2.7 Polynomial Functions

Question Paper

Course	DP IB Maths
Section	2. Functions
Topic	2.7 Polynomial Functions
Difficulty	Hard

Time allowed: **80**

Score: **/66**

Percentage: **/100**

Question 1a

Consider the function $f(x) = 3x^3 + px^2 + 22x + q$, where p and q are constants. It is given that $(x^2 - x + 6)$ is a factor of $f(x)$.

(a)
Find the values of p and q.

[4 marks]

Question 1b

(b)
Find the roots of $f(x)$.

[3 marks]

Question 2a

$\frac{3}{4}$ is a zero of the function $f(x) = 4x^3 - 19x^2 + kx - 12$, where k is a constant.

(a)
As well as finding the value of k, find all the solutions to the equation $f(x) = 0$.

[5 marks]

Question 2b

(b)
Sketch the graph of $y = f(x)$.

[3 marks]

Question 2c

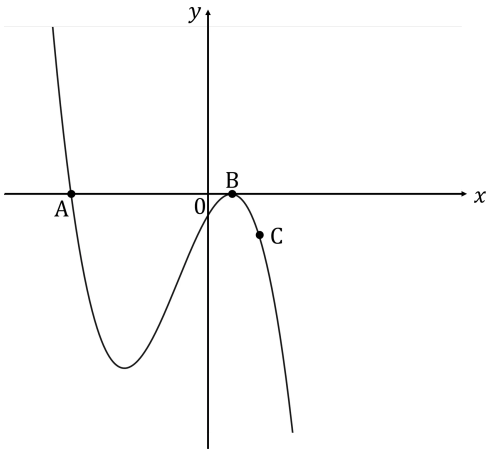
The point $\left(\frac{7}{6}, \frac{125}{108}\right)$ is a turning point on the graph $y = f(x)$.

(c)
Given that $f(x) = p$ has three distinct real solutions, where p is a real constant, find the set of possible values of p.

[2 marks]

Question 3a

The graph of $y = f(x)$ is shown below, where $f(x)$ is a polynomial function. The graph passes through the points $\mathrm{A}(-3,0)$, $\mathrm{B}\left(\frac{1}{2},0\right)$ and $\mathrm{C}(1,-12)$.



(a)
Given that the degree of f is as small as possible, find an equation for $f(x)$.

[3 marks]

Question 3b

The graph is translated by the vector $\begin{pmatrix} k \\ 0 \end{pmatrix}$ to form the graph $y = g(x)$, where k is a constant and $g(x)$ is a polynomial.

(b)
Given that is a factor of $g(x)$, find the possible values of k.

[3 marks]

Question 4

Given that $(x+4)$ is a factor of the function $f(x) = px^3 + (5p+1)x^2 + 5qx - 2q - 2$ and that the remainder when $f(x)$ is divided by $(x+1)$ is -12, find the values of the constants p and q.

[6 marks]

Question 5

Show that $3x^3 + 16x^2 - 22x$ can be written in the form $(3x+1)(ax^2 + bx + c) + d$, where a, b, c and d are constants to be found.

[4 marks]

Question 6

For the function $f(x) = (3x - 1)(x^2 + x - 1)(ax^2 + bx + c)$, the sum of the roots is $\frac{1}{3}$ and the product of the roots is $-\frac{31}{36}$.

Find all five roots of $f(x)$.

[6 marks]

Question 7

α and β are non-real solutions of the equation $2x^2 - (2k - 3)x + 2k = 0$.

Given that $\alpha^2 + \beta^2 = \frac{9}{4}$ and $k \neq 0$, find the value of k.

[6 marks]

Question 8

The function $f(x) = x^2 - mx + 3m - 4$ has two integer solutions, one of which is double the other one.

Find the value of m.

[6 marks]

Question 9a

Consider the function $f(x) = px^6 + qx^4 + rx^2 + 1$, where p, q and r are real constants.

(a)
Show that if α is a zero of $f(x)$ then $-\alpha$ is also a zero.

[2 marks]

Question 9b

(b)

Given that $\sqrt{5}$ and $-7-6\mathrm{i}$ are roots of the equation $f(x)=0$, find the value of p.

[4 marks]

Question 10a

Let f be a polynomial defined by $f(x)=8x^3-24x^2-72x+385.$

(a)

Use algebra to show that:

(i)

$(2x+7)$ is a factor of $f(x)$,

(ii)

$f(x)=0$ has exactly one real root.

[6 marks]

Question 10b

Consider the function g defined by $g(x) = f(x) + k$, where k is a real constant.

(b)
Given that the equation $g(x) = 0$ has exactly three real roots, find the set of possible values of k.

[3 marks]