5.4 Further Integration

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.4 Further Integration
Difficulty	Medium

Time allowed: 110

Score: /90

Percentage: /100

Question 1a

(a) Find the indefinite integral for

$$\int \sin x \ \mathrm{d}x$$

[1 mark]

Question 1b

(b) Show that the exact value of the definite integral

$$\int_1^4 \frac{1}{x} \ \mathrm{d}x$$

is $2 \ln 2$.

[3 marks]

Question 1c

(c) Find the indefinite integral for

$$\int 7e^{7x} \ \mathrm{d}x$$

[2 marks]

Question 2a

(a) Integrate

$$\int \cos 2x \ \mathrm{d}x$$

[2 marks]

Question 2b

(b) Show that

$$\int (3x-1)^3 \ \mathrm{d}x = \frac{1}{12}(3x-1)^4 + c$$

where c is a constant of integration.

[3 marks]

Question 2c

(c) Find an expression for y given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = e^{5x}$$

and that $y = 1$ when $x = 0$.

[3 marks]

Question 3a

(a) Find the indefinite integral for

$$\int \left(\sqrt{x} + \frac{3}{\sqrt{x}} \right) \, \mathrm{d}x$$

[3 marks]

Question 3b

(b) Find the indefinite integral for

$$\int \frac{x^{\frac{2}{3}} + x^{\frac{11}{6}}}{x^2} \, dx$$

[3 marks]

Question 4a

(a) Given that $f(x) = 2x^3 + 4x,$ find $f'(x)$.

[2 marks]

Question 4b

(b) Hence, or otherwise, find

$$\int \frac{3x^2 + 2}{2x^3 + 4x} \, dx$$

[4 marks]

Question 5a

Consider the function $f(x) = \ln(2x^2 + 1)$.

(a) Find $f'(x)$.

[3 marks]

Question 5b

(b) Hence, find

$$\int \frac{x}{2x^2 + 1} \, dx$$

[3 marks]

Question 6

Let $f'(x) = x^2 \cos(x^3 + 1)$.

Find $f(x)$ given that $f(-1) = 1$.

[5 marks]

Question 7a

(a) Show that

$$\frac{\tan x}{\sin x \cos x} = \frac{1}{\cos^2 x}$$
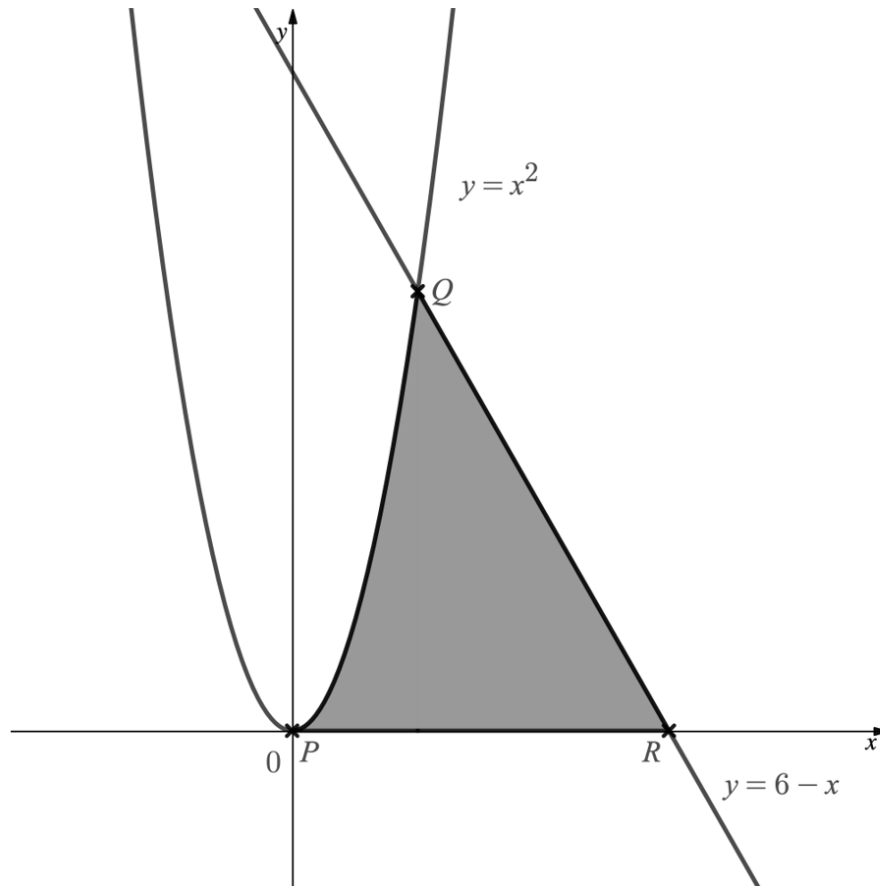
[2 marks]

