NOTES

Worked Example

A sequence can be defined by $u_n = 2 \times 3^{n-1}$ for $n \in \mathbb{Z}^+$.

a)
Write an expression for $u_1 + u_2 + u_3 + \ldots + u_6$ using sigma notation.

$u_n = 2 \times 3^{n-1}$, $n \in \mathbb{Z}^+$ ← n is the set of all positive integers

Using sigma notation

$$u_1 + u_2 + \ldots + u_6 = \sum_{k=1}^{6} u_k$$

$$\boxed{\sum_{k=1}^{6} (2 \times 3^{k-1})}$$

b)
Write an expression for $u_7 + u_8 + u_9 + \ldots + u_{12}$ using sigma notation.

$u_n = 2 \times 3^{n-1}$, $n \in \mathbb{Z}^+$ ← n is the set of all positive integers

Using sigma notation

$$u_7 + u_8 + \ldots + u_{12} = \sum_{k=7}^{12} u_k$$

$$\boxed{\sum_{k=7}^{12} (2 \times 3^{k-1})}$$

1.2.2 Arithmetic Sequences & Series

YOUR NOTES
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Arithmetic Sequences

What is an arithmetic sequence?

- In an **arithmetic sequence,** the difference between consecutive terms in the sequence is constant
- This **constant difference** is known as the **common difference, *d*,** of the sequence
  - For example, 1, 4, 7, 10, … is an arithmetic sequence with the rule '*start at one and add three to each number*'
    - The **first term, u_1,** is 1
    - The **common difference, *d*,** is 3
  - An arithmetic sequence can be **increasing** (positive common difference) or **decreasing** (negative common difference)
  - Each term of an arithmetic sequence is referred to by the letter *u* with a subscript determining its place in the sequence

How do I find a term in an arithmetic sequence?

- The n^{th} term formula for an arithmetic sequence is given as

$$u_n = u_1 + (n-1)d$$

  - Where u_1 is the first term, and d is the common difference
  - This is given in the formula booklet, you do not need to know how to derive it
- Enter the information you have into the formula and use your GDC to find the value of the term
- Sometimes you will be given a term and asked to find the first term or the common difference
  - Substitute the information into the formula and solve the equation
    - You could use your GDC for this
- Sometimes you will be given two terms and asked to find both the first term and the common difference
  - Substitute the information into the formula and set up a **system of linear equations**
  - Solve the simultaneous equations
    - You could use your GDC for this

Exam Tip

- Simultaneous equations are often needed within arithmetic sequence questions, make sure you are confident solving them with your GDC

YOUR NOTES
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Worked Example

The fourth term of an arithmetic sequence is 10 and the ninth term is 25, find the first term and the common difference of the sequence.

$u_4 = 10$, $u_9 = 25$

Formula for n^{th} term of an arithmetic series:

$u_n = u_1 + (n-1)d$

Sub in $u_4 = 10$ and $u_9 = 25$

$u_4 = u_1 + (4-1)d = u_1 + 3d = 10$

$u_9 = u_1 + (9-1)d = u_1 + 8d = 25$

Solve using GDC:

let $u_1 = x$ and $d = y$

$x + 3y = 10$

$x + 8y = 25$

$x = 1$, $y = 3$

$$\boxed{\begin{aligned} u_1 &= 1 \\ d &= 3 \end{aligned}}$$

Arithmetic Series

YOUR NOTES
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How do I find the sum of an arithmetic series?

- An **arithmetic series** is the sum of the terms in an **arithmetic sequence**
  - For the arithmetic sequence 1, 4, 7, 10, ... the arithmetic series is 1 + 4 + 7 + 10 + ...
- Use the following formulae to find the sum of the first n terms of the arithmetic series:

$$S_n = \frac{n}{2}(2u_1 + (n-1)d) \quad ; \quad S_n = \frac{n}{2}(u_1 + u_n)$$

    - u_1 is the first term
    - d is the common difference
    - u_n is the last term
  - Both formulae are given in the formula booklet, you do not need to know how to derive them
- You can use whichever formula is more convenient for a given question
  - If you know the first term and common difference use the first version
  - If you know the first and last term then the second version is easier to use
- A question will often give you the sum of a certain number of terms and ask you to find the value of the first term or the common difference
  - Substitute the information into the formula and solve the equation
    - You could use your GDC for this

Exam Tip

- The formulae you need for arithmetic series are in the formula book, you do not need to remember them
  - Practice finding the formulae so that you can quickly locate them in the exam

YOUR NOTES

Worked Example

The sum of the first 10 terms of an arithmetic sequence is 630.

a)
Find the common difference, *d*, of the sequence if the first term is 18.

