3.4 Voronoi Diagrams

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.4 Voronoi Diagrams
Difficulty	Medium

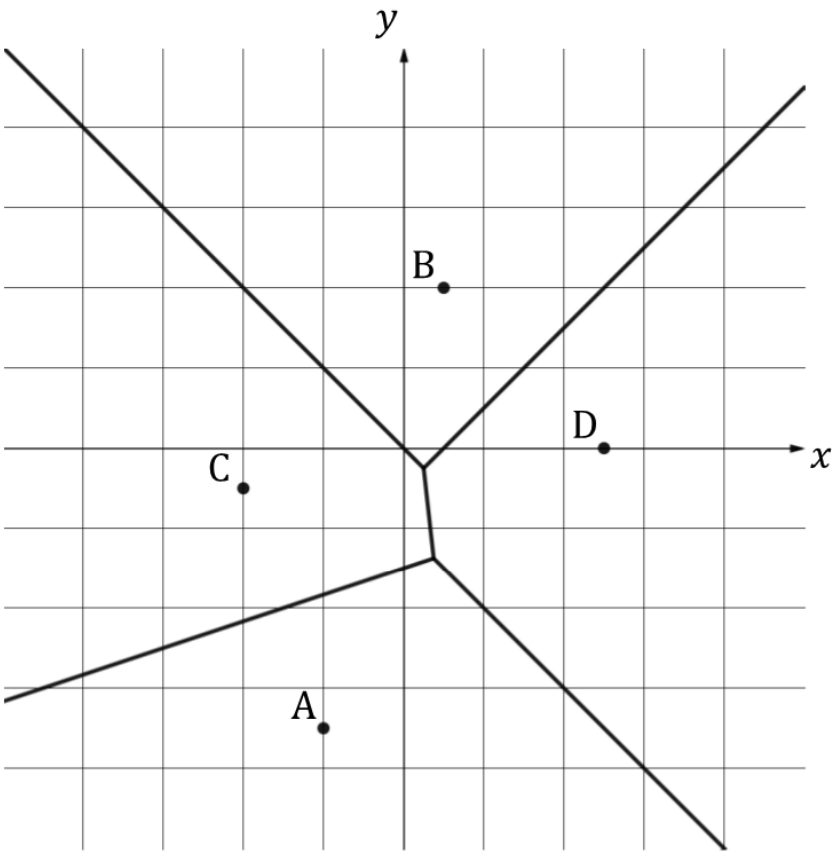
Time allowed: 60

Score: /44

Percentage: /100

Question 1a

Points A$(-2,-7)$, B$(1,4)$, C$(-4,-1)$ and D$(5,0)$ on the Voronoi diagram below represent the locations of four cinemas in Berlin, Germany.
Horizontal scale: 1 unit represents 1 km. Vertical scale: 1 unit represents 1 km.



Amy wants to go to a cinema and her house is located at $(-1, 1)$.

(a) (i) Determine which cinemas Amy's house is closest to.

(ii) Calculate the distance from Amy's house to these two cinemas.

[4 marks]

Question 1b

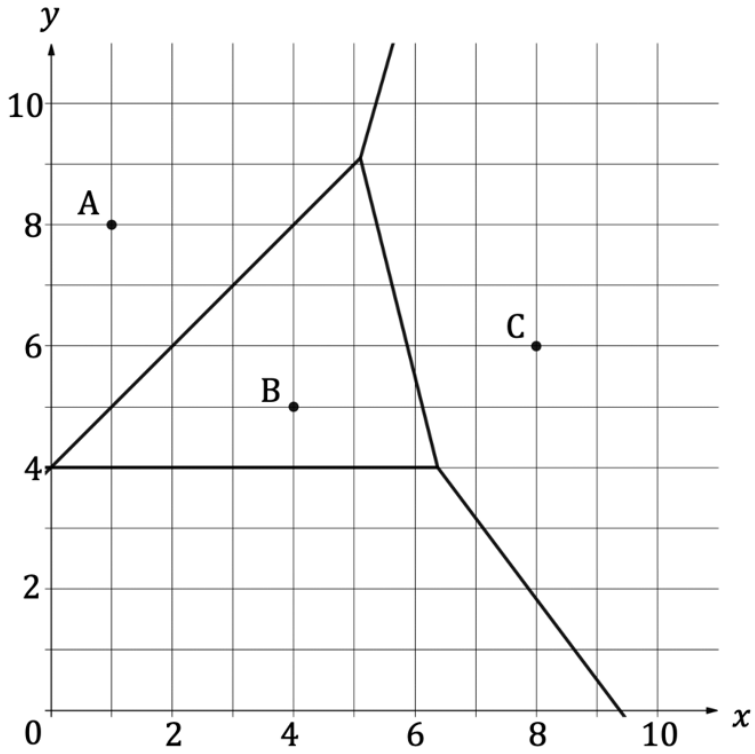
Kayla's apartment is an equal distance from cinemas A and C.