Question 7b

(b) Hence find

$$\int \frac{3 \tan x}{5 \sin x \cos x} \, \mathrm{d}x$$

[3 marks]

Question 8a

The diagram below shows the graphs of the line $y = 6 - x$ and the curve $y = x^2$.

Point P is the point of intersection of the curve $y = x^2$ with the x-axis. Point Q is the point of intersection of the curve $y = x^2$ with the line $y = 6 - x$ for which $x > 0$. Point R is the point of intersection of the line $y = 6 - x$ with the x-axis.

(a) Write down the x-coordinates of points P, Q and R.
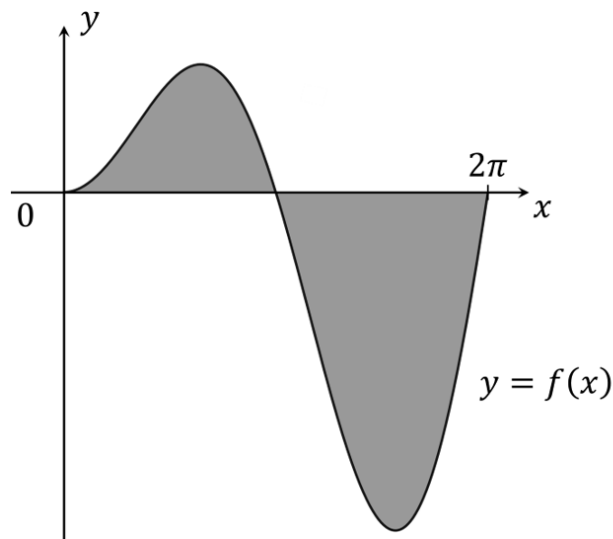
[3 marks]

Question 8b

(b) Calculate the area of the shaded region.

[2 marks]

Question 9a

The diagram below shows the graph of the function f which is defined by

$$f(x) = x \sin x, \quad 0 \le x \le 2\pi$$

The shaded region in the diagram is the region enclosed by the x-axis and the graph of $y = f(x)$.

(a) Find the area of the part of the shaded region that lies above the x-axis.

[4 marks]

Question 9b

(b) Find the area of the entire shaded region.

[3 marks]