$S_{10} = 630$

Formula for the sum of an arithmetic series:

$$S_n = \frac{n}{2}(2u_1 + (n-1)d)$$

Sub in $S_{10} = 630$, $u_1 = 18$

$$S_{10} = \frac{10}{2}(2(18) + (10-1)d) = 630$$

$$5(36 + 9d) = 630$$

Solve: $36 + 9d = 126$

$9d = 90$

$d = 10$

$$\boxed{d = 10}$$

b)
Find the first term of the sequence if the common difference, *d*, is 11.

Sub in $S_{10} = 630$, $d = 11$

$$S_{10} = \frac{10}{2}(2u_1 + (10-1)(11)) = 630$$

$$5(2u_1 + 99) = 630$$

Solve: $2u_1 + 99 = 126$

$2u_1 = 27$

$$\boxed{u_1 = 13.5}$$

YOUR NOTES
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1.2.3 Geometric Sequences & Series

YOUR NOTES

Geometric Sequences

What is a geometric sequence?

- In a **geometric sequence**, there is a **common ratio, *r*,** between consecutive terms in the sequence
    - For example, 2, 6, 18, 54, 162, ... is a sequence with the rule '*start at two and multiply each number by three*'
        - The **first term, u_1,** is 2
        - The **common ratio, *r*,** is 3
- A geometric sequence can be **increasing** ($r > 1$) or **decreasing** ($0 < r < 1$)
- If the common ratio is a **negative number** the terms will alternate between positive and negative values
    - For example, 1, -4, 16, -64, 256, ... is a sequence with the rule '*start at one and multiply each number by negative four*'
        - The **first term, u_1,** is 1
        - The **common ratio, *r*,** is -4
- Each term of a geometric sequence is referred to by the letter *u* with a subscript determining its place in the sequence

How do I find a term in a geometric sequence?

- The n^{th} term formula for a geometric sequence is given as

$$u_n = u_1 r^{n-1}$$

    - Where u_1 is the first term, and r is the common ratio
    - This formula allows you to find **any term** in the geometric sequence
    - It is given in the formula booklet, you do not need to know how to derive it
- Enter the information you have into the formula and use your GDC to find the value of the term
- Sometimes you will be given a term and asked to find the first term or the common ratio
    - Substitute the information into the formula and solve the equation
        - You could use your GDC for this
- Sometimes you will be given two or more consecutive terms and asked to find both the first term and the common ratio
    - Find the common ratio by dividing a term by the one before it
    - Substitute this and one of the terms into the formula to find the first term
- Sometimes you may be given a term and the formula for the n^{th} term and asked to find the value of *n*
    - You can solve these using **logarithms** on your GDC

Exam Tip

- You will sometimes need to use logarithms to answer geometric sequences questions
  - Make sure you are confident doing this
  - Practice using your GDC for different types of questions

YOUR NOTES

YOUR NOTES

Worked Example

The sixth term, u_6, of a geometric sequence is 486 and the seventh term, u_7, is 1458.

Find,

i)
the common ratio, r, of the sequence,

$u_6 = 486$, $u_7 = 1458$

The common ratio, r, is given by

$$r = \frac{u_2}{u_1} = \frac{u_3}{u_2} = \ldots = \frac{u_{n+1}}{u_n}$$

Sub in $u_6 = 486$, $u_7 = 1458$

$$r = \frac{u_7}{u_6} = \frac{1458}{486} = 3$$

$$\boxed{r = 3}$$

ii)
the first term of the sequence, u_1.

Formula for n^{th} term of a geometric series:

$$u_n = u_1 r^{n-1}$$

Sub in $r = 3$ and either $u_6 = 486$ or $u_7 = 1458$

$$u_6 = u_1(3)^{6-1} = 486$$

Solve: $243 u_1 = 486$

$u_1 = 2$

$$\boxed{u_1 = 2}$$

YOUR NOTES
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Geometric Series

How do I find the sum of a geometric series?