(b) Find the shortest possible distance Kayla's apartment could be from cinemas A and C.

[2 marks]

Question 2a

Sites A, B and C on the Voronoi diagram below represent the location of solar panels.
Horizontal scale: 1 unit represents 10 km. Vertical scale: 1 unit represents 10 km.



(a) A fourth site, D, is missing from the diagram. Write down the coordinates for site D.

[1 mark]

Question 2b

(b) The perpendicular bisectors surrounding B intersect at points $(0, 4)$, $(6.375, 4)$ and $(5.1, 9.1)$. Calculate the area of cell B.

[4 marks]

Question 2c

(c) A point E is located at (5, 10). Find the distance from E to the nearest solar panel.

[2 marks]

Question 2d

The daily average number of watts produced by each solar panel is given in the table below.

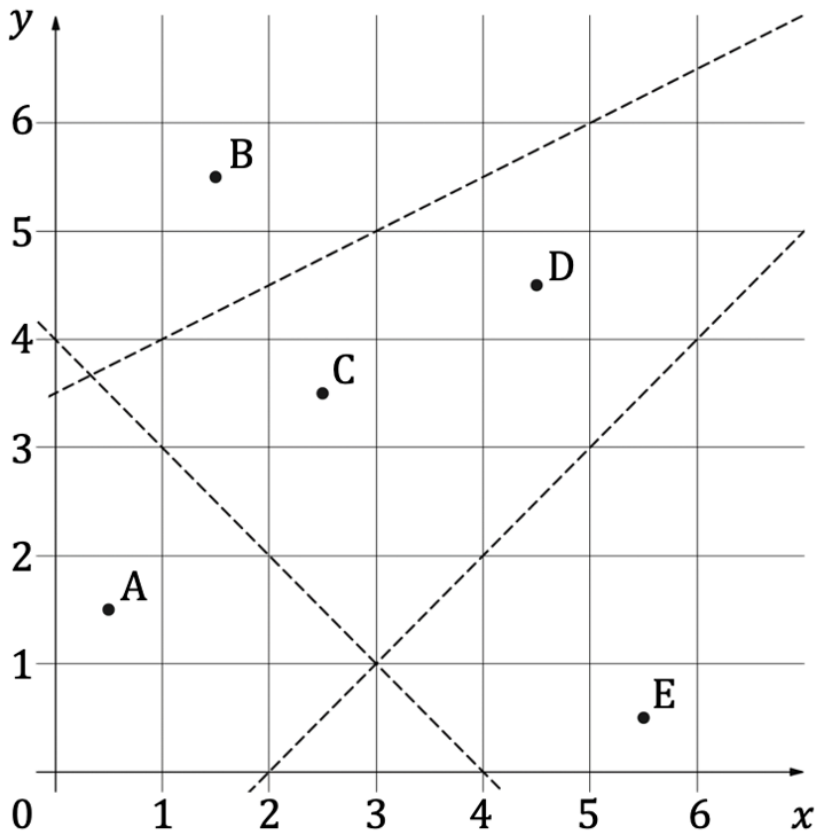
Site	A	B	C	D
Watts per day	276	293	312	322

(d) Estimate the watts produced at F(9, 1).

