1.6 Further Complex Numbers

Question Paper

Course	DP IB Maths
Section	1. Number & Algebra
Topic	1.6 Further Complex Numbers
Difficulty	Medium

Time allowed: 80

Score: /62

Percentage: /100

Question 1a

Consider $w = \frac{z_1}{z_2}$, where $z_1 = 2 + 2\sqrt{3}\,\mathrm{i}$ and $z_2 = 2 + 2\mathrm{i}$.

(a)
Express w in the form $w = a + b\mathrm{i}$.

[2 marks]

Question 1b

(b)
Write the complex numbers z_1 and z_2 in the form $re^{\mathrm{i}\theta},\ r \geq 0,\ -\pi < \theta < \pi$.

[2 marks]

Question 1c

(c)
Express w in the form $re^{i\theta},\ r \geq 0,\ -\pi < \theta < \pi$.

[2 marks]

Question 2a

Consider the equation $z^4 - 1 = 15$, where $z \in \mathbb{C}$.

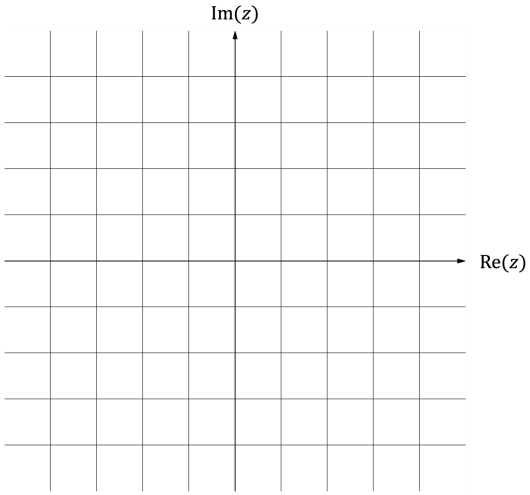
(a)
Find the four distinct roots of the equation, giving your answers in the form $a + b\mathrm{i}$, where $a, b \in \mathbb{R}$.

[4 marks]

Question 2b

(b)

Represent the roots found in part (a) on the Argand diagram below.



[2 marks]

Question 2c

(c)

Find the area of the polygon whose vertices are represented by the four roots on the Argand diagram.

[2 marks]

Question 3a

Let $z_1 = 6cis(\frac{\pi}{6})$ and $z_2 = 3\sqrt{2}e^{i\left(\frac{\pi}{4}\right)}$.

a)
Giving your answers in the form $rcis\theta$, find

(i)
$z_1 z_2$

(ii)
$\frac{z_1}{z_2}$.

[4 marks]

Question 3b

b)
Write z_1 and z_2 in the form $a + bi$.

[2 marks]

Question 3c

c)
Find $z_1 + z_2$, giving your answer in the form $a + b\mathrm{i}$.

[2 marks]

Question 3d

It is given that z_1^* and z_2^* are the complex conjugates of z_1 and z_2 respectively.

d)
Find $z_1^* + z_2^*$, giving your answer in the form $a + b\mathrm{i}$.

[2 marks]

Question 4a

Let $z_1 = 2cis(\frac{\pi}{3})$ and $z_2 = 2 + 2\mathrm{i}$.

a)
Express

(i)
z_1 in the form $a + b\mathrm{i}$

(ii)
z_2 in the form $r\,cis\theta$

[2 marks]

Question 4b

b)
Find $w_1 = z_1 + z_2$, giving your answer in the form $a + b\mathrm{i}$.

[2 marks]

Question 4c

c)
Find $w_2 = z_1 z_2$, giving your answer in the form $r\,cis\theta$.

[3 marks]

Question 4d

d)
Sketch w_1 and w_2 on a single Argand diagram.

[2 marks]

Question 5a

It is given that that $z_1 = 2e^{\mathrm{i}\left(\frac{\pi}{3}\right)}$ and $z_2 = 3\,cis\left(\frac{n\pi}{12}\right)$, $n \in \mathbb{Z}^+$.

a)
Find the value of $z_1 z_2$ for $n = 3$.

[3 marks]

Question 5b

b)

Find the least value of n such that $z_1 z_2 \in \mathbb{R}^+$.

[3 marks]

Question 6a

Consider the complex number $w = \frac{z_1}{z_2}$ where $z_1 = 3 - \sqrt{3}\,\mathrm{i}$ and $z_2 = 2cis\left(\frac{2\pi}{3}\right)$.

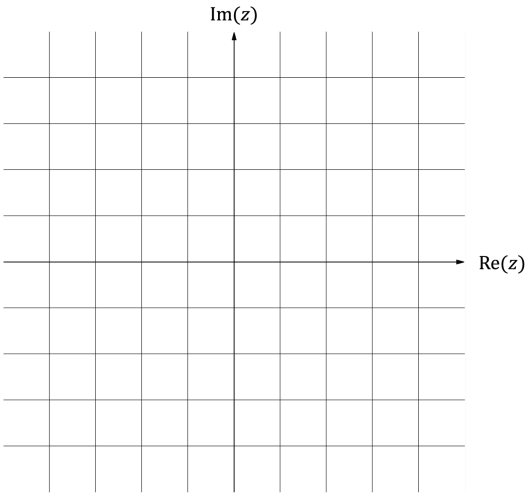
a)

Express w in the form $r\,cis\theta$.

[5 marks]

Question 6b

b)
Sketch z_1, z_2 and w on the Argand diagram below.



[3 marks]

Question 6c

c)
Find the smallest positive integer value of n such that w^n is a real number.

[2 marks]

Question 7a

Consider the complex numbers $w = 3\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$ and $z = 3 - \sqrt{3}i$.

(a)
Write w and z in the form $r\,\text{cis}\,\theta$, where $r > 0$ and $-\pi < \theta \leq \pi$.

[4 marks]

Question 7b

(b)
Find the modulus and argument of zw.

[2 marks]

Question 7c

(c)
Write down the value of zw.

[2 marks]

Question 8

Write $5\cos(2t+3)+4\cos(2t+5)$ in the form $A\cos(2t+B)$ where $A>0,\ -\pi<B<\pi$.

[5 marks]