- A **geometric series** is the sum of a certain number of terms in a **geometric sequence**
  - For the geometric sequence 2, 6, 18, 54, ... the geometric series is 2 + 6 + 18 + 54 + ...
- The following formulae will let you find the sum of the first n terms of a geometric series:

$$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}$$

    - u_1 is the first term
    - r is the common ratio
  - Both formulae are given in the formula booklet, you do not need to know how to derive them
- You can use whichever formula is more convenient for a given question
  - The first version of the formula is more convenient if $r > 1$ and the second is more convenient if $r < 1$
- A question will often give you the sum of a certain number of terms and ask you to find the value of the first term, the common ratio, or the number of terms within the sequence
  - Substitute the information into the formula and solve the equation
    - You could use your GDC for this

Exam Tip

- The geometric series formulae are in the formula booklet, you don't need to memorise them
  - Make sure you can locate them quickly in the formula booklet

YOUR NOTES

Worked Example

A geometric sequence has $u_1 = 25$ and $r = 0.8$. Find the value of u_5 and S_5.

$u_1 = 25, \quad r = 0.8$

Formula for n^{th} term of a geometric series:

$$u_n = u_1 r^{n-1}$$

Sub in $u_1 = 25, \quad r = 0.8$

$$u_5 = 25(0.8)^4 = 10.24$$

Formula for the sum of a geometric series:

$$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}$$

$r < 1$ so this version is easier to use.

Sub in $u_1 = 25, \quad r = 0.8$

$$S_5 = \frac{u_1(1 - r^5)}{1 - r} = \frac{25(1 - 0.8^5)}{1 - 0.8} = 84.04$$

$$\boxed{\begin{aligned} u_5 &= 10.24 \\ S_5 &= 84.04 \end{aligned}}$$

1.2.4 Applications of Sequences & Series

YOUR NOTES

Applications of Arithmetic Sequences & Series

Many real-life situations can be modelled using sequences and series, including but not limited to: patterns made when tiling floors; seating people around a table; the rate of change of a population; the spread of a virus and many more.

What do I need to know about applications of arithmetic sequences and series?

- If a quantity is changing repeatedly by having a fixed amount **added to** or **subtracted from** it then the use of **arithmetic sequences** and **arithmetic series** is appropriate to **model** the situation
  - If a sequence seems to fit the pattern of an arithmetic sequence it can be said to be **modelled** by an arithmetic sequence
  - The scenario can be **modelled** using the given information and the formulae from the formula booklet
- A common application of arithmetic sequences and series is **simple interest**
  - Simple interest is when an initial investment is made and then a percentage of the initial investment is added to this amount on a regular basis (usually per year)
- Arithmetic sequences can be used to make estimations about how something will change in the future

Exam Tip

- Exam questions won't always tell you to use sequences and series methods, practice spotting them by looking for clues in the question
- If a given amount is repeated periodically then it is likely the question is on arithmetic sequences or series

YOUR NOTES

Worked Example

Jasper is saving for a new car. He puts USD $100 into his savings account and then each month he puts in USD $10 more than the month before. Jasper needs USD $1200 for the car. Assuming no interest is added, find,

i)
the amount Jasper has saved after four months,

Identify the arithmetic sequence :

$u_1 = 100 \quad , \quad d = 10$

After 4 months Jasper will have saved :

$u_1 + u_2 + u_3 + u_4 = S_4$

Formula for the sum of an arithmetic series :

$$S_n = \frac{n}{2}(2u_1 + (n-1)d)$$

$$S_4 = \frac{4}{2}(2u_1 + (4-1)d)$$

Sub in $u_1 = 100$ and $d = 10$

$$S_4 = \frac{4}{2}(2(100) + (4-1)(10))$$

$$= 2(200 + 30)$$

$$= 2(230)$$

$$\boxed{S_4 = \$460}$$

ii)
the month in which Jasper reaches his goal of USD $1200.

YOUR NOTES
⬇

Sub $S_n = 1200$, $u_1 = 100$, $d = 10$ into formula:

$$1200 = \frac{n}{2}(2(100) + (n-1)(10))$$

Solve using algebraic solver on GDC:

$n = 8.67\ldots$ or $n = -27.67\ldots$

disregard as n cannot be negative.