Question 10a

The diagram below depicts the design for a new company logo. The logo is formed by a circle centred on the origin, which is divided into two regions by the curve $y = f(x)$ where f is the function defined by $f(x) = \cos\dfrac{3x}{2}$, $-\pi \le x \le \pi$. The points where the circle and the curve intersect lie on the x-axis, as shown.
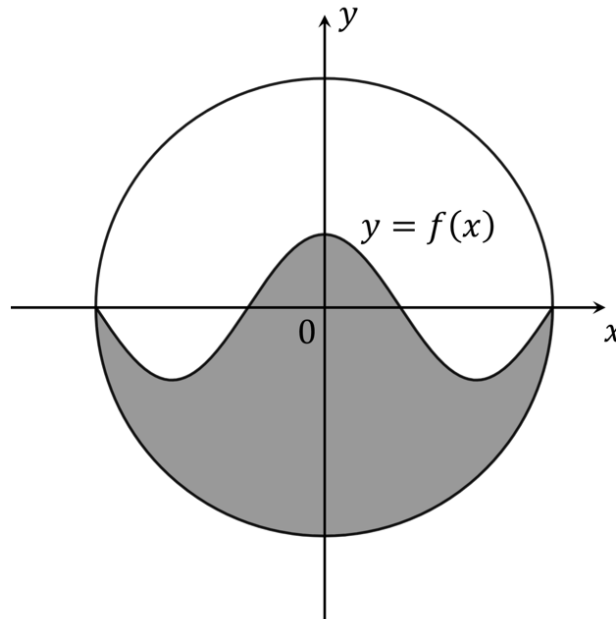
The shaded region in the diagram is the region inside that circle that lies below the curve $y = f(x)$.

(a) (i) Write down the radius of the circle that forms the outer border of the logo.

(ii) Hence determine the exact area of the shaded region.
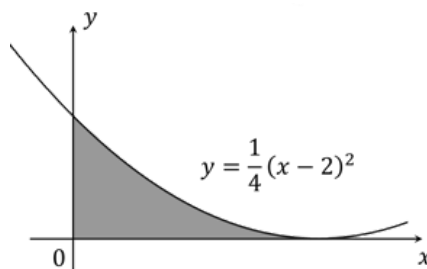
[6 marks]

Question 10b

(b) Find the percentage of the circular logo that is shaded.

[2 marks]

Question 11a

The following diagram shows a part of the graph of the curve $y = \frac{1}{4}(x-2)^2$. The shaded region is the region enclosed by the graph and the positive x- and y-axes.

(a)

(i)

Find the coordinates of the points where the graph intersects the coordinate axes.

(ii)

For the part of the curve that forms the boundary of the shaded region, show that $x = 2 - 2\sqrt{y}$.

[3 marks]

Question 11b

(b)
Find the area of the shaded region

(i)
by calculating it as an area between the curve and the x-axis.

(ii)
by calculating it as an area between the curve and the y-axis.

[6 marks]

Question 11c

(c)
Find the volume of the solid formed when the shaded region is rotated 2π radians about the x-axis.
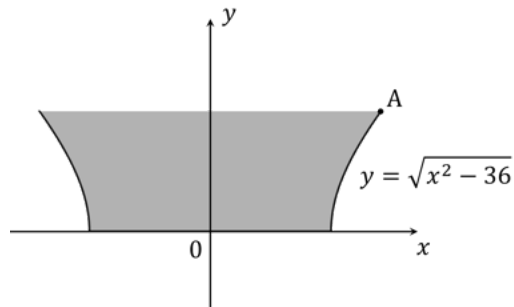
[5 marks]

Question 11d

(d)

Find the volume of the solid formed when the shaded region is rotated 2π radians about the y-axis.

[5 marks]

Question 12a

The diagram below shows the cross-section of a bowl that a company is planning to begin producing.

As indicated on the diagram, one of the sides of the bowl in the cross-section may be described by the curve $y = \sqrt{x^2 - 36}$, where units for x and y are centimetres. The cross-section is entirely symmetrical about the y-axis. The flat circular bottom of the bowl has a diameter of 12 cm, and the vertical depth of the bowl is 6 cm. For purposes of answering this question, the thickness of the bottom and sides of the bowl may be regarded as negligible.

(a)
Find the exact coordinates of the point marked A on the diagram.

[3 marks]

Question 12b

(b)
Show that the capacity of the bowl in cm^3 is given by

$$\pi \int_0^b (y^2 + 36)\,\mathrm{d}y$$

where b is a constant to be determined.

[4 marks]

Question 12c

(c)

Hence find the capacity of the bowl.

[2 marks]