[1 mark]

Question 3a

Points A(0.5, 1.5), B(1.5, 5.5), C(2.5, 3.5), D(4.5, 4.5) and E(5.5, 0.5) represent mechanics in a city. The mechanics are shown below on an incomplete Voronoi diagram.
Horizontal scale: 1 unit represents 1 km. Vertical scale: 1 unit represents 1 km.



(a) Calculate the gradient of the line segment CD.

[2 marks]

Question 3b

(b) Find the equation of the line which would complete the Voronoi cell containing site C. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

[3 marks]

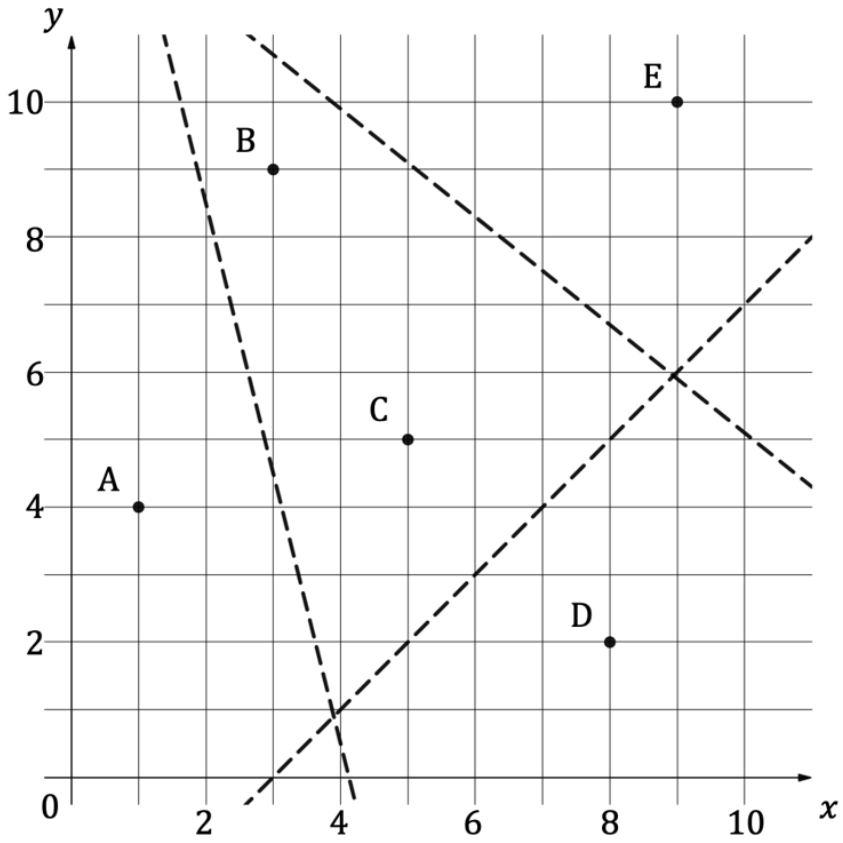
Question 3c

(c) In the context of the question, explain the significance of the Voronoi cell containing site C.

[1 mark]

Question 4a

Rangers use aerial imagery to help locate wolfpacks in Yellowstone National Park. This week the plane is not available so they must use last week's image which shows the last known locations of five wolfpacks at points $A(1, 4)$, $B(3, 9)$, $C(5, 5)$, $D(8, 2)$ and $E(9, 10)$ as illustrated on the following coordinate axes.
Horizontal scale: 1 unit represents 10 km. Vertical scale: 1 unit represents 10 km.



Wolfpacks stick to very rigid territories keeping their distance from other packs to avoid confrontation. Using the ariel image, rangers draw three straight lines to form an incomplete Voronoi diagram.

(a) Calculate the gradient of the line segment BC.