$\therefore$ $S_8 < 1200$

$S_9 > 1200$ reaches total in 9^{th} month

Jasper will reach USD \$1200 in the 9^{th} month.

YOUR NOTES
⬇

Applications of Geometric Sequences & Series

What do I need to know about applications of geometric sequences and series?

- If a quantity is changing repeatedly by a fixed **percentage,** or by being **multiplied** repeatedly by a fixed amount, then the use of **geometric sequences** and **geometric series** is appropriate to **model** the situation
  - If a sequence seems to fit the pattern of a geometric sequence it can be said to be **modelled** by a geometric sequence
  - The scenario can be **modelled** using the given information and the formulae from the formula booklet
- A common application of geometric sequences and series is **compound interest**
  - Compound interest is when an initial investment is made and then interest is paid on the initial amount **and on the interest already earned** on a regular basis (usually every year)
- Geometric sequences can be used to make estimations about how something will change in the future
- The questions won't always tell you to use sequences and series methods, so be prepared to spot 'hidden' sequences and series questions
  - Look out for questions on savings accounts, salaries, sales commissions, profits, population growth and decay, spread of bacteria etc

Exam Tip

- Exam questions won't always tell you to use sequences and series methods, practice spotting them by looking for clues in the question
- If a given amount is changing by a percentage or multiple then it is likely the question is on geometric sequences or series

YOUR NOTES

Worked Example

A new virus is circulating on a remote island. On day one there were 10 people infected, with the number of new infections increasing at a rate of 40% per day.

a)
Find the expected number of people newly infected on the 7th day.

Identify the geometric sequence:

$u_1 = 10$, $r = 1.4$

40% increase so 140% of the day before

New infections : u_7

Formula for n^{th} term of a geometric series :

$$u_n = u_1 r^{n-1}$$

Sub in $u_1 = 10$, $r = 1.4$

$$u_7 = 10(1.4)^6 = 75.29...$$

$\boxed{\text{Expected number of new infections} = 75}$

b)
Find the expected number of infected people after one week (7 days), assuming no one has recovered yet.

Total infections : S_7

Formula for the sum of a geometric series :

$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$

$r > 1$ so this version is easier to use.

Sub in $u_1 = 10$, $r = 1.4$

$$S_7 = \frac{10(1.4^7 - 1)}{1.4 - 1} = 238.53...$$

$\boxed{\text{Expected number of total infections} = 239}$

YOUR NOTES
⬇

1.3 Financial Applications

1.3.1 Compound Interest & Depreciation

Compound Interest

What is compound interest?

- Interest is a small percentage paid by a bank or company that is added on to an initial investment
  - Interest can also refer to an amount paid on a loan or debt, however IB compound interest questions will always refer to interest on **investments**
- **Compound interest** is where interest is paid on **both the initial investment** and any interest that has **already been paid**
  - Make sure you know the difference between compound interest and simple interest
    - Simple interest pays interest only on the initial investment
- The interest paid each time will increase as it is a percentage of a higher number
- Compound interest will be paid in instalments in a given timeframe
  - The interest rate, *r*, will be per annum (per year)
    - This could be written *r* % p.a.
  - Look out for phrases such as **compounding annually** (interest paid yearly) **or compounding monthly** (interest paid monthly)
    - If $\alpha\%$ p.a. (per annum) is paid compounding monthly, then $\frac{\alpha}{12}\%$ will be paid each month
    - The formula for compound interest allows for this so you do not have to compensate separately

How is compound interest calculated?

- The formula for calculating compound interest is:

$$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$$

  - Where
    - *FV* is the future value
    - *PV* is the present value
    - *n* is the number of years
    - *k* is the number of compounding periods per year
    - *r* % is the nominal annual rate of interest
  - This formula is **given in the formula booklet**, you do not have to remember it
- Be careful with the *k* value
  - Compounding annually means *k* = 1
  - Compounding half-yearly means *k* = 2
  - Compounding quarterly means *k* = 4
  - Compounding monthly means *k* = 12
- Your GDC will have a finance solver app on it which you can use to find the future value
  - This may also be called the TVM (time value of money) solver

- You will have to enter the information from the question into your calculator
- Be aware that many questions will be set up such that you will have to use the formula
  - So for compound interest questions it is better to use the formula from your formula booklet than your GDC

YOUR NOTES

Exam Tip

- Your GDC will be able to solve some compound interest problems so it is a good idea to make sure you are confident using it, however you must also familiarise yourself with the formula and make sure you can find it in the formula booklet

Worked Example

Kim invests MYR 2000 (Malaysian Ringgit) in an account that pays a nominal annual interest rate of 2.5% **compounded monthly**. Calculate the amount that Kim will have in her account after 5 years.

