4.4 Probability Distributions

Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.4 Probability Distributions
Difficulty	Medium

Time allowed: 70

Score: /51

Percentage: /100

Question 1a

Three biased coins are tossed.

(a) Write down all the possible outcomes when the three coins are tossed.

[1 mark]

Question 1b

For each coin the probability of getting heads is $\frac{2}{3}$. A random variable, X, is defined as the number of heads when the three coins are tossed.

(b) Complete the following probability distribution table for X:

x	0	1	2	3
$\mathbf{P}(X = x)$				

[3 marks]

Question 1c

(c) Hence, by inserting the relevant probabilities, represent the probability distribution for X as a piecewise function in the form

$$\mathrm{P}(X = x) = f(x) = \begin{cases} & x = 0 \\ & x = 1 \\ & x = 2 \\ & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

[2 marks]

Question 1d

(d) Represent the probability distribution for X as a bar chart.

[2 marks]

Question 2

The random variable X has the probability function

$$P(X = x) = \begin{cases} \frac{x}{3k} & x = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

Show that $k = 5$.

[2 marks]

Question 3a

The random variable X has the probability function

$$P(X = x) = \begin{cases} kx & x = 1, 3, 5, 7 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k.

[2 marks]

Question 3b

(b) Find $P(X > 3)$.

[2 marks]

Question 3c

(c) State, with a reason, whether or not X is a discrete random variable.

[1 mark]

Question 4a

The random variable X has the probability function

$$\mathrm{P}(X = x) = \begin{cases} 0.23 & x = -1, 4 \\ k & x = 0, 2 \\ 0.13 & x = 1, 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k.

[2 marks]

Question 4b

(b) Construct a table giving the probability distribution of X.

[2 marks]

Question 4c

(c) Find $\mathrm{P}(0 \leq X < 3)$.

[1 mark]

Question 5

A discrete random variable X has the probability distribution shown in the following table:

$\boldsymbol{x}$	0	1	2	3	4
$\mathbf{P}(\boldsymbol{X} = \boldsymbol{x})$	$\frac{5}{24}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{8}$

Find:

(i) $\mathrm{P}(X < 4)$

(ii) $\mathrm{P}(X > 1)$

(iii) $\mathrm{P}(2 < X \leq 4)$

(iv) $\mathrm{P}(0 < X < 4)$

[6 marks]

Question 6a

Leonardo has constructed a biased spinner with six sectors labelled 0, 1, 1, 2, 3 and 5. The probability of the spinner landing on each of the six sectors is shown in the following table:

number on sector	0	1	1	2	3	5
probability	$\frac{6}{20}$	p	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

(a) Find the value of p.

[1 mark]

Question 6b

Leonardo is playing a game with his biased spinner. The score for the game, X, is the number which the spinner lands on after being spun.

(b) Leonardo plays the game twice and adds the two scores together. Find the probability that Leonardo has a *total* score of 5.

[3 marks]

Question 6c

(c) Complete the following cumulative probability function table for X:

Score x	0	1	2	3	5
$\mathbf{P}(X \leq x)$	$\frac{6}{20}$				1

[2 marks]

Question 6d

(d) Find the probability that X is

(i) no more than 1

(ii) at least 3.

[2 marks]

Question 7a

A discrete random variable X has the following probability distribution:

x	-3	-1	0	1	3
$\mathbf{P}(X = x)$	0.11	k^2	0.1	$2k$	0.1

where k is a positive constant.

(a) Show that $k^2 + 2k - 0.69 = 0$.

[2 marks]

Question 7b

(b) Hence find the value of k.

[1 mark]

Question 7c

(c) Find $\mathrm{E}(X)$.

[3 marks]

Question 8

A spinner is spun on a circle that is divided up into five sections, A, B, C, D and E.

The probability of the spinner landing on each section is given by the following table:

Region	A	B	C	D	E
Probability	0.55	0.15	0.15	0.1	0.05

A person who rotates the spinner scores points depending on which section the spinner lands on. These points are shown below.

Region	A	B	C	D	E
Points	-5	2	3	10	k

Given that the game is fair, find the value of k.

[4 marks]

Question 9a

A discrete random variable X has the following probability distribution:

x	0	1	2	3	4
$\mathbf{P}(X = x)$	0.1	0.05	a	b	0.1

The value of $\mathrm{E}(X) = 2.3$.

(a) Show that a and b must satisfy the following two simultaneous equations:

$$a + b = 0.75$$
$$2a + 3b = 1.85$$

[3 marks]

Question 9b

(b) Hence find the value of a and the value of b.

[2 marks]

Question 9c

(c) Find $\mathrm{P}(1 \leq X < 4)$.

[2 marks]