[2 marks]

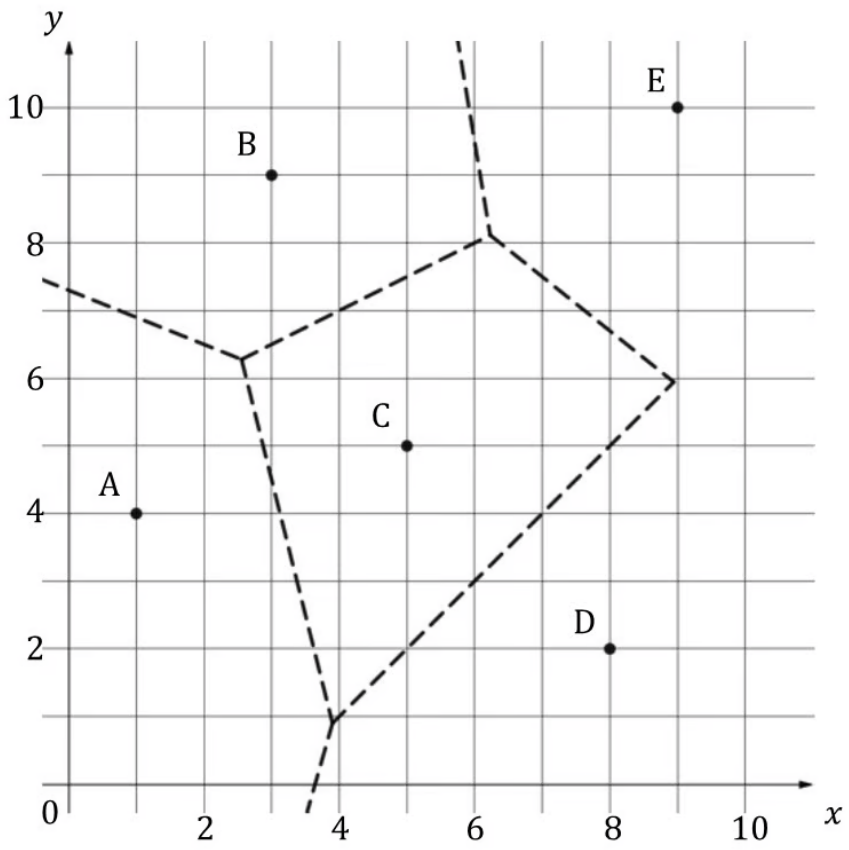
Question 4b

(b) Find the equation of the line which would complete the Voronoi cell containing site C. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

[3 marks]

Question 4c

There is one straight line missing from the Voronoi diagram below.



(c) Find the equation of the missing line. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

[3 marks]

Question 4d

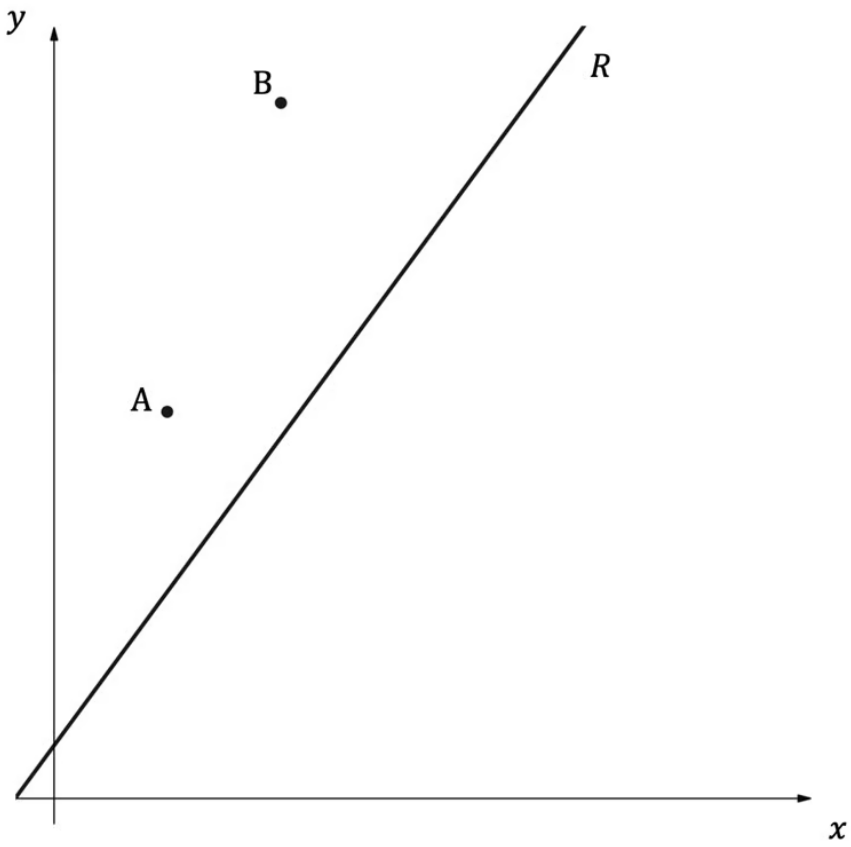
A family of rabbits are sighted at point F(10, 5).