Compound interest formula:

$$FV = PV\left(1 + \frac{\overset{\text{interest rate}}{r}}{100\underset{\text{compounding periods}}{k}}\right)^{k\overset{\text{number of years}}{n}}$$

future value, present value

Substitute values in:

$PV = 2000$ (initial investment)

$k = 12$ (compounding monthly)

$r = 2.5\%$

$n = 5$ (number of years)

$$FV = 2000\left(1 + \frac{2.5}{(100)(12)}\right)^{(12\times 5)}$$

$$= 2266.002\ldots$$

$$\boxed{FV \approx \text{MYR } 2270 \text{ (3sf)}}$$

Depreciation

YOUR NOTES
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What is depreciation?

- Depreciation is when something looses value over time
  - The most common examples of depreciation are the value of cars and technology or the temperature of a cooling cup of coffee

How is compound depreciation calculated?

- The formula for calculating compound depreciation is:

$$FV = PV \times \left(1 - \frac{r}{100}\right)^n$$

  - Where
    - FV is the future value
    - PV is the present value
    - n is the number of years
    - r% is the rate of depreciation
  - This formula is **not** given in the formula booklet, however it is almost the same as the formula for compound interest but
    - with a **subtraction** instead of an addition
    - the value of k will always be 1
- Your GDC **could** again be used to solve some compound depreciation questions, but watch out for those which are set up such that you will have to use the formula

Exam Tip

- Although the formula is not the same as the one given in the formula booklet for compound interest, it is very similar
  - Practice finding this formula and recognising the differences

YOUR NOTES

Worked Example

Kyle buys a new car for AUD $14 999. The value of the car depreciates by 15% each year.

a)
Find the value of the car after 5 years.

Depreciation formula :

rate of depreciation

$$FV = PV\left(1 - \frac{r}{100}\right)^n \leftarrow \text{number of years}$$

future value — present value

Substitute values in :

$PV = 14999$ (initial cost)

$r = 15\%$

$n = 5$ (number of years)

$$FV = 14999\left(1 - \frac{15}{100}\right)^5$$

$$= 6655.13\ldots$$

$$\boxed{FV \approx \text{AUD } \$6660 \text{ (3sf)}}$$

b)
Find the number of years and months it will take for the value of the car to be approximately AUD $9999.

$$FV = PV\left(1 - \frac{r}{100}\right)^n$$

$FV \approx 9999$

$PV = 14999$

$r = 15\%$

Substitute values in:

$$9999 \approx 14999\left(1 - \frac{15}{100}\right)^n$$

Use GDC to solve:

$n = 2.495\ldots$

2 years 0.495^{th} of a year

Convert to years and months:

2 years $+ \; 0.495\ldots \times 12$ months

$\boxed{\approx 2 \text{ years and } 6 \text{ months}}$

YOUR NOTES

YOUR NOTES
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Amortisation

What is amortisation?

- Amortisation is the process of repaying a loan over a fixed period of time
  - Most commonly questions will be about mortgages (loans taken out to buy a home) or loans taken out for a large purchase
- Interest will be paid on the original amount
  - Each repayment that is made will partly repay the original loan and partly pay the interest on the loan
  - As payments are made the amount owed will decrease and so the interest paid will decrease
    - As you continue to repay a loan more of the repayment goes on the loan and less on the interest

How can the GDC be used to make calculations involving loans?

