3.5 Trigonometric Functions & Graphs

Question Paper

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.5 Trigonometric Functions & Graphs
Difficulty	Very Hard

Time allowed: 80

Score: /65

Percentage: /100

Question 1

Let $f(x) = \sin\big(2(x+5)\big) - 4$

Sketch the graph of $2f(x-3)$ for the interval $0 \leq x \leq 2\pi$. Label the coordinates of the local maxima and minima. The coordinates should be given as exact values.

[6 marks]

Question 2

(i) Sketch the graph of $y = 4\sin(\theta + 45) - \frac{4}{\sqrt{2}}$ in the interval $-180° \leq \theta \leq 360°$.

(ii) Write down all the values for θ, where $4\sin(\theta + 45) - \frac{4}{\sqrt{2}} = 0$ in the given interval.

[7 marks]

Question 3

A particle is travelling horizontally whilst moving in and out of a body of water at a constant speed. The particle reaches a maximum height of 1.3 m above the water level and and a depth of 2.6 m.

The particle starts at a depth of $\frac{13}{8}$ m and takes $\frac{4}{3}\pi$ seconds to move up through the water, reach the maximum height, dive to the minimum depth and return to its starting depth.

Write down a model for the height, h m, of the particle, relative to water level, at a time t seconds, in the form $h = A\sin(Bt + C) + D$, where A, B, C and D are constants to be found.

[7 marks]

Question 4a

A section of a model railway track has a series of rises and falls. The vertical displacement of a train carriage moving along the track, h in m, relative to the horizontal floor, can be modelled by

$$h(t) = \sin(6t^3 - 2t^2 - 10t) + 1.5, \quad \text{for } 0 \leq t \leq \frac{\pi}{3}$$

where t is the time in minutes.

(a) Find the average vertical speed of the train carriage when it experiences the maximum change in height within the given section of track.

[3 marks]

Question 4b

Vertical metal supports are required to ensure that the track is stable. A support is required at either end of the track, as well as at each local maximum and minimum.

(b) Given that there is 7.9 m of metal available to create the supports, show that this is not sufficient to place the supports in the locations required.

[3 marks]

Question 4c

A support attached to the side of the track is also required. The support must attach to the model at four separate points, including the end point of the section of the track and can be modelled by the function $y = x + c$, where c is a constant.

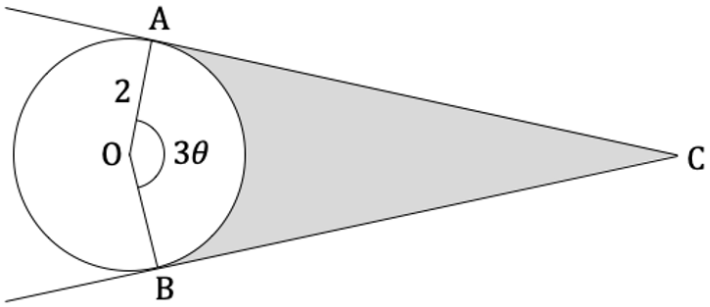
(c) Find the length of the side support from the first point of connection with the track to the fourth.

[4 marks]

Question 5

The diagram below shows a circle with centre O and radius 2 cm. Points A and B lie on the circumference of the circle and angle $A\widehat{O}B = 3\theta$, where $0 < \theta < \frac{\pi}{3}$.

The tangents to the circle at points A and B intersect at point C.



Find the value of θ when the shaded area is equal to the area of sector OAB.

[7 marks]

Question 6a

A particle, A, starts at a fixed point, O, before being set into motion. The vertical displacement of the particle, h_A cm, from point O can be modelled by the equation

$$h_A(x) = x^2 \cos 5x \sin x, \quad \text{for } 0 \leq x \leq 5$$

where x is the horizontal displacement, in cm, of the particle from O.

(a) Find the straight line distance of the particle from O at end of the motion.

[3 marks]

Question 6b

A second particle, B, starts moving at the same time as particle A. The motion of particle B can be described by the function

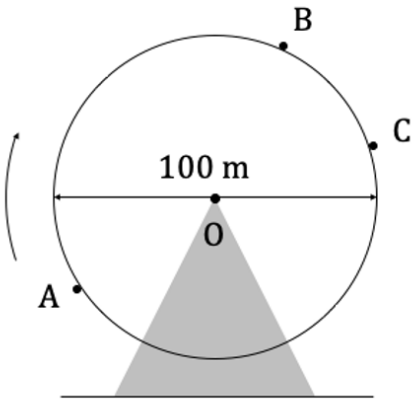
$$h_B(x) = 2x + a, \quad \text{for } 0 \leq x \leq 5, \quad a \in \mathbb{R}$$

(b) Given that the particles stop moving at the point of collision, find the value of a.

[4 marks]

Question 7a

A Ferris wheel with centre O and diameter 100 metres, comprises 32 individual compartments and rotates clockwise at a constant speed of 0.96 kilometres per hour.



Passengers board the Ferris wheel at point A. The height, h metres, of a compartment above the ground after it passes through point A is modelled by the function

$$h(t) = 58 + 40\cos\left(\frac{25}{72}(t+8)\right) - 30\sin\left(\frac{25}{72}(t+8)\right), \text{for } t \geq 0$$

where t is the time elapsed in minutes.

(a) Find the height of point A above the ground.

[2 marks]

Question 7b

(b) (i) Calculate the number of minutes it takes for the Ferris wheel to complete one revolution.

(ii) Hence find the number of revolutions the Ferris wheel makes in one hour.

[3 marks]

Question 7c

Points B and C indicate the edges of the region in which a person gains the best view of the city during the rotation of the Ferris wheel. B is reached 10 mins after boarding the wheel. Point C is located at a vertical distance of 26.2 m below point B.

(c) Find

(i) the length of time for which the person experiences the best view of the city.

(ii) Find the angle $B\hat{O}C$.

[4 marks]

Question 8a

A large clock face is mounted on a tower, with the centre of the clock face at a height of 19 m above ground level and the tip of the minute hand reaching the circumference of the clock face.

The clock is started at 12 pm and the tip of the minute hand travels a total distance of 5.8 m from its initial position in 35 minutes.

(a) Find a model for the height of the tip of the minute hand, h m, above ground level, in the form $h(x) = A\sin(Bx) + C$, where x is the angle measured clockwise between the number 12 and the minute hand and A, B and C are constants to be found.

[4 marks]

Question 8b

(b) Given that the minute hand has travelled a total distance of 31.8 m before it is stopped, find

(i) the final angle between the hour hand and the minute hand,

(ii) the final height of the tip of the minute hand above the ground.

[8 marks]