(d) Write down the wolfpack territory the family of rabbits are in.

[1 mark]

Question 5a

Two schools are represented by points $A(3, 15)$ and $B(6, 27)$ on the graph below. A road, represented by the line R with equation $y - 2x - 2 = 0$, passes near the schools. An architect is asked to determine the location of a new bus stop on the road such that it is the same distance from the two schools.



(a) Find the equation of the perpendicular bisector of [AB]. Give your equation in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

[5 marks]

Question 5b

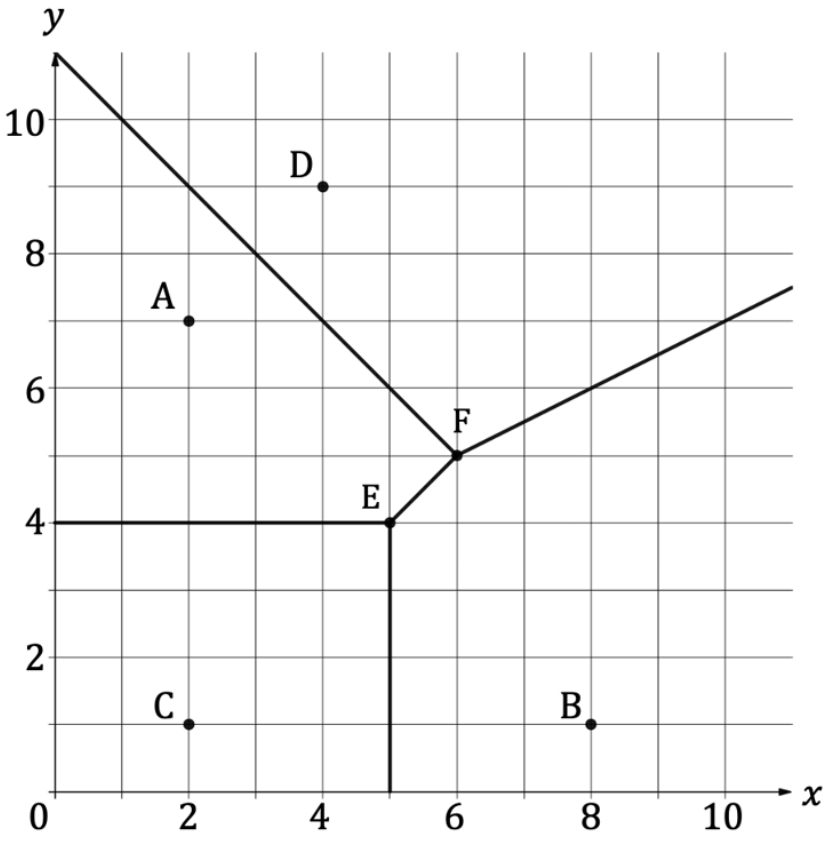
(b) Determine the coordinates of the point on R where the bus stop should be located.

[2 marks]

Question 6a

The sites A, B, C and D in the Voronoi diagram below represent the locations of active volcanos in an Indonesian region and points E and F are intersections.

Horizontal scale: 1 unit represent 10 km. Vertical scale: 1 unit represent 10 km.



There is a population centre at P(7, 5).

(a) Calculate the distance from the population centre to the closest volcano.

[2 marks]

Question 6b

(b) Determine the optimal position for a central shopping centre in the region, such that it is as far away from a volcano as possible.

[3 marks]

Question 6c

(c) A geologist says a safe distance for a shopping centre from a volcano is 45 km. Determine whether the position of the shopping centre will be safe.

[3 marks]