- Your GDC should be used to solve questions involving loans
  - Use the **finance solver mode** (sometimes called the TVM (time value of money) solver)
    - *N* will be the number of **repayment periods** (remember to include months and years if necessary)
    - *I (%)* is the interest rate
    - *PV* is the amount that was borrowed at the start – as this has been received it will be entered as a **positive** number
    - *PMT* is the payments made per period – this is repaying the loan so will be a **negative** number
    - *FV* is the future value (this will be zero as the loan will be paid off at the end of the period)
    - *P/Y* is the number of payments per year, usually 12 as payments are made monthly
    - *C/Y* is the **compounding periods** per year
    - *PMT@* is the time of the year or month the payment is made (assume this is the end unless told otherwise)
  - Leave the section that you need to find out blank and fill in all other sections
  - Your GDC will fill in the last part for you
- It is sensible to check your final answer, you can do this by finding the total amount paid back overall and comparing it to the original loan
  - The total amount repaid will be **a little more** than the original loan plus *I* % of the original loan

Exam Tip

- Be sure to write down the values that you put into the financial solver on your GDC, don't just write down the final answer as if it is incorrect you won't get any marks if there is no working shown!
- Make sure that you are clear on what the signage of any monetary value is, if it's positive then money is coming in to you, if it's negative then you are paying money out

YOUR NOTES

Worked Example

Olivia takes a mortgage of EUR €280 000 to purchase a house at a nominal annual interest rate of 3.2%, **compounded monthly**. She agrees to pay the bank EUR €1500 at the end of every month to amortise the loan. Find

i)

the number of years and months it will take Olivia to pay back the loan,

Use the finance / TVM solver on your GDC:

N	I%	PV	PMT	FV	P/Y	C/Y	PMT@
	3.2	280 000	-1500	0	12	12	END

GDC will fill this in for you. negative because paying this back each month. paid monthly. compounding monthly. paid at the end of each month.

N = 258.61

Convert to years:

Number of years $= \frac{258.61}{12} = 21.55$

$\boxed{\text{21 years and 7 months}}$

ii)

the total amount Olivia will pay to purchase the house.

Total amount paid = N × PMT

Total amount paid = 258.61 × 1500

$\boxed{\text{Total amount paid} = €387\,915}$

YOUR NOTES
⬇

Annuities

What is an annuity?

- An annuity is a fixed sum of money paid to someone at specified intervals over a fixed period of time
  - Most commonly this will be because of an initial lump sum investment which will be returned at fixed intervals of time with a fixed interest rate
  - Either from personal savings or from receiving an inheritance

How are annuities calculated?

- Your GDC should be used to solve questions involving annuities
  - Use the **finance solver mode** (sometimes called the TVM (time value of money) solver)
    - *N* will be the number of **payment periods** (remember to include months and years if necessary)
    - *I (%)* is the interest rate
    - *PV* is the amount that was invested – as this has been invested it will be entered as a **negative number**
    - *PMT* is the amount paid per period – as this is being received it will be a **positive number**
    - *FV* is the future value (for an annuity this will be **zero** as the balance at the end of the payment period will be zero)
    - *P/Y* is the number of payments per year
    - *C/Y* is the **compounding periods** per year
    - *PMT@* is the time of the year or month the payment is made (usually the start)
  - Leave the section that you need to find out blank and fill in all other sections
  - Your GDC will fill in the last part for you
- Although you are unlikely to need to use it, the formula for calculating an annuity is:

$$FV = A\frac{(1 + r^n) - 1}{r}$$

  - Where
    - *FV* is the future value
    - *A* is the amount invested
    - *n* is the number of years
    - *r%* is the interest rate
  - This formula is **not** given in the formula booklet, however your GDC will work out annuities for you so you do not need to remember it

Exam Tip

- Be sure to write down the values that you put into the financial solver on your GDC, don't just write down the final answer as if it is incorrect you won't get any marks if there is no working shown!
- Try to remember the difference between amortization and annuities:
  - with **amortization** you are **paying** money out
  - with **annuities** you are **receiving** money

YOUR NOTES

Worked Example

Janni invests 2 million DKK (Danish krone) into an annuity for her retirement. The annuity returns 3% compounded annually. Find the monthly payments Janni will receive if she wants the annuity to last for 25 years.

Use the finance / TVM solver on your GDC:

N	I%	PV	PMT	FV	P/Y	C/Y	PMT@
300	3	-2 000 000		0	12	1	START
25 years × 12		negative because this was invested	GDC will fill this in for you.		paid monthly	compounding annualy	paid at the start of each month

PMT = 9418.95

$\boxed{\text{Janni receives DKK 9419 